Bargaining and Delay in Thin Markets

Francesc Dilmé*

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Abstract

Many markets are thin: they have few traders at any given moment in time. For example, traders in job and housing markets are typically constrained, both by their geographical location and their individual characteristics. In these markets, entry and exit make trading opportunities stochastically change over time, affecting the bargaining position of traders. This paper presents a model of a thin market with endogenous arrival of traders and characterizes the timing and prices of the transactions. Trade delay and price dispersion are found to persist even when buyers and sellers are homogeneous and bargaining frictions are small. Our results underscore that properly incorporating the submarket structure into the study of decentralized markets is necessary in order to correctly assess some properties of their outcomes.

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1 Introduction

Many markets are thin: at any given moment in time, they have few active traders. For example, there is increasing evidence that workers are locked into job markets determined by their commuting areas and their specific skills.[1] Similarly, people looking for renting or buying housing units typically focus on geographically reduced areas and a narrow range of characteristics. In these markets, traders trade-off between trading soon (if possible) and waiting for the arrival of new trade opportunities. These markets evolve over time, and so the timing and prices of transactions depend not only on current trade opportunities, but also on the endogenous expectation about future ones.

Our goal is to characterize the trade outcome of a thin market (i.e., the timing and pricing of transactions) and compare it to those previously obtained for big decentralized markets. We first develop a general canonical model of a thin market. We then analyze how the bargaining protocol and endogenous arrival of traders determine the endogenous market evolution, equilibrium transaction prices, and trade delay. We obtain that thin markets exhibit trade delay and price dispersion even when traders are homogeneous and bargaining frictions are small. Also, we argue that prices are mostly determined by the stochastic dynamics of the market composition, and not the particular bargaining protocol used to set the prices. Our results show that incorporating a submarket structure in the study of decentralized markets has significant novel implications for the predicted trade outcome.

We construct a thin-market version of the Gale (1987) model with an endogenously evolving market composition. At any given moment in time, the market consists of a finite number of sellers who own one unit of a homogeneous indivisible good, and a finite number of homogeneous buyers with a unit demand. Once in the market, each trader keeps meeting traders from the other side of the market. In the base model, within each meeting, one of the traders is randomly chosen to make a take-it-or-leave-it offer. The other trader either accepts the offer, and both traders leave the market, or rejects it, and both traders continue. Importantly, the arrival of buyers and sellers, the matching rate, and the probability of making offers all depend on the market composition, that is, the numbers of buyers and sellers in the market. We later show that our results apply to more general arrival and exit processes and to more general bargaining protocols. We focus on Markov perfect equilibria using market composition as the state variable, where all buyers and all sellers play the same strategy.

Our first result establishes that trade delay may occur in equilibrium; that is, some equilibrium offers may be rejected. Delay occurs when traders on the short side of the market benefit from the arrival of new traders, while traders on the long side of the market benefit

[1] See, for example, Artuç, Chaudhuri, and McLaren (2010), Dix-Carneiro (2014), Artuç and McLaren (2015),Marinescu and Rathelot (2018), Manning and Petrongolo (2017), and Azar, Marinescu, Steinbaum, and Taska (2018).

from other traders' transactions. The option of waiting for arrival gives traders on the short side of the market a high bargaining power, and this induces traders on the long side of the market to let other traders trade first. We prove that, nonetheless, there is never a "market breakdown"; there is a strictly positive probability of trade in every meeting.

The second result establishes that price dispersion remains sizable when bargaining frictions are small. In the limit where traders in the market meet increasingly often, there is a unique transaction price for each state of the market, called *market price*. Such a price depends on the bargaining positions of the traders in the market. When the market is imbalanced, traders on the long side of the market Bertrand compete, and the market price is close to their endogenous continuation value from not trading. When, instead, the market is balanced, the relative positions of the traders depend on the bargaining protocol. Each trader gets a share of the trade surplus equal to the equilibrium share they would get in the Rubinstein bargaining game with stochastic outside options. We characterize the price process using a *risk-neutral measure* where the market evolves *as if* a trader on the long side of the market deviated to not trading. Market prices are shown to be proportional to the discounted future time the market exhibits excess demand under the risk-neutral measure, adjusted by the bargaining power of the sellers when the market is balanced.

We obtain some general properties of the market price process when bargaining frictions are small. First, we show that even when the market composition does not drift toward being balanced, the market price always tends, in expectation, toward the price of a balanced market. That is, on average, the market price increases when there is excess supply, and it decreases when there is excess demand. Second, trade delay disappears in the limit where the arrival rate of traders to the market increases. In this case, the distribution of transaction prices degenerates into a "competitive price" proportional to the ergodic probability of the market exhibiting excess demand. Finally, we provide conditions that ensure immediate trade, which take the form of bounds on the effect of individual transactions on the arrival rates of traders to the market. We show that, under these conditions, an increase in the interest rate results in an increase the spread of the equilibrium distribution of market prices.

Our results are robust to some extensions of our model. We first show that they hold for arrival processes following a general multidimensional Markov chain. We allow some components, such as the economic cycle of the economy and legislation changes, to be exogenously evolving, and some others, such as idiosyncratic demand and supply shocks and the visibility of the market to new potential traders, to evolve endogenously. We also consider the effect of changing the bargaining protocol to a general Nash bargaining one. In this case, when bargaining frictions are small, different bargaining protocols affect the price only through the relative bargaining power of sellers and buyers when the market is balanced. Thus, for example, the bargaining protocol does not affect prices significantly if the market is rarely balanced. We finally discuss the effect of endogenizing entry and exit of traders.

1.1 Literature review

Our paper contributes to the literature on thin markets with arrival of traders. The paper closest to ours, Taylor (1995), analyzes a centralized market where buyers and sellers arrive over time. In every period, traders on the short side of the market make price offers, and when the market is balanced one side the market is chosen at random. Coles and Muthoo (1998) consider a similar model where buyers and sellers arrive in pairs, and they allow for heterogeneity in both buyers and goods. Similarly, Said (2011) studies a dynamic market in which buyers compete in a sequence of private-value second-price auctions. These papers analyze price dynamics under different price mechanisms in centralized markets with either constant arrival rates or immediate replacement of traders. Our focus is, instead, on analyzing decentralized bargaining with an endogenous arrival process. We characterize how the arrival process and bargaining asymmetries affect price dynamics and trade delay. This allows us to compare our results with some of the literature on big markets (see below).

Our paper is also related to the extensive literature on bargaining and matching in large markets, reviewed in Osborne and Rubinstein (1990) and Gale (2000).[2] Models in this literature feature a continuum of traders and non-stochastic population dynamics, which is often assumed to be in a stationary state.[3] By contrast, we focus, on how the endogenous stochastic process determining the number of traders on each side of the market affects and is affected by the trade outcome, and how both of these are determined by the bargaining protocol.

Finally, there has been some recent interest in thin markets in a network of traders. For example, Condorelli, Galeotti, and Renou (2016), Talamàs (2016), and Elliott and Nava (forthcoming) look at bargaining in networks with immediate replacement of traders, and allow for differences in the valuation of the good by sellers and buyers. By contrast, we focus on how the dynamics of the population determines the price process and bargaining outcomes in an endogenously growing complete network.

The rest of the paper is organized as follows. Section 2 introduces our model, and Section 3 provides the equilibrium analysis. Section 4 analyzes the equilibrium outcome when the bargaining frictions are small. Section 5 discusses general market processes, bargaining protocols, and entry/exit of traders, and Section 6 concludes. The Appendix provides an example of a thin market exhibiting trade delay in equilibrium and the proofs of the results.

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[2] Important contributions are Rubinstein and Wolinsky (1985), Gale (1987), Burdett and Coles (1997), Shimer and Smith (2000), Atakan (2006), Satterthwaite and Shneyerov (2007), Manea (2011) and Lauermann (2012). Section 4.2 considers the limit where traders arrive increasingly frequently, which is interpreted as the market growing by replication, and we compare the results on convergence to the competitive outcome of this literature.

[3] An exception is Manea (2017a), who studies a non-stationary market with a continuum of agents. Our model analyzes how the fact that each agent's can change the evolution of a thin market affects the trade outcome and generates delay, and we characterize the resulting endogenous and stochastic price dynamics.

2 The model

In this section, we introduce a model similar to Rubinstein and Wolinsky (1985) and Gale (1987). There are two key distinguishing features. The first is that the market is assumed to be "thin"; that is, the number of traders in the market at any given moment in time is a non-negative integer (instead of a mass) that stochastically changes over time. The second is that we allow the arrival process, the matching process, and the bargaining process to be state-dependent. Thus, we do not impose any restrictions on the matching technology, and account for the case where entry decisions are endogenous (see Section 5.3 below).

State of the market. Time is continuous with an infinite horizon, $t \in \mathbb{R}_+$. There are an infinite number of potential buyers and sellers. At any given moment in time t, there are $B_t \in \{0, ..., \bar{B}\}$ buyers and $S_t \in \{0, ..., \bar{S}\}$ sellers in the market, for some large $\bar{B}, \bar{S} > 0$. The state (of the market) at time t is defined to be[4] (B_t, S_t).

Arrival process. Buyers arrive to the market at a Poisson rate $\gamma_b \equiv \gamma_b(B_t, S_t) \in \mathbb{R}_+$, and sellers arrive to the market at a Poisson rate $\gamma_s \equiv \gamma_s(B_t, S_t) \in \mathbb{R}_+$. The total rate at which the state exogenously changes is denoted by $\gamma \equiv \gamma_b + \gamma_s$. Section 5.1 considers a more general arrival process. Note that $\gamma_b(\bar{B}, \cdot) \equiv \gamma_s(\cdot, \bar{S}) \equiv 0$.

Bargaining. For ease of exposition, our base model uses a simplistic (yet canonical) bargaining protocol. As noted in Section 5.2, our results can be straightforwardly generalized to allow for general Nash bargaining.

If, at time t, there are buyers and sellers in the market (i.e., $B_t, S_t > 0$), meetings occur at a Poisson arrival rate $\lambda(B_t, S_t) > 0$. When a meeting occurs, nature selects one of the buyers and one of the sellers in the market uniformly randomly, and also chooses the trader who makes a price offer. The probability of the seller being chosen is[5] $\xi(B_t, S_t) \in (0, 1)$. The trading counter-party decides whether to accept the offer or not. If the offer is accepted, the good is transacted and the traders leave the market, whereas if the offer is rejected they continue in the market.

Payoffs. Both buyers and sellers discount the future at rate $r > 0$. If a buyer and a seller trade at time t at price p they obtain, respectively, $e^{-rt}(1 - p)$ and $e^{-rt}p$. If they never trade they both obtain 0. Both buyers and sellers are risk-neutral and expected-utility maximizers. Even though the formal expressions for the payoffs (and the conditions for the optimality of a strategy profile) are obtained using a standard recursive analysis, their length makes it convenient to defer them to Appendix B.1.

[4]The assumption that the number of traders in the market is bounded is technical and simplifies the intuitions and the proofs. Standard arguments—that is, taking sequences of models where $\bar{B}$ and $\bar{S}$ tend to $+\infty$—permit showing that our results apply when $\bar{B} = \bar{S} = \infty$ and the arrival rates to be bounded.

[5]The assumption that $\xi(B_t, S_t) \notin \{0, 1\}$ enables us to bypass the Diamond's paradox (see Remark 4.1).

Strategies. To simplify the model setting, we focus directly on Markov strategies using the state of the market as the state variable. Thus, the strategy of a trader (buyer or seller) maps each state (B,S) with $B,S>0$ both to a price offer distribution in $\Delta(\mathbb{R}_+)$ and to a probability of acceptance for each possible offer received. These shall interpreted to be his/her strategy in the bargaining game if he/she is matched and the market state is[6] (B,S).

Equilibrium concept. We focus on *symmetric Markov perfect equilibria*, where all traders on each side of the market use the same strategy (see Appendix B.1 for the formal definition). For simplicity, we will refer to symmetric Markov perfect equilibria as just "equilibria."

3 Equilibrium analysis

We begin this section by presenting the equations that the continuation value of each type of trader satisfies in an equilibrium, and then stating the existence of an equilibrium. We will then use these expressions to obtain some preliminary results, and to provide some intuition on why they hold.

3.1 Equilibrium continuation values and existence

Fix an equilibrium. Our first goal is to characterize the equations that the continuation value of a buyer and a seller satisfies in each state. We use $V_b(B,S)$ to denote a buyer's continuation value at some state (B,S) with $B>0$, and $V_s(B,S)$ to denote a seller's continuation value at some state (B,S) with $S>0$, both defined in Appendix B.1.

To ease notation, we will sometimes use N_b and N_s to denote B and S, respectively. We will sometimes refer to buyers and sellers as, respectively, b-traders and s-traders. Also, for a fixed trader's type $\theta \in \{b,s\}$, we use $\bar{\theta}$ to denote the complementary type, where $\{\theta,\bar{\theta}\}=\{b,s\}$. For $\theta \in \{b,s\}$, the continuation value of a θ-trader at some state (B,S) can be written as

$$V_\theta = \overbrace{\frac{\frac{1}{N_\theta}\lambda}{\lambda+\gamma+r}}^{\text{match}} V_\theta^{\mathrm{m}} + \overbrace{\frac{\frac{N_\theta-1}{N_\theta}\lambda}{\lambda+\gamma+r}}^{\text{others match}} V_\theta^{\mathrm{o}} + \overbrace{\frac{\gamma}{\lambda+\gamma+r}}^{\text{arrival}} V_\theta^{\mathrm{a}}\,, \tag{3.1}$$

where we omit the dependence of all V_θ's, λ, and γ on the state of the market, and where V_θ^{m}, V_θ^{o}, and V_θ^{a} are as defined below.[7] The continuation value is divided into the following three components:

[6] We implicitly assume that traders observe the state of the market. Markov perfect equilibria (see the definition below) remain equilibria independently of the information structure as long as the current state of the market is known to the traders in the market.

[7] To ease notation, we will omit the dependence of some variables on the state of the market (B,S) when the state is clear.

1. **Match:** Consider, for example, a seller who is matched with a buyer. If she is chosen to make the offer, she can make an unacceptable price offer (say above 1), which provides her with a continuation value of V_s. The seller can alternatively make an offer intended to be acceptable to the buyer. Since the continuation value of a buyer from rejecting the offer is V_b, he accepts for sure price offers strictly lower than $1 - V_b$, and rejects offers strictly above $1 - V_b$. Using the standard argument for take-it-or-leave-it offers, equilibrium offers by the seller that are accepted with positive probability are equal to $1 - V_b$. If, instead, the buyer is chosen to make the offer, the seller receives a payoff of V_s: in equilibrium, if the offer is acceptable, the buyer makes her indifferent whether to accept it or not. Hence, we have

$$V_s^{\mathrm{m}} = \xi \max\{V_s, 1-V_b\} + (1-\xi)\, V_s\ . \tag{3.2}$$

   The analogous equation for a buyer is given by

$$V_b^{\mathrm{m}} = \xi\, V_b + (1-\xi) \max\{V_b, 1-V_s\}\ . \tag{3.3}$$

2. **Others match:** The continuation value of a θ-trader if other traders match depends on the acceptance probability of the equilibrium offers. This value can be written as

$$V_\theta^{\mathrm{o}}(B,S) = \alpha\, V_\theta(B-1, S-1) + (1-\alpha)\, V_\theta(B,S)\ , \tag{3.4}$$

   where $\alpha \equiv \alpha(B,S)$ is the equilibrium probability that a buyer and a seller trade in a meeting in state (B,S). It is important to notice that, if the net surplus from trade is positive (i.e., $1 - V_s - V_b > 0$), the equilibrium offer is accepted for sure in any meeting in state (B,S) (hence $\alpha = 1$), whereas if it is negative (i.e., $1 - V_s - V_b < 0$), the equilibrium offer is rejected for sure (hence $\alpha = 0$).

3. **Arrival:** An arriving trader is a buyer with probability $\frac{\gamma_b}{\gamma_s+\gamma_b}$, and is a seller with probability $\frac{\gamma_s}{\gamma_s+\gamma_b}$. This implies that the continuation value of a θ-trader conditional on the arrival of a trader to the market can be written as

$$V_\theta^{\mathrm{a}}(B,S) = \tfrac{\gamma_b}{\gamma_b+\gamma_s}\, V_\theta(B+1, S) + \tfrac{\gamma_s}{\gamma_b+\gamma_s}\, V_\theta(B, S+1) \tag{3.5}$$

   for both $\theta \in \{b, s\}$.

Our first result establishes the existence of an equilibrium.[8]

Proposition 3.1. *An equilibrium exists. The continuation values in an equilibrium are uniquely determined by the probability of agreement α, and satisfy equations* (3.1)–(3.5).

[8]Given our focus on symmetric MPE, the existence of an equilibrium is not immediate. In the proof, we first establish that the probability of agreement α uniquely determines the continuation values, and then obtain that there exists α such that $\alpha(B,S) = 1$ when $V_b(B,S) + V_s(B,S) < 1$ and $\alpha(B,S) = 0$ when $V_b(B,S) + V_s(B,S) < 1$.

3.2 Preliminary results

We continue our analysis with some preliminary results that characterize some important features of equilibrium behavior. The first result establishes that there is no equilibrium and state of the market where equilibrium offers are rejected for sure. Hence, even though equilibrium offers may be rejected with strictly positive probability, there is never a "market breakdown." In other words, in equilibrium, there are no periods of time where trade happens with zero probability even though there are both buyers and sellers in the market.

Result 1. *In any equilibrium, there is strictly positive probability of trade in every meeting; that is, $\alpha(B,S) > 0$ whenever $B,S > 0$.*

The proof of the result proceeds by contradiction. Assume that there is an equilibrium and a state (B,S) where equilibrium offers are rejected for sure. This implies that the joint continuation value of a buyer and a seller in state (B,S) is weakly higher than the trade surplus:

$$V(B,S) \equiv V_b(B,S) + V_s(B,S) \geq 1 \ .$$

There thus exists a state (B',S') (possibly equal to (B,S)) in which $V(B',S')$ is maximal across all states and $\alpha(B',S') = 0$. Nevertheless, in this case, we have a contradiction:

$$V(B',S') = \tfrac{\gamma(B',S')}{\gamma(B',S')+r}\, V^{\mathrm{a}}(B',S') \leq \tfrac{\gamma(B',S')}{\gamma(B',S')+r}\, V(B',S') < V(B',S') \ .$$

The next result establishes that when there is a meeting and the market is balanced (i.e., $B = S$), there is trade with probability one.

Result 2. *In any equilibrium, there is trade for sure when the market is balanced; that is, $\alpha(B,S) = 1$ when $B = S$.*

When the market is balanced, a buyer and a seller "agree" on the relative likelihood of the three events that potentially change the state (matching, others matching, and arrival). Since their joint surplus is never higher than 1 (by Result 1), we have that

$$V = \tfrac{\frac{1}{S}\lambda}{\lambda+\gamma+r} \underbrace{V^{\mathrm{m}}}_{=1} + \tfrac{\frac{S-1}{S}\lambda}{\lambda+\gamma+r} \underbrace{V^{\mathrm{o}}}_{\leq 1} + \tfrac{\gamma}{\lambda+\gamma+r} \underbrace{V^{\mathrm{a}}}_{\leq 1} \leq \tfrac{\lambda+\gamma}{\lambda+\gamma+r} < 1 \ ,$$

As we see, their joint surplus from not agreeing is strictly lower than 1 since they discount the time when the next event occurs.

Our last result in this section establishes that, if equilibrium offers are rejected with a positive probability at some state (B,S), then a trader on the long side of the market benefits from other traders' transactions in such state.

Result 3. *Fix an equilibrium. Assume a state (B,S) such that $\alpha(B,S) < 1$ exists. Then, if the θ-traders are on the long side of the market, $V^{\mathrm{o}}_{\theta}(B,S) > V_{\theta}(B,S)$ and $V^{\mathrm{o}}_{\bar\theta}(B,S) < V_{\bar\theta}(B,S)$.*

To shed some light on Result 3, consider the case where sellers are on the long side of the market, that is, $S > B$. As equation (3.1) shows, the rate at which there is a match involving other traders is, from a seller's perspective, $\frac{S-1}{S}\lambda$. This rate is lower from a buyer's perspective, for whom it equals $\frac{B-1}{B}\lambda$. Thus, the weight of the event "other traders match" is larger in determining the sellers' continuation value than in determining the buyers' (see equation (3.1)). If, for example, state (B,S) is such that there is positive probability that the equilibrium offer is rejected (i.e., $\alpha < 1$), then $V(B,S) = 1$. Also, we know from Result 1 that the joint continuation value of a buyer and a seller is weakly lower than 1 at any given state. Hence, we can write

$$1 = V = \frac{\lambda}{\lambda+\gamma+r}\Big(\overbrace{\tfrac{1}{S}V_s + \tfrac{S-1}{S}V_s^{\mathrm{o}} + \tfrac{1}{B}V_b + \tfrac{B-1}{B}V_b^{\mathrm{o}}}^{(*)}\Big) + \frac{\gamma}{\lambda+\gamma+r}V^{\mathrm{a}}\,. \tag{3.6}$$

Since V^{m}, V^{o}, and V^{a} are weakly lower than 1, the previous equation holds only if $V_s^{\mathrm{o}} > V_s$ and $V_b^{\mathrm{o}} < V_b$. In this case, the greater weight that a seller assigns to the event that two other traders meet makes the term $(*)$ in the previous expression strictly greater than 1 (which is necessary for V to be equal to 1). In fact, it can be written as

$$1 < (*) = \underbrace{\tfrac{S-B}{BS}(V_s^{\mathrm{o}} - V_s)}_{>0} + \underbrace{\tfrac{1}{B}V + \tfrac{B-1}{B}V^{\mathrm{o}}}_{\leq 1} = \underbrace{\tfrac{S-B}{BS}(V_b - V_b^{\mathrm{o}})}_{>0} + \underbrace{\tfrac{1}{S}V + \tfrac{S-1}{S}V^{\mathrm{o}}}_{\leq 1}\,.$$

See Appendix A for an illustrative example of a thin market where all equilibria exhibit trade delay. In the example, the state of the market is $(1,2)$. Each seller obtains a large continuation payoff if the other seller trades, but sellers are more likely to arrive to the market before a trade occurs. As a result, sellers engage in a war of attrition before any of them trade or another seller arrives.

4 Small bargaining frictions

We now turn to the case where the bargaining frictions are small, that is, where traders in the market meet frequently. This may be a plausible assumption in some thin markets such as localized housing markets or job markets for specific occupations, where the rate at which traders (can) meet once they are in the market is much higher than the arrival rate to the market. As in the large markets literature, studying the case where frictions are small will allow us to provide a sharper characterization of the equilibrium outcome.

In order to analyze the case where bargaining frictions are small, we separate each state's meeting rate $\lambda(B,S)$ into two parts. The first is a state-independent common factor $k > 0$, which will be big. The second is a function $\ell(B,S)$, measuring the relative rate at which traders meet in each state. Thus, from now on, we use $\lambda(B,S)$ and $k\,\ell(B,S)$ interchangeably.

The difficulty of characterizing how some properties of equilibrium outcomes change "when bargaining frictions are small" is that our model may have multiple equilibria. The following notation is then convenient in comparing the properties of equilibrium outcomes as k increases. The notation "$\simeq$" indicates that terms on each of the sides are equal in any equilibrium, except for terms that go to 0 as k goes to[9] $+\infty$. Our first result establishes that when bargaining frictions are small, the joint continuation value of a buyer and a seller is close to the joint surplus they obtain from trade.

Result 4. $V_b(B,S)+V_s(B,S)\simeq 1$ *for all states* (B,S) *with* $B,S>0$.

To get an intuition for Result 4 note that, for a fixed equilibrium, there are three kinds of states. The first kind comprises all states where $B,S\geq 1$ and equilibrium offers are rejected with a positive probability (so $V=1$). The second kind comprises all states where either $B=1$ or $S=1$ (or both), and there is trade for sure in every meeting. If for example there is one buyer, so $S\geq B=1$, his continuation value can be approximated as follows:

$$V_b \simeq \xi\, V_b + (1-\xi)\,(1-V_s) \;\;\Rightarrow\;\; V_b \simeq 1-V_s\,.$$

Intuitively, given that meetings happen very frequently, the buyer can almost costlessly wait until he makes the offer and obtain $1-V_s\geq V_b$. Finally, the third kind comprises states where $B,S>1$ and there is immediate trade. In a state in this set, a buyer has the option to wait for a transaction to occur, and so $V_b \succeq V_b(B-1,S-1)$, where "$\succeq$" means that the terms on left side are bigger than those on the right hand side except for terms that vanish as $k\to\infty$. By the same argument we have $V_s\succeq V_s(B-1,S-1)$. Define m as the lowest value $m\geq 1$ such that $(B-m,S-m)$ belongs to one of the first two sets of states. Since we just argued that the net surplus from trade in state $(B-m,S-m)$ is small (i.e., $1-V(B-m,S-m)\simeq 0$), we can iteratively use the previous argument to show that

$$1\geq V(B,S)\succeq V(B-m,S-m)\simeq 1 \;\;\Rightarrow\;\; V(B,S)\simeq 1$$

An immediate and important consequence of Result 4 is that, when bargaining frictions are low, a seller is approximately indifferent between wether to trade or not in all states (B,S) where $S>B$. This is obviously true if $\alpha<1$ (the first set of states defined above). When, instead, $\alpha=1$ and $S>B=1$, the payoff of a seller is

$$V_s(1,S)\simeq \tfrac{1}{S}\,V_s(1,S)+\tfrac{S-1}{S}\,V_s(0,S-1)\,.$$

Thus, from the previous equation, it follows that $V_s(1,S)\simeq V_s(0,S–1)$, and therefore not trading is close to optimal for a seller. As we argued above, when bargaining frictions are low, states in the third set (where $\alpha=1$ and $S>B\geq 1$) change very fast to a state in one of the first two stets and hence the result holds.

[9] For example, the statement of Result 4 should be read as "For all $\varepsilon>0$ there is a $\bar{k}>0$ such that, if $k>\bar{k}$, then for any equilibrium and state (B,S) we have that $|V_b(B,S)+V_s(B,S)-1|<\varepsilon$."

Another implication of Result 4 is that the price dispersion of the transactions that occur in a given state is low when the bargaining frictions are small. Indeed, the equilibrium price in state (S,B) is either $V_s(S,B)$ (if the buyer makes the offer) or $1-V_b(S,B)$ (if the seller makes the offer). Since $V(S,B)\simeq 1$, we have that $V_s(S,B)\simeq 1-V_b(S,B)$. We therefore let the *market price* denote the approximate price at which transactions take place in each state (S,B); that is, $V_s(S,B)$. The following section shows that there is market price dispersion across states.

4.1 Characterization of the market price

As we argued above, Result 4 establishes that, when bargaining frictions are small, traders on the long side of the market are close to indifferent between trading and letting other traders trade as long as there are traders on both sides of the market. We use such indifference to provide a characterization of the equilibrium price by changing the probability measure that determines the evolution of the state of the market. This approach is in the same spirit as the use of risk-neutral measures in the study of financial markets. The main difference, apart from the thinness of our market, is that here the side of the market with more traders changes over time.

Fix an equilibrium. Consider a measure for which the state of the market (B_t,S_t) evolves according to a Markov chain as follows. At Poisson rates $\gamma_b(B_t,S_t)$ and $\gamma_s(B_t,S_t)$, the state changes to $(B_t{+}1,S_t)$ and $(B_t,S_t{+}1)$, respectively. Additionally, the state changes to $(B_t{-}1,S_t{-}1)$ at rate $\tilde\delta(B_t,S_t)$, where

$$\tilde\delta(B_t,S_t)\equiv\begin{cases}\frac{B_t-1}{B_t}\,\alpha(B_t,S_t)\,\lambda(B_t,S_t) & \text{if } B_t\geq S_t,\\ \frac{S_t-1}{S_t}\,\alpha(B_t,S_t)\,\lambda(B_t,S_t) & \text{if } B_t< S_t.\end{cases} \tag{4.1}$$

By a slight abuse of language, we call this measure the *risk-neutral measure* (of the fixed equilibrium). Note that the evolution of (B_t,S_t) under the risk-neutral measure corresponds to the evolution of the state of the market "when, at each time, one trader on the long side of the market deviates to not trading". Note also that the dynamics of the state of the market under the risk-neutral measure can be entirely determined from the equilibrium dynamics of the state of the market (and therefore uniquely obtained by an external observer who only observes the evolution of the state).

Proposition 4.1. *For any state (B_0,S_0) we have that*

$$V_s(B_0,S_0)\simeq\tilde{\mathbb{E}}\Big[\int_0^\infty e^{-rt}\big(\mathbb{I}_{B_t>S_t}+\xi(1,1)\,\mathbb{I}_{B_t=S_t}\big)\,r\,\mathrm{d}t\Big]\,, \tag{4.2}$$

where $\tilde{\mathbb{E}}$ is the expectation using the risk-neutral measure.

Proposition 4.1 gives an approximation of the transaction price at each state (B,S) (which is approximately equal to the market price $V_s(B,S)$) in terms of the equilibrium dynamics of the state and the probability that a seller makes an offer when there is only one buyer and one seller in the market. As we see, it is a discounted average (under the risk-neutral measure) of the future time the market exhibits excess supply, adjusted by the times it is balanced.

To obtain some intuition for Proposition 4.1, consider a state (B,S) where the market is imbalanced. If there are more sellers than buyers (i.e., $B<S$), sellers are approximately indifferent whether to trade or not, and this implies that

$$V_s \simeq \frac{\frac{S-1}{S}\,\alpha\lambda}{\frac{S-1}{S}\,\alpha\lambda+\gamma+r}\,V_s^{\mathrm{m}} + \frac{\gamma}{\frac{S-1}{S}\,\alpha\lambda+\gamma+r}\,V_s^{\mathrm{a}}\ . \tag{4.3}$$

A similar equation can be obtained when there are more buyers than sellers in the market (replacing s by b and S by B). Using Result 4 we can write, when $B>S$,

$$\overbrace{1-V_b}^{\simeq V_s} \simeq \frac{r}{\frac{B-1}{B}\,\alpha\lambda+\gamma+r} + \frac{\frac{B-1}{B}\,\alpha\lambda}{\frac{B-1}{B}\,\alpha\lambda+\gamma+r}\,\overbrace{(1-V_b^{\mathrm{m}})}^{\simeq V_s^{\mathrm{m}}} + \frac{\gamma}{\frac{B-1}{B}\,\alpha\lambda+\gamma+r}\,\overbrace{(1-V_b^{\mathrm{a}})}^{\simeq V_s^{\mathrm{a}}}\ . \tag{4.4}$$

Hence, when the market is imbalanced, the outcome of the market resembles the outcome typically obtained in models of Bertrand competition. Indeed, in a match, the payoff of a trader on the long side of the market if he/she trades is very close to his/her continuation value if he/she does not trade and, instead, waits until the state of the market changes. Importantly, in a dynamic market, the continuation value is endogenous, and driven by the expectation about future trade opportunities.[10]

When the market is balanced, Result 2 establishes that there is trade in every meeting. Consequently, when $S>1$, we have

$$V_s(S,S) \simeq \tfrac{1}{S}\,V_s(S,S) + \tfrac{S-1}{S}\,V_s(S-1,S-1)\ ,$$

where $V_s(S,S) \simeq V_s(S-1,S-1)$. Each seller is close to indifferent between trading and letting other traders trade until she is alone in the market with a single buyer. When there are only one buyer and one seller in the market, the reservation value of the seller (i.e., her value from not trading) is $\frac{\gamma(1,1)}{\gamma(1,1)+r}\,V_s^{\mathrm{a}}(1,1)$. Similarly, the reservation value of the buyer is $\frac{\gamma(1,1)}{\gamma(1,1)+r}\,V_b^{\mathrm{a}}(1,1)$. As the bargaining frictions become small, the transaction price is determined by the limit outcome of a two-player bargaining game à la Rubinstein (1982) with randomly arriving outside options (given by the potential arrival of other traders). The "size of the pie" over which they bargain is not 1, but the trade surplus net of the sum of the outside options, which is

$$1-\tfrac{\gamma(1,1)}{\gamma(1,1)+r}\,(V_b^{\mathrm{a}}(1,1)+V_s^{\mathrm{a}}(1,1)) \simeq \tfrac{r}{\gamma(1,1)+r}\ .$$

[10]This result can be interpreted as micro-founding, using a decentralized approach, the assumption in Taylor (1995) that, at any given time when the market is imbalanced, the transaction price is equal to the one obtained in a static market with Bertrand competition between the traders on its long side.

As in the standard Rubinstein bargaining game, the seller obtains, on top of her reservation value, a fraction of the pie equal to the probability with which she makes offers, $\xi(1,1)$. Hence, the Rubinstein payoff of the seller, which is approximately equal to the transaction price, is given by

$$V_s(1,1) \simeq \frac{r}{\gamma(1,1)+r}\,\xi(1,1) + \frac{\gamma(1,1)}{\gamma(1,1)+r}\,V_s^{\mathrm{a}}(1,1)\ . \tag{4.5}$$

Equations (4.3)–(4.5) indicate that, under the risk-neutral measure, V_s approximately follows the same equations as the continuation payoff of a fictitious agent who receives a flow payoff equal to 1 when there is excess supply (i.e., $B_t > S_t$), a flow payoff equal to 0 when there is excess demand (i.e., $B_t < S_t$), and a flow payoff equal to $\xi(1,1)$ when the market is balanced (i.e., $B_t = S_t$). The right-hand side of equation (4.2) gives an expression of such a continuation value.

An implication of Proposition 4.1 is that only the evolution of the sign of the net supply in the market (i.e., the number of sellers minus the number of buyers, which we call the *balancedness of the market*), is relevant for determining the market price. The reason is that the intensity of the competition between traders on the long side of the market is irrelevant for determining the price when the market is unbalanced: the price equals their reservation value independently of their number. Thus, the price is not directly determined by future trade opportunities of the traders in the market, but rarther by the expected evolution of the balancedness of the market. The price only dependends on the details of the bargaining protocol through the relative bargaining power of a seller when only she and a buyer are present in the market.

Remark 4.1 (Diamond's paradox). Corollary 4.1 shows that, in the limit where bargaining frictions disappear, the payoff of each trader in each state is strictly positive as long as there is a positive probability that his or her side of the will become the short side of the market in the future. This may be surprising since, in bargaining models with one-sided offers (which in our model would correspond to $\xi \equiv 0$ or $\xi \equiv 1$), the side of the market making the offers obtains all the surplus from trade, independently of the degree of balancedness of the market. This is usually known as Diamond's paradox (see Diamond, 1971). In our model, the order of setting these limits matters: our claim implicitly sets the limit of small bargaining frictions first, and the limit of one-sided offers second. The claim would not hold if we first assumed that $\xi(\cdot,\cdot)$ is constant and equal to either 0 or 1, and then we set the limit where the bargaining frictions disappear. In such a case, the type of traders making all the offers would obtain all gains from trade.

Changes in continuation values

The risk-neutral measure is typically defined as such that the current value of a financial asset is equal to its expected payoffs in the future discounted at the risk-free rate. If $B_0 < S_0$ it is easy to see that, indeed, the transaction price in state (B_0, S_0) (which is approximately equal to the market price $V_s(B_0, S_0)$) is approximately equal to the discounted price at which, in equilibrium, a seller at time 0 expects to sell the good (if she follows an optimal strategy). As we argued before, a close-to-optimal strategy for a seller at time 0 when the bargaining frictions are small is not to trade until the market is balanced. If, however, instead $B_0 > S_0$, a similar argument used to obtain the continuation value of a seller can be used to obtain the continuation value of a buyer. It then follows that

$$V_s(B_0, S_0) \simeq \begin{cases} \tilde{\mathbb{E}}[e^{-r\,\tau_0}\, V_s(1,1)] & \text{if } B_0 \le S_0\,, \\ 1 - \tilde{\mathbb{E}}[e^{-r\,\tau_0}\,(1 - V_s(1,1))] & \text{if } B_0 > S_0\,, \end{cases} \tag{4.6}$$

where τ_0 is the (stochastic) time it takes for the market to balance; see the proof of Corollary 4.1. Hence, the risk-neutral measure makes the current continuation value of a trader on the long side of the market equal to his/her expected surplus from waiting until the market is balanced before trading. This allows us to establish the following result:

Corollary 4.1. *Fix some state (B_t, S_t) where θ-traders are on the long side of the market. Then, for all $\varepsilon > 0$ there exists some $\bar{\Delta} > 0$ such that if $\Delta < \bar{\Delta}$ then*

$$r\,V_{\theta,t} - \varepsilon \precsim \frac{\tilde{\mathbb{E}}_t[V_{\theta,t+\Delta}] - V_{\theta,t}}{\Delta} \precsim \min\left\{\frac{\mathbb{E}_t[V_{\theta,t+\Delta}] - V_{\theta,t}}{\Delta}, r\,V_{\theta,t}\right\} + \varepsilon\,, \tag{4.7}$$

where $V_{\theta,t} \equiv V_\theta(B_t, S_t)$.

As we argued before, traders on the long side of the market are approximately indifferent between trading or not. As a result, the expected increase in their continuation value under the risk neutral measure has to approximately grow at rate r. Such expected increase is typically lower under the equilibrium measure. To see this, assume that at time t there is excess supply, i.e., $B_t < S_t$. Assume also that there is trade delay in state (B_t, S_t). Then, the rate at which transactions happen when all sellers follow the equilibrium strategy is higher than the transaction rate when one the sellers deviates and decides not to trade ($\alpha\,\lambda$ vs $\frac{S_t - 1}{S_t}\,\alpha\,\lambda$). Given that sellers are approximately indifferent to trading, and since by Result 3 they benefit from other sellers' transactions, the expected increase in the continuation payoff of the sellers is larger under the equilibrium measure than under the risk-neutral measure.

An important implication of Corollary 4.1 is that when the market is imbalanced, the market price tends toward the market price of a balanced market in expectation. Remarkably, this result is independent of whether the state of the market tends towards being balanced. That is, the market price increases in expectation when $B_t < S_t$ (and so the sellers are indifferent whether to trade or not), and decreases in expectation when $B_t > S_t$ (and so the buyers are indifferent whether to trade or not).

Equation (4.6), toghether with the converse inequality when buyers are on the long side of the market, is useful in setting approximate bounds on the transaction prices when bargaining frictions are small. For example, setting an approximate bound for the discounted transaction price in a state (B_0, S_0) only requires knowing the equilibrium dynamics of the state of the market and the price when the market is balanced, which is approximately[11] $V_s(1,1)$:

$$V_s(B_0, S_0) \begin{cases} \leq \mathbb{E}\left[e^{r\tau_0} V_s(1,1)\right] & \text{if } B_0 \leq S_0, \\ \geq 1 - \mathbb{E}\left[e^{r\tau_0}(1 - V_s(1,1))\right] & \text{if } B_0 > S_0. \end{cases} \tag{4.8}$$

The previous inequalities can be replaced by "$\simeq$" if $\mathbb{E}$ is replaced by $\tilde{\mathbb{E}}$ (see equation (4.6)).

4.2 No delay

We now study the equilibrium outcome of equilibria without trade delay when the bargaining frictions disappear. To this end, we first present a condition on the primitives of the model that guarantees that trade delay disappears as the bargaining frictions vanish.

Condition 1. For each state (B, S) with $B, S > 0$, $\frac{\gamma_\theta(B-1,S-1)}{\gamma(B-1,S-1)+r} \leq \frac{\gamma_\theta}{\gamma+r} + \frac{r}{\gamma+r}\frac{1}{2}$.

Condition 1 requires that single transactions do not accelerate by much the expected time until the arrival of each type of traders. This condition is satisfied if the arrival rates are state-independent, for example. It limits the possibility that traders on the long side of the market benefit significantly from transactions of other traders—see Appendix A for an example with delay when Condition 1 does not hold. Consequently, our condition prevents trade delay from occurring in equilibrium. The next proposition will establish this.

Proposition 4.2. *Under Condition 1 there exists some $\bar{k}$ such that if $k > \bar{k}$ then there is no equilibrium with trade delay. Furthermore, there exists an increasing function $p : \mathbb{Z} \to [0,1]$ such that $V_s(B,S) \simeq p(S-B)$ for all states (S,B).*

Proposition 4.2 establishes that Condition 1 is sufficient to ensure that trade delay disappears when bargaining frictions are small. Intuitively, by Result 3, trade delay occurs in a given state (B, S) with $S > B$ only if sellers gain from other traders' transactions, that is, $V_s(B-1, S-1) - V_s > 0$. The proof of Proposition 4.2 shows that since by Condition 1 the arrival

[11] Result 2 establishes that when the market is balanced (so $B = S$) there is trade for sure in every meeting. This implies that, when k is large, $V_\theta(S,S) \simeq \frac{1}{S} V_\theta(S,S) + \frac{S-1}{S} V_\theta(S-1,S-1)$, and so $V_\theta(S,S) \simeq V_\theta(S-1,S-1)$. Since transactions happen fast, it follows that $V_\theta(S,S) \simeq V_\theta(1,1)$.

of buyers cannot increase by much after a transaction, this gain is small when k is large, and so there is no equilibrium with trade delay.

Under Condition 1 trade delay vanishes as bargaining frictions disappear and so the time it takes the short side of the market to clear is increasingly small. The evolution of the state of the market is mostly determined by the evolution of the net supply, $N_t \equiv S_t - B_t$. As k increases, the limit process for the evolution of the net supply is as follows. If $N_t \geq 0$ then N_t decreases by one unit and increases by one unit, respectively, at a Poisson rates equal to $\gamma_b(0, N_t)$ and $\gamma_s(0, N_t)$. If $N_t < 0$, instead, then N_t decreases by one unit and increases by one unit, respectively, at a Poisson rates equal to $\gamma_b(-N_t, 0)$ and $\gamma_s(-N_t, 0)$. This reduction of the dimensionality of the state of the market adds tractability to the analysis of thin markets.

The net supply also evolves approximately autonomously under the risk-neutral measure when k is high. Yet, the equilibrium and risk-neutral dynamics of the net supply do not necessarily coincide in the limit where bargaining frictions vanish. If the market is imbalanced, their law of motions do coincide. Hence, equation (4.8) now is replaced by

$$p(N) = \begin{cases} 1 - \mathbb{E}^{\infty}[e^{-r\tau_0}|N_0 = N]\,(1 - p(0)) & \text{if } N \leq 0, \\ \mathbb{E}^{\infty}[e^{-r\tau_0}|N_0 = N]\,p(0) & \text{if } N > 0, \end{cases} \tag{4.9}$$

where $\mathbb{E}^{\infty}$ is the expectation using the limiting dynamics (as $k \to \infty$) for the net supply described in the previous paragraph. If the market is balanced, however, the arrival rate of θ-traders is $\gamma_\theta(0,0)$ in equilibrium, whereas it is $\gamma_\theta(1,1)$ under the risk-neutral measure.

Note that Condition 1 holds trivially in the big markets studied by Rubinstein and Wolinsky (1985) and Gale (1987), which exhibit no trade delay. Indeed, the equilibrium arrival rate of traders—which, in their models, is a discrete-time flow—is independent of whether a given trader trades or not. Thus, delaying trade does not change the continuation value of the traders, but it postpones the realization of the gains from trade. In general, this argument cannot be applied to a thin market: as each transaction affects the aggregate state of the market, traders may have the incentive to let other traders trade, and to trade only when their bargaining power is higher.

Changes in r

In this section, we consider the effect that changing r has on the distribution of market prices. We start by presenting a condition that will ensure that N_t has an ergodic distribution. It requires the arrival rate of agents on the short side of the market be higher than the arrival rate of agents on the long side of the market.

Condition 2. $\gamma_s(B,S) - \gamma_b(B,S) > 0$ if $B > S$ and $\gamma_b(B,S) - \gamma_s(B,S) > 0$ if $B < S$.

It is not difficult to see that if Condition 2 holds then the equilibrium dynamics generate an ergodic distribution of the state of the market in all equilibria. The following proposition establishes that trade delay shrinks as the arrival rates increase.

Under Conditions 1 and 2, the limit process for N_t as $k \to \infty$ (described above) also has an ergodic distribution F. Such distribution is such that, for each net supply value N, $\lim_{t\to\infty} \Pr(N_t = N|N_0) = F(\{N\})$ independently of[12] N_0. Note that since by Condition 1 there is no trade delay when k is large, the limit ergodic distribution of N_t does not depend on the discount rate, but the ergodic distribution of market prices does.

Corollary 4.2. *Assume that Conditions 1 and 2 hold. Then, an increase in r generates a spread of the limit ergodic distribution of market prices. If, additionally, $\gamma_\theta(0,0) = \gamma_\theta(1,1)$ for both $\theta \in \{b,s\}$, such a spread is mean-preserving.*

The first result in Corollary 4.2 establishes that an increase in r raises the ergodic market price dispersion.[13] It relies on the fact that under Condition 1 there is no trade delay when bargaining frictions are small, so the dynamics of the state of the market are independent of r. As a result, as equation (4.9) indicates, an increase in the discount rate r lowers the discount factor of the time it takes the market to become balanced. Hence, the market price for a given N_t tends to become more extreme when r increases: it tends to decrease when $N_t > 0$, and it tends to increase when $N_t < 0$ tends to become higher.[14] For instance, in the limit where $r \to \infty$, we have that $p(N_t) \to 0$ for all $N_t > 0$, and $p(N_t) \to 1$ for all $N_t < 0$.

The second result in Corollary 4.2 can be understood as follows. Assume that Condition 1 holds and $\gamma_\theta(0,0) = \gamma_\theta(1,1)$ for both $\theta \in \{b,s\}$. As we argued above, these conditions guaratnee that the equilibrium and the risk-neutral measures coincide in the limit where the bargaining frictions disappear. Furthermore, from equation (4.2) we have that the ergodic mean of the market price can be approximated by the ergodic probability that the market exhibits excess demand plus the ergodic probability that the market is balanced multiplied by $\xi(1,1)$. Since under Condition 1 there is no trade delay when bargaining frictions are small, such a long-run expected market price is independent of the discount rate r, and so the second result in Corollary 4.2 holds. In general, if the arrival rates in states $(0,0)$ and $(1,1)$ are close (but not equal), or if the ergodic likelihood that the market is balanced is low, Corollary 4.2 establishes

[12] Many assumptions ensure that the state of the market has an ergodic distribution in any equilibrium. An example is Condition 4.3 below.

[13] Recall that a CDF F_1 in $\mathbb{R}$ is a spread of another CDF F_2 if they satisfy the "single-crossing condition": there exists a value $\bar{x} \in \mathbb{R}$ such that $F_1(x) \geq F_2(x)$ for all $x < \bar{x}$ and $F_1(x) \leq F_s(x)$ for all $x > \bar{x}$.

[14] The negatice direct effect of an increase in r in $p(N_t)$ when $N > 0$ (on the right hand side of equation (4.9)) may sometimes be compensated by an increase in $p(0)$. Nevertheless, as the proof of Corollary 4.2 shows, the direct effect dominates if N_t is large enough.

that increases in the interest rate will increase the spread of prices, and will keep its mean approximately unchanged.

4.3 Large market limit

One of the salient questions in the literature on decentralized bargaining in large markets is whether lowering frictions leads to a competitive outcome. The answer to this question sheds light on whether and how frictions may be magnified or mitigated by the equilibrium behavior of the traders in the market, and therefore shed light on how robust the predictions of models with markets without frictions are.[15] In this section, we ask a similar question about thin markets. Specifically, we analyze the role of the friction that remains in the market when the meeting frequency is high; that is, the time that a trader has to wait for trading when he or she is on the long side of the market owed to the slow arrival of traders. In other words, we analyze the equilibrium outcome in the limit where the arrival rates of buyers and sellers are increasingly big.

Fix some functions $\tilde{\gamma}_b$ and $\tilde{\gamma}_s$. For each value $M > 0$, we consider the model with arrival rates $\gamma_b = M\tilde{\gamma}_b$ and $\gamma_s = M\tilde{\gamma}_s$. Increasing M can then be interpreted as unifying similar markets into bigger ones. In practice, such market unificaton may correspond to the launch of a website providing information on job offerings, rental prices, or housing prices in close locations, since such a website may make it easier for buyers and sellers in different markets to meet each other, which may de facto unify the different markets into a single market. Market unification may also be the result of improvements in the transportation infrastructure that reduce the commuting time, such as new metro stations or new roads. We will show that even though the unification of markets may not change the ergodic distribution of the market composition much, it reduces trade delay, it makes prices fluctuate faster (in the sense that changes in the market price happen more frequently), and that it makes ergodic distribution of market prices more concentrated around a given value.

Proposition 4.3. *Assume that Condition 2 holds. For all $M' > 0$ there is some $\bar{M} > 0$ such that if $M > \bar{M}$ and k is big enough, then any equilibrium in the model with arrival rates $(M\tilde{\gamma}_b, M\tilde{\gamma}_s)$ satisfies that the trade rate $\alpha(B,S)\,\lambda(B,S)$ is bigger than M' in any state (B,S).*

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[15] For example, Gale (1987) characterizes the trade outcome in the large-market version of our model in the limit where the discount rate tends to 0, and obtains that the trade outcome converges to that of a competitive market. In this limit, the price is either 0 (if there are more buyers than sellers) or 1 (if there are more buyers than sellers). Other papers have identified settings where the trade outcome fails to convergence to the competitive one. Such failure of convergence may be due, among other reasons, to symmetric information between traders (Satterthwaite and Shneyerov, 2007; Lauermann and Wolinsky, 2016), the heterogeneity on each side of the market (Lauermann, 2012), or lack of knowledge about the state of the market (Lauermann, Merzyn, and Virág, 2017). See also Lauermann (2013) for an analysis of other causes of trade delay.

An intuition for Proposition 4.3 is obtained as follows. As the arrival of traders increases, the current state of the market becomes progressively less relevant in determining the price, since each trader in the market can wait for the state of the market to change without incurring a big cost. In particular, the delay cost of not trading until the net state reaches some given state in the support of the ergodic distribution of the state of the market tends to 0 as $M \to 0$ (see, for example, equation (4.6)). As a result, as the arrival rates increase, it may seem that, for each given state, the payoff from not trading becomes increasingly attractive to each of the traders in the market. However, this is not possible when bargaining frictions are small: the sum of the continuation values of a seller and a buyer in the market is always close to 1, independently of the value of the arrival rates. Even though waiting is increasingly cheap, it is also increasingly worthless, in the sense that the price variation across states becomes increasingly small.

In a thin market there is no natural analogue of a "competitive outcome", since the number of traders on each side of the market, at any given moment in time, is finite. Still, if traders arrive more frequently, it becomes less costly for them wait to trade (and compete) with future traders. Consequently, the effective market that each trader has access to is increasingly large. As we will see, this implies that the outcomes of a thin market with frequent arrival and a competitive market share many features in common: their prices are (approximately) constant, and depend only on the (expected) balancedness of the market. In contrast to a competitive market, a thin market has an endogenous arrival process whereby, in general, no side of the market obtains the full surplus from trade.

The following corollary characterizes how the waiting options of buyers and sellers affect the market's outcome when the arrival rates of buyers and of sellers increase.

Corollary 4.3. *For all $\varepsilon > 0$, there is some $\bar{M} > 0$ such that if $M > \bar{M}$, then, in any equilibrium in the model with arrival rates $(M\tilde{\gamma}_b, M\tilde{\gamma}_s)$ and for any state (B_0, S_0),*

$$\left| V_s(B_0, S_0) - \lim_{t\to\infty} \tilde{\mathbb{E}}\left[\mathbb{I}_{B_t > S_t} + \xi(1,1)\,\mathbb{I}_{B_t = S_t}\right] \right| \leq \varepsilon \ .$$

Corollary 4.3 establishes that, as the arrival rates increase, the current transaction price is determined by the long-run distribution of market prices. In particular, if the equilibrium dynamics generate an ergodic distribution of the state of the market, the distribution of transaction prices becomes degenerate at some "competitive" price p^*. This is intuitive: since waiting for the state to change (instead of trading now) is increasingly cheap as M increases, the market price in all states of the market converges to a single price. Such a price can be obtained using equation (4.2), from which it is clear that when M is big the market price is close to the ergodic probability of the market having an excess demand, adjusted by the probability with which the market is balanced.[16] It is then immediate to see that, as $M \to \infty$, the

[16] It is easy to verify (from equations (3.1)–(3.5)) that a model with primitives $(\lambda, M\tilde{\gamma}_b, M\tilde{\gamma}_s, r, \xi)$ has the same set

distribution of transaction prices also degenerates to the competitive price p^*.

The fact that trade delay shrinks to 0 when the arrival rates increase does not necessarily imply that delay is not relevant in determining the competitive price. Even though the trade rate at each state of the market increases, it may remain similar (in relative terms) to the arrival rates and, therefore, different equilibria can have different ergodic distributions of states.

To provide further intuition for the previous results, let B_t^Σ and S_t^Σ denote, respectively, the number of buyers and sellers who arrived to the market between time 0 and time t, including the ones who "arrived" (or were present) at time 0. Then, trivially, $N_t = S_t - B_t = S_t^\Sigma - B_t^\Sigma$; thus equation (4.2) holds replacing B_t and S_t by B_t^Σ and $\bar{S}_t^\Sigma$, respectively. As traders become more patient, the price (at time 0, for example) approximates the (ergodic) probability that more sellers than buyers arrive in the future. Hence, as M increases, the effective market that a trader (at time 0) has access to grows intertemporally. Given that the endogenous arrival process may tend to keep the thin market balanced (if, for example, Condition 2), the competitive price is not necessary either 0 (when there is excess supply) or 1 (when there is excess demand). Instead, in contrast to the big market case, the competitive price of a thin market is a convex combination of the two extremes, each of them weighted according to the probability that the market features excess supply and demand.

5 Generalizations and extensions

This section discusses different generalizations and extensions of our model. These generalizations and extensions illustrate how our results can be extended beyond the particular assumptions made for the sakes of concreteness and simplicity. They also indicate that different specifications of a thin market give rise to the same set of results, and hence give us a sense of the robustness of our findings.

5.1 General market process

In our base model, we assume that the arrival rates of traders depend only on the current number of buyers and sellers in the market. In practice, the arrival of traders to markets may depend on other factors, like the state of the economy (economic booms or downturns), changes in similar markets, idiosyncratic demand/supply shocks in the market, changes in legislation affecting the bargaining power of the different types of traders, etc. In this section, we argue that our results are robust to allowing the arrival process to depend on a multidimensional state.

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of equilibria as a model with primitives $(\lambda/M, \tilde{\gamma}_b, \tilde{\gamma}_s, r/M, \xi)$. Hence, when M is large, the weight that the value of the state of the market at high values of t has on determining the market price increases; see equation (4.2).

Assume that the state of the market at time t is (B_t, S_t, ω_t), where ω_t is the value of a stochastic process taking values in some set $\Omega \subset \mathbb{R}^n$ for some $n \in \mathbb{N}$. We denote ω_t the *market's cycle* at time t. Hence, by a slight abuse of notation, we let $\gamma_b \equiv \gamma_b(B_t, S_t, \omega_t)$ and $\gamma_s \equiv \gamma_s(B_t, S_t, \omega_t)$ denote, respectively, the arrival rates of buyers and sellers of buyers to the market at time t. We denote by $\xi \equiv \xi(B_t, S_t, \omega_t)$ the probability that the seller makes an offer in a meeting when the state of the market is (B_t, S_t, ω_t).

We assume that the market's cycle ω_t changes when there is a transaction, when there is an arrival, or exogenously at a Poisson rate $\eta \equiv \eta(B_t, S_t, \omega_t)$, where $\eta : \mathbb{Z}_+ \times \mathbb{Z}_+ \times \Omega \to \mathbb{R}_+$. In each of these events, the new state is determined by a random variable $\tilde{\omega}$ that depends only on (B_t, S_t, ω_t) and the type of event. Some components of the market's cycle can be assumed to evolve exogenously (e.g., the state of the economy or regulation changes) and others endogenously (e.g., the number of traders in the market, the visibility of the market to other traders, or regional economic conditions if the market is geographically located).

Equations (3.1)–(3.5) can be adapted to the general market process. For example, transactions and arrivals have the potential to change (some components) of the market's cycle. Also, at each moment in time, there is the possibility of an exogenous change in the market's cycle. In equilibrium, the continuation value of a θ-trader in market state (B, S, ω) satisfies

$$V_\theta = \frac{\frac{1}{N_\theta}\lambda}{\lambda+\gamma+r+\eta} V_\theta^{\mathrm{m}} + \frac{\frac{N_\theta-1}{N_\theta}\lambda}{\lambda+\gamma+r+\eta} V_\theta^{\mathrm{o}} + \frac{\gamma}{\lambda+\gamma+r+\eta} V_\theta^{\mathrm{a}} + \frac{\eta}{\lambda+\gamma+r+\eta} V_\theta^{\mathrm{c}} , \tag{5.1}$$

where we omit the dependence of the different V_θ's, λ, γ, and η on the state of the market (B, S, ω), where V_θ^{m} is defined as in (3.2) and (3.3), and where

$$\begin{aligned}
&V_\theta^{\mathrm{o}} \equiv \alpha\, \mathbb{E}_{\tilde{\omega}}\big[V_\theta(B-1, S-1, \tilde{\omega}) \big| \text{trade}\big] + (1-\alpha)\, V_\theta ,\\
&V_\theta^{\mathrm{a}} \equiv \tfrac{\gamma_b}{\gamma_b+\gamma_s}\, \mathbb{E}_{\tilde{\omega}}\big[V_\theta(B+1, S, \tilde{\omega}) \big| \text{buyer arrives}\big] + \tfrac{\gamma_s}{\gamma_b+\gamma_s}\, \mathbb{E}_{\tilde{\omega}}[V_\theta(B, S+1, \tilde{\omega}) \big| \text{seller arrives}] , \text{ and}\\
&V_\theta^{\mathrm{c}} \equiv \mathbb{E}_{\tilde{\omega}}\big[V_\theta(B, S, \tilde{\omega}) \big| \text{exogenous change}\big]
\end{aligned}$$

are the expected continuation value if the market's cycle changes (with all expectations being conditional on the state (B, S, ω)).

Using the same arguments as in our base model, we can show that Results 1–4 and Corollary 4.1 still hold under a general market process. Indeed, the last two terms on the right-hand side of equation (5.1) play a similar role: they provide the effect of exogenous changes of the state of the market. One can then see that the arguments for Results 1–4 and Corollary 4.1 are independent of the particular form of these terms. Proposition 4.1, however, has to be adapted as follows: when the market is balanced, the flow payoff of the fictitious agent described in the paragraph that comes after equation (4.5) is equal to $\xi(1, 1, \omega_t)$; that is, it potentially depends on the market cycle.

The generalized process for the state of the market expands the range of settings in which trade delay may occur in equilibrium. Indeed, as we will see in the example in Appendix

A, the crucial feature for trade delay to occur is that, for a fixed state (B, S), traders on the short side of the market benefit from the arrival of traders, while traders on the long side of the market benefit from the endogenously determined transactions. Hence, trade delay may occur when traders on the short side of the market expect to benefit from exogenous changes in the market evolution, such as changes in the economic cycle or in legislation regarding their relative bargaining power. Conversely, endogenous changes in the market, driven for example by transactions, may change the arrival rates of new traders, as they may make the market more visible.

5.2 Nash bargaining

In this section, we argue that our results can be straightforwardly generalized to allowing the outcome of each meeting to be the outcome of a general Nash bargaining problem.

In our base model, the bargaining protocol for each meeting consists of a take-it-or-leave-it offer by a randomly chosen trader. In a more general bargaining protocol, such as Nash bargaining, a meeting results in some (potentially stochastic) transfers, and a probability of agreement. Thus, we can write the payoffs for traders when they meet as

$$V_s^{\mathrm{m}} = \alpha\,\mathbb{E}[p|\text{agree}] + (1-\alpha)\,(V_s + \mathbb{E}[p|\text{disagree}])\,,$$
$$V_b^{\mathrm{m}} = \alpha\,\mathbb{E}[1-p|\text{agree}] + (1-\alpha)\,(V_b - \mathbb{E}[p|\text{disagree}])\,,$$

where $\alpha \equiv \alpha(B, S)$ is an endogenous probability of agreement in state (B, S). The assumption individual rationality by buyers and sellers (that is, the assumption that they can opt out from bargaining and obtain their continuation value instead) requires that $V_\theta^{\mathrm{m}} \geq V_\theta$ for both $\theta \in \{b, s\}$ (i.e., $V_\theta^{\mathrm{m}} \in [V_\theta, 1 - V_{\bar{\theta}}]$). Consequently, $\alpha = 0$ and $\mathbb{E}[p|\text{disagree}] = 0$ whenever $V_s + V_b > 1$. Our results rely on the fact that α is "high" when $V_b + V_s > 1$, but not on the fact that it is 1. Indeed, if there is a cap $\bar{\alpha}$ to the probability of agreement, the meeting frequency λ can be readjusted accordingly.

The above properties permit obtaining results to those in Sections 3 and 4 for a generalized bargaining protocol. Indeed, the particular structure of the bargaining protocol is not used to show Results 1–4 and Corollary 4.1, only the individual rationality of the traders. In Proposition 4.1, $\xi(1, 1)$ has to be replaced by the expected fraction of the net surplus captured by a seller when there are only one seller and one buyer in the market. Finally, the "size of the pie" over which traders bargain in every meeting, $1 - V_b - V_s$, can be shown to shrink when bargaining frictions disappear; thus, the results in Section 4.2 also hold.

5.3 Outside options and attention frictions

The arrival and matching rates of our model (i.e., λ, γ_b and γ_s) are kept general throughout our analysis. This has allowed us to establish general properties of the outcomes of thin markets. In this section, we discuss different extensions of our model that partially endogenize the arrival and matching rates, giving them some additional structure. In addition, we discuss the case where traders may exit the market before they trade. In all cases, results similar to those obtained in the previous sections apply.

Endogenous entry. In some cases, entering a market may require a sunk investment. For example, home sellers may have to condition and advertise their housing units, and workers may have to update their submarket-specific knowledge and to prepare some documentation (CV, cover/reference letters, etc.) before entering the market. Hence, the decision to enter a market may be the result of a cost-benefit analysis, where potential traders compare the cost of entering the market with the expected gains from trade. To accommodate such a possibility, we could extend our model in the following way. (In the large markets literature, Manea (2017b) shows the existence of steady states in a similar specification.) Consider an extended model where buyers and sellers become active at some respective (state-independent) rates $\bar{\gamma}_b$ and $\bar{\gamma}_s$ instead of directly entering the market. Once a buyer or a seller becomes active, he/she draws a cost c from some distribution F_b or F_s, respectively. In an equilibrium of this model, if for example a seller becomes active and the state is (B,S), she enters the market if the net payoff from doing so, $V_s(B,S+1)-c$, is above some fixed outside option normalized to be 0 (choosing to sell in another market or keeping the good for herself). This implies that $\gamma_s(B,S)=\bar{\gamma}_s F_s(V_s(B,S+1))$. Given that our results hold for general arrival rates (further generalized in Section 5.1), any equilibrium outcome in such a model corresponds to an equilibrium outcome of some specification of our model. In the model with endogenous arrival, the arrival of agents on the short side of the market will tend to be higher than the arrival of agents on the long side, and hence the market will tend to remain approximately balanced.

Exogenous and endogenous exit. A common assumption in the large-market literature is that θ-traders leave the market at some (typically state-independent) Poisson rate $\rho_\theta > 0$, for each $\theta \in \{b,s\}$. This assumption is often made to keep the size of the market stationary when the arrival rates are constant, and incorporates the observation that traders some times exit the market for exogenous reasons. Making such an assumption in our model adds an extra term equal to $B\rho_b + S\rho_s$ to each denominator in equation (3.1), as well as the term

$$\tfrac{1}{\lambda+\gamma+r+B\rho_b+S\rho_s}\big(B\rho_b V_s(B-1,S)+(S-1)\rho_s V_s(B,S-1)\big)$$

on the right-hand side of the equation when $\theta = s$ (and a similar term when $\theta = b$). The additional term plays a role similar to that of the term “arrival” in equation (3.1): it also corresponds to an exogenous (i.e., equilibrium-independent) change in the state of the market.

More generally, our model can be adapted to accommodate endogenous exit. In a job market, endogenous exit may correspond to workers moving to other commuting areas when wages are low in their current commuting areas. Similarly, house-seekers may end up looking for houses with different characteristics when the prices of houses in their desired sub-market are high (families may end up buying/renting an apartment instead of a house with a garden, for example). To incorporate endogenous exit, we can add to our model a stochastic process determining the decision times (arriving at some Poisson rate) where a trader can decide whether to leave the market or not. Leaving the market gives a θ-trader an exogenous continuation value equal to $\underline{V}_\theta \geq 0$ (with $\underline{V}_b + \underline{V}_s < 1$). Equilibria of this extended model would feature some states (typically highly imbalanced) with exit of the traders on the long side of the market, and other states with no exit. If the arrival rate of decision times was high enough, the state of the market would remain approximately balanced most of the time (and would feature no exit, as in our model). Additionally, there would be some highly unbalanced states such that, after the arrival of a trader on the long side of the market, a trader on the long side of the market would almost immediately leave. As in the previous case with endogenous entry, the state of the market would tend to stay approximately balanced in a model with either exogenous or endogenous exit.

Attention frictions. The matching rate of our model can be interpreted as an attention friction faced by traders. Consider, for example, a thin market like the one in Section 2 where traders draw "attention times" instead of "meeting" other traders. In this model, a θ-trader in the market draws attention times at a (possibly state-independent) Poisson rate $\lambda_\theta > 0$, for $\theta \in \{b, s\}$. When a trader draws an attention time, he/she chooses a trader on the other side of the market (if any) and makes her/him an offer. This model with attention frictions would generate the same (symmetric Markov perfect) equilibria as our model with

$$\lambda \equiv S\lambda_s + B\lambda_b \text{ and } \xi \equiv \tfrac{S\lambda_s}{S\lambda_s + B\lambda_b}\ .$$

The limit "where bargaining frictions vanish" considered in Section 4 corresponds, in the model with attention frictions, to the limit "where attention frictions vanish."

6 Conclusions

We have studied decentralized bargaining in dynamic thin markets. In these markets, the bargaining powers and the arrival of traders are endogenous and depend on both the current market conditions and the expectations about future market conditions.

Our results stress that modeling big decentralized markets as the sum of small thin markets has important implications for the predicted trade outcome. For example, in a thin market, trade delay and price dispersion arise even when bargaining frictions are small and traders are not significantly heterogeneous. Delay occurs because traders on different sides of the market assign different relative likelihoods to arrivals or transactions by other traders. Price dispersion arises from the fact that each trader's reservation value depends on the e-volving composition of the market. The slow arrival of traders makes their reservation value depend on the current market composition in a nontrivial way even when the bargaining frictions are small. Still, even though the market is thin, the particular bargaining protocol used to set prices in the market has a small effect on the equilibrium price. Our characterization of the price in terms of the evolution of the market may serve as a guide for future empirical work.

We obtain some general novel implications for the price process that have the potential of being tested using disaggregated data from individual buyers (employers/house seekers) and sellers (workers/house owners). First, market prices drift toward the price of a balanced market. Second, increases in the interest rate result in increases in the spread of the distribution of market prices. Finally, if different markets are unified, the distribution of transaction prices degenerates into a competitive price. Unlike in a big markets, such a competitive price typically is not extreme in thin markets, that is, no side of the market obtains all surplus from trade.

Our model can be generalized in multiple directions. Of particular interest is the possibility of allowing buyers and sellers to be heterogeneous in terms of the quality of their goods or their valuations for them. This would make the analysis much less tractable, as it would enlarge the dimensionality of the state of the market.[17] Another possible extension consists on explicitly modeling costly relocation of traders between different sub-markets. This would allow providing new insights on endogenous gentrification (see Guerrieri, Hartley, and Hurst (2013) for a centralized large-market approach) or sectorial mobility of workers (see Artuç and McLaren (2015) for evidence), as well as analyzing the effects that idiosyncratic and common shocks have on mobility across markets. The analysis of these and other extensions is left to future research.

[17] Abreu and Manea (2012b,a) and Elliott and Nava (forthcoming) show that, in a model of bargaining in networks, the outcome of bargaining is stochastic even in the limit where bargaining frictions vanish, as sometimes transactions with low gains from trade are realized in the presence of more beneficial trade opportunities.

A An example with trade delay

In our setting, all sellers and buyers are homogeneous and do not have private information. Thus, given our focus on symmetric equilibria, it is a priori unclear whether there exist equilibria where some equilibrium offers are rejected with positive probability. In this section we illustrate how equilibria with trade delay may arise. In order to keep the example simple, we focus on a given state of the market and exogenously fix the continuation payoffs when such a state changes, without explicitly modeling the continuation play. This "reduced version" of our model simplifies the expressions and arguments, while enabling us to verify that there exist full specifications of our model with the same equilibrium features.

Consider the following reduced version of our model. Initially, there is one buyer and two sellers in the market. The buyer could be a raising Hollywood actor looking for a house in a neighborhood in Los Angeles. The sellers could be two fading stars looking for selling their houses in this particular neighborhood. We assume that $\gamma_s(1,2) > \gamma_b(1,2) = 0$, and denote $\gamma \equiv \gamma_s(1,2)$ and $\lambda \equiv \lambda(1,2)$. If a transaction occurs before the arrival of a seller, the sale is publicized by tabloids, and the neighborhood becomes "trendy." This attracts other raising actors (buyers). The remaining seller obtains a high continuation payoff, which for simplicity is assumed to be equal to[18] 1. If, instead, a seller arrives, the strong competition between sellers gives the buyer a high continuation payoff, which is again assumed to be 1 (see footnote 18), and the sellers obtain[19] 0. It is then clear that traders on the long side of the market benefit from other traders' transactions, which by Result 3 is a necessary condition for trade delay to occur in equilibrium.

We first compute the continuation values of the buyer and sellers under the assumption that, in each meeting, the price offer is equal to the continuation value of the trader receiving the offer, and that such an offer is accepted for sure (i.e., equations (3.1)–(3.5) hold with $\alpha = 1$). The continuation values of the buyer and the seller in state $(1,2)$ solve the following system of equations:

$$V_b(1,2) = \tfrac{\lambda}{\lambda+\gamma+r}\left(\xi\, V_b(1,2) + (1-\xi)\,(1-V_s(1,2))\right) + \tfrac{\gamma}{\lambda+\gamma+r}\,1\,,$$
$$V_s(1,2) = \tfrac{\lambda/2}{\lambda+\gamma+r}\left(\xi\,(1-V_b(1,2)) + (1-\xi)\,V_s(1,2)\right) + \tfrac{\lambda/2}{\lambda+\gamma+r}\,1\,.$$

[18] All values can be perturbed while keeping the same features of the example. For example, A continuation payoff of the seller that is arbitrarily close to 1 after the buyer and the other seller trade (that is, in state is $(0,1)$) can be supported assuming that $\gamma_b(B,1) \gg \gamma_s(B,1)$ for all $B \geq 0$. Analogously, a high continuation value of the buyer in state $(1,3)$ can be supported if, for example, $\gamma_b(B,S) \ll \gamma_s(B,S)$ whenever $B < S-1$.

[19] In Section 5.1, using a more general state of the market, we argued that trade delay arises in a wider set of situations. The crucial requirement is that traders on the short side of the market benefit from some events (arrival of traders, changes in the economic cycle, legislation reforms, etc.), while traders on the short side of the market benefit from transactions of other traders (as they can make the market more "trendy").

Solving for the previous system of equations, and using simple algebra, it is easy to show that

$$V_b(1,2) + V_s(1,2) = 1 + \frac{\gamma\,(\lambda-2\,r)-2\,r^2}{(\gamma+\lambda+r)\,(2\,\gamma+(1-\xi)\,\lambda+2\,r)}\,.$$

If λ is high or r is low (i.e., the right-hand side of the above equation is strictly greater than 1), an equilibrium where there is trade in every meeting does not exist. In this case, any equilibrium in this reduced version of the base model has the property that offers are rejected with positive probability. Using α to denote the probability of agreement in a meeting in state $(1,2)$, in any equilibrium of the (reduced) game, we have

$$\alpha = \min\left\{1, \frac{2\,r\,(\gamma+r)}{\gamma\,\lambda}\right\}.$$

Notice that the rate at which an agreement occurs in state $(1,2)$ (which equals $\alpha\,\lambda$) converges to $\frac{2\,r\,(\gamma+r)}{\gamma}$ as λ becomes big; that is, a significant trade delay remains even in the limit where bargaining frictions disappear.

Our example illustrates that trade delay may occur when traders on one side of the market benefit from other traders' transactions, while traders on the other side of the market benefit from the arrival of new traders. In the example, sellers obtain a high continuation payoff if a transaction occurs, and the buyer gets a high payoff if a trader arrives. If there was immediate trade, a seller would obtain $\frac{\lambda/2}{\lambda/2+\gamma+r}$ from not trading and letting the other seller trade with the buyer. The buyer is unwilling to accept a price above $\frac{\gamma}{\gamma+r}$, given that he has the option of waiting for the arrival of another seller and then obtaining a high payoff. As a result, if either λ is large or r is small, immediate agreement is not possible: otherwise, each seller would have the incentive to let the other seller trade at a low price and to obtain a high continuation payoff afterward. The equilibrium behavior of the sellers in the market thus resembles a war of attrition: each sellers trades at the rate that makes the other seller (and the buyer) indifferent whether to trade at price $\frac{\gamma}{\gamma+r}$ or not. From each seller's perspective, such delay lowers the value of making unacceptable offers, since doing so comes with the risk of another seller arriving. As time passes, either one of the sellers trades (and the remaining seller obtains a high payoff), or another seller arrives (and all sellers obtain a low continuation payoff).

Remark A.1. Inefficient delay can also be found in other bargaining models with complete information. For example, Cai (2000) analyzes a model of one-to-many bargaining between farmers and a railroad company, where the gains from trade are realized only if all the farmers agree. Similar to our model, the farmers want other farmers to trade, to gain monopsony power. Also, Abreu and Manea (2012b,a) and Elliott and Nava (forthcoming) analyze bargaining models in networks in which delay may happen because traders are heterogeneous, in terms of their value from trade or their position in the network. Our example illustrates that trade delay may occur even when bargaining is decentralized and traders are homogeneous, the reason being that some traders on one side of the market may benefit from other traders' trades, while on one side of the market benefit from other traders' arrivals.

B Omitted expressions and proofs of the results

B.1 Payoffs and equilibria

In the Appendix we use $\mathscr{B} \equiv \{0,\dots,\bar{B}\}$ and $\mathscr{S} \equiv \{0,\dots,\bar{S}\}$ and $\mathscr{B}^* \equiv \mathscr{B}\backslash\{0\}$ and $\mathscr{S}^* \equiv \mathscr{S}\backslash\{0\}$. Fix a strategy for the buyers (π_b, α_b) and a strategy for the sellers (π_s, α_s). For each type $\theta \in \{b,s\}$ and state $(B,S) \in \mathscr{B}^* \times \mathscr{S}^*$, $\pi_\theta(B,S) \in \Delta(\mathbb{R})$ is the distribution of price offers that θ-traders make if they are matched and chosen to make the offer in state (B,S), and $\alpha_\theta(\,\cdot\,;B,S) : \mathbb{R} \to [0,1]$ maps each price offer received to a probability of acceptance.

Fix a strategy profile $\{(\pi_\theta,\alpha_\theta)\}_{\theta\in\{b,s\}}$ and state (B,S). We compute the continuation values the strategy gives to a buyer (denoted $V_b(B,S)$) and to a seller (denoted $V_s(B,S)$) using standard recursive analysis. They satisfy equation (3.1) (for both $\theta \in \{b,s\}$), where now the expected continuation values conditional on being selected in the match given by

$$\begin{aligned} V_b^{\mathrm{m}}(B,S) \equiv{}& \xi\,\mathbb{E}_{\tilde{p}}\big[\,\alpha_b(\tilde{p})\,(1-\tilde{p}) + (1-\alpha_b(\tilde{p}))\,V_b(B,S)\,\big|\,\pi_s\,\big] \\ &+ (1-\xi)\,\mathbb{E}_{\tilde{p}}\big[\,\alpha_s(\tilde{p})\,(1-\tilde{p}) + (1-\alpha_s(\tilde{p}))\,V_b(B,S)\,\big|\,\pi_b\,\big] \quad \text{and} \qquad \text{(B.1)} \\ V_s^{\mathrm{m}}(B,S) \equiv{}& \xi\,\mathbb{E}_{\tilde{p}}\big[\,\alpha_b(\tilde{p})\,\tilde{p} + (1-\alpha_b(\tilde{p}))\,V_s(B,S)\,\big|\,\pi_s\,\big] \\ &+ (1-\xi)\,\mathbb{E}_{\tilde{p}}\big[\,\alpha_s(\tilde{p})\,\tilde{p} + (1-\alpha_s(\tilde{p}))\,V_s(B,S)\,\big|\,\pi_b\,\big] \qquad \text{(B.2)} \end{aligned}$$

instead of equations (3.2) and (3.3), where the continuation value of the type-θ trader conditional on some other traders being selected in the match is given by

$$\begin{aligned} V_\theta^{\mathrm{o}}(B,S) \equiv{}& \xi\,\mathbb{E}_{\tilde{p}}\big[\,\alpha_b(\tilde{p})\,V_\theta(B-1,S-1) + (1-\alpha_b(\tilde{p}))\,V_\theta(B,S)\,\big|\,\pi_s\,\big] \\ &+ (1-\xi)\,\mathbb{E}_{\tilde{p}}\big[\,\alpha_s(\tilde{p})\,V_\theta(B-1,S-1) + (1-\alpha_s(\tilde{p}))\,V_\theta(B,S)\,\big|\,\pi_b\,\big] \qquad \text{(B.3)} \end{aligned}$$

instead of by equation (3.4), and where V_θ^{a} satisfies equation (3.5).[20] It is convenient to set $V_b(0,S) = V_s(B,0) = 0$ for all $B \in \mathscr{B}$ and $S \in \mathscr{S}$, so both V_b and V_s have $\mathscr{B} \times \mathscr{S}$ as a domain.

The system of equations determining the continuation values buyers and sellers has a unique solution by the standard fixed-point argument. Indeed, we can replace V_b by $W_s \equiv 1 - V_b$ and interpret the previous equations as an operator which maps any pair of functions $(V_s, W_s) : \mathscr{B} \times \mathscr{S} \to \mathbb{R}^2$ to another pair of similar functions. It is then easy to verify that such an operator satisfies the sufficient Blackwell conditions for a contraction.

We use the principle of optimality to define our equilibrium concept. More concretely, we say that $\{(\pi_\theta,\alpha_\theta)\}_{\theta\in\{b,s\}}$ is a *symmetric Markov perfect equilibrium* if the corresponding continuation values—solving the system of equations $\big\langle(3.1),(\text{B}.1),(\text{B}.2),(\text{B}.3),(3.5)\big\rangle$—are such that, for each state (B,S) and $\theta \in \{b,s\}$, the pair $(\pi_\theta(B,S), \alpha_\theta(\cdot;B,S))$ maximizes the right-hand side of equation (B.1) if $\theta = b$ and right-hand side of (B.2) if $\theta = s$.

[20] As in the main text we keep keep notation short by omitting the explicit dependence of γ and ξ on the state (B,S), and we use $\alpha_\theta(\tilde{p})$ to denote $\alpha_\theta(\tilde{p};B,S)$.

B.2 Proofs of the results

Proposition 3.1

Proof. Fix an equilibrium. Standard arguments imply that if there is a positive probability that offers made by a seller are accepted in state (B, S), then the equilibrium probability that such offers are equal to $1 - V_b(B, S)$ is one. Similarly, an equilibrium offer by a buyer in state (B, S) is accepted with positive probability in equilibrium if and only if it is equal to $V_s(B, S)$. Since these offers make the receiver of the offer indifferent on accepting them or not, it is without loss of generality (to prove existence of equilibria) to focus on equilibria where, in state (B, S) and for all $\theta \in \{b, s\}$, buyers offer $V_s(B, S)$ and sellers offer $1 - V_b(B, S)$ for sure, and a θ-trader accepts the equilibrium offer with some probability $\alpha_\theta(B, S)$. Thus, equations (B.1) and (B.2) can be replaced by equations (3.2) and (3.3). Note that the continuation values of a seller and a buyer only depend on α_b and α_s through

$$\alpha \equiv (1-\xi)\,\alpha_b + \xi\,\alpha_s$$

(see equation (B.3)), with the convention that $\alpha(B, S) = 0$ whenever $B = 0$ or $S = 0$, and so equation (B.3) can be replaced by equation (3.4). Hence, equations (3.1)–(3.5) determine the continuation payoffs in an equilibrium.

Fix some $\alpha \in [0, 1]^{\mathscr{B}^* \times \mathscr{S}^*}$, interpreted as a putative equilibrium probability of trade. We can compute the equilibrium continuation value by solving equations in (3.1)–(3.5), and let $V_b(\cdot;\alpha)$ and $V_s(\cdot;\alpha)$ denote the corresponding solutions. Note also that a buyer and a seller are indifferent on accepting the equilibrium offer at state (B, S) if and only if $V_b(B, S;\alpha) + V_s(B, S;\alpha) = 1$. Hence, there is no $\theta \in \{b, s\}$ such that the θ-trader has a profitable deviation at a given state $(B, S) \in \mathscr{B}^* \times \mathscr{S}^*$ if and only if

$$\alpha(B,S) \in \begin{cases} \{0\} & \text{if } V_b(B,S;\alpha) + V_s(B,S;\alpha) > 1, \\ [0,1] & \text{if } V_b(B,S;\alpha) + V_s(B,S;\alpha) = 1, \\ \{1\} & \text{if } V_b(B,S;\alpha) + V_s(B,S;\alpha) < 1. \end{cases}$$

To see this assume, for example, that $V_b(B, S;\alpha) + V_s(B, S;\alpha) > 1$ and that $\alpha_s(B, S) > 0$ (if, instead, $\alpha_s(B, S) < 1$ the argument is analogous). If a buyer makes the equilibrium offer (equal to $V_s(B, S;\alpha)$) at state (B, S) he obtains

$$\begin{aligned} &\alpha\,(1 - V_s(B,S;\alpha)) + (1-\alpha)\,V_b(B,S;\alpha) \\ &\quad = V_b(B,S;\alpha) - \alpha\big(V_s(B,S;\alpha) + V_b(B,S;\alpha) - 1\big) < V_b(B,S;\alpha)\,. \end{aligned}$$

If, instead, he offers $V_s(B, S;\alpha) - \varepsilon$, for some $\varepsilon > 0$, the seller rejects the offer for sure, and so the buyer obtains $V_b(B, S;\alpha)$, which makes him strictly better off.

To conclude the proof of existence of equilibria, we define $A: [0,1]^{\mathscr{B}^* \times \mathscr{S}^*} \rightrightarrows [0,1]^{\mathscr{B}^* \times \mathscr{S}^*}$ as follows:

$$A(\alpha)(B,S) = \begin{cases} \{0\} & \text{if } V_b(B,S;\alpha) + V_s(B,S;\alpha) > 1, \\ [0,1] & \text{if } V_b(B,S;\alpha) + V_s(B,S;\alpha) = 1, \\ \{1\} & \text{if } V_b(B,S;\alpha) + V_s(B,S;\alpha) < 1. \end{cases}$$

Standard arguments apply to show that $A(\cdot)$ has a closed graph, and that $A(\alpha)$ is, for all $\alpha \in [0,1]$, non-empty and convex. Hence, the existence of equilibria follows from Kakutani's fixed point theorem.

Existence when $\bar{B} = \bar{S} = +\infty$. Fix some functions $\xi, \lambda, \gamma_s, \gamma_b : \mathbb{Z}_+^2 \to (0,1) \times \mathbb{R}_{++} \times \mathbb{R}_+ \times \mathbb{R}_+$, where γ_s and γ_b are bounded. Consider a sequence $(\bar{B}_n, \bar{S}_n)_n$ strictly increasing in both arguments. For each n, we can construct a model with a finite state space as follows ("the n-th model"). In the n-th model, ξ^n and λ^n coincide with ξ and λ, now with domain $\mathscr{B}^n \times \mathscr{S}^n \equiv \{0,\ldots,\bar{B}_n\} \times \{0,\ldots,\bar{S}_n\}$. The arrival rates in the n-th model is, for each $(B,S) \in \mathscr{B}^n \times \mathscr{S}^n$, are

$$\gamma_b^n(B,S) = \begin{cases} \gamma_s(B,S) & \text{if } B < \bar{B}^n, \\ 0 & \text{if } B = \bar{B}^n, \end{cases} \quad \text{and} \quad \gamma_s^n(B,S) = \begin{cases} \gamma_s(B,S) & \text{if } S < \bar{S}^n, \\ 0 & \text{if } S = \bar{S}^n. \end{cases}$$

For each n, let α^n characterize an equilibrium of the n-th model. Let $\mu : \mathbb{N} \to \mathbb{N}^n$ be a bijective ordering of $\mathbb{N}$. Initialize $(\alpha_0^n)_n = (\alpha^n)_n$. Then, for each $m \in \mathbb{N}$, we use $(\alpha_{m-1}^n)_n$ to recursively construct $(\alpha_m^n)_n$ as follows. Let $\underline{\alpha}_m$ be the minimum cluster point of $(\alpha_{m-1}^n(\mu(m)))_n$ (recall that the ser of cluster points of a sequence is closed). If there is an increasing subsequence of $(\alpha_{m-1}^n(\mu(m)))_n$ converging to $\underline{\alpha}_m$, then we let $(\alpha_m^n)_n$ be the biggest subsequence of $(\alpha_{m-1}^n)_n$ such that $(\alpha_{m-1}^n(\mu(m)))_n$ is increasing and converging to $\underline{\alpha}_m$. If no increasing subsequence of $(\alpha_{m-1}^n(\mu(m)))_n$ converging to $\underline{\alpha}_m$ exists, then there must exist a decreasing subsequence converging to of $(\alpha_{m-1}^n(\mu(m)))_n$ converging to $\underline{\alpha}_m$. In this case, $(\alpha_m^n)_n$ be the largest decreasing subsequence of $(\alpha_m^n)_n$ such that $(\alpha_{m-1}^n(\mu(m)))_n$ is decreasing and converges to $\underline{\alpha}_m$.

Note that, as explained above, for each m there exists some $\underline{\alpha}_m \in [0,1]$ and a subsequence $(\alpha_m^n)_n$ of $(\alpha^n)_n$ such that $(\alpha_m^n(\mu(m')))_n$ converges to $\underline{\alpha}_{m'}$ for all $m' \in \{1,\ldots,m\}$. It is then easy to prove that, in fact, $\alpha : \mathbb{N}^2 \to [0,1]$ given by $\alpha(B,S) = \underline{\alpha}_{\mu^{-1}(B,S)}(B,S)$ characterizes an equilibrium in the infinite model. □

Proof of Results 1-4

Proof. The proofs follow from the arguments in the main text. □

Proof Proposition 4.1

Proof. Throughout the proof we fix a sequence $(k_n)_n$ tending to $+\infty$ and, for each n, an equilibrium for the model in which the matching rate is $\lambda = k_n\,\ell$. For each n and fixed state (B,S), we let $V_{\theta,n} \equiv V_{\theta,n}(B,S)$ denote the continuation value of a θ-trader in the n-th equilibrium in state (B,S), for $\theta \in \{b,s\}$, and $\alpha_n \equiv \alpha_n(B,S)$ denote the probability of trade in a meeting in this equilibrium.

Preliminary result: We first note that using the standard analysis in Rubinstein (1982), we have that equation (4.5) holds for $V_{s,n}$. Indeed, for state $(B,S)=(1,1)$ we can write

$$V_{\theta,n} = \tfrac{k_n\,\ell}{k_n\,\ell+\gamma+r}\,(\xi_\theta\,(1-V_{\bar\theta,n}) + (1-\xi_\theta)\,V_{\theta,n}) + \tfrac{\gamma}{k_n\,\ell+\gamma+r}\,V^{\mathrm{a}}_{\theta,n}$$

for all $\theta\in\{b,s\}$, where $\xi_b = 1-\xi$ and $\xi_s = \xi$. The previous equations coincide with the equations for the continuation payoffs in a two-player Rubinstein bargaining where the "threat point" (i.e., value from not trading) for a θ-trader is $\frac{r}{\gamma+r}\,V^{\mathrm{a}}_{\theta,n}$. Solving for $V_{b,n}$ and $V_{s,n}$, it is easy to show that

$$\lim_{n\to\infty}\left|\tfrac{r}{\gamma+r}\,\xi_\theta + \tfrac{\gamma}{\gamma+r}\,V^{\mathrm{a}}_{\theta,n} - V_{\theta,n}\right| = 0\,. \tag{B.4}$$

Defininitions: For each n, define a function $\tilde V_{\theta,n} : \mathscr{B}\times\mathscr{S} \to [0,1]$, interpreted as the payoff of a θ-trader when he/she decides to trade only when the market is balanced, as follows. It is obtained solving equations (3.1), (3.4) and (3.5) (adding tildes to all V's) and, instead of equations (3.2) and (3.3), and requiring that $\tilde V^{\mathrm{m}}_\theta = \tilde V_\theta$ when $B\neq S$ (no trade when the market is imbalanced) and $\tilde V_\theta{}^{\mathrm{m}} = V^{\mathrm{m}}_\theta$ when $B=S$ (trade for sure when the market is balanced). Note that for $\theta=s$ and $S\neq B$, equation (3.1) can be rewritten as equation (4.3) replacing "$\simeq$" by "$=$", adding tildes to all V's, and replacing α by α_n. We can obtain an analogous equation for $\tilde V_{b,n}$.

Define $W_n(B,S)$ to be equal to $\tilde V_{s,n}(B,S)$ when $B\le S$, and to be equal to $1-\tilde V_{b,n}(B,S)$ when $B>S$. Our goal is to show that, independently of the choice of the pair sequences $(k_n)_n$ and corresponding equilibria, W_n is approximated by the right-hand side of equation (4.2) as $n\to\infty$, and that

$$\lim_{n\to\infty}|W_n - V_{s,n}| = 0 \text{ for all } B\le S \;\text{ and }\; \lim_{n\to\infty}|W_n - V_{b,n}| = 0 \text{ for all } B>S\,.$$

In our notation, $W \simeq 1-V_b$ for all $B\le S$ and $W\simeq 1-V_b$ for all $B>S$. (As in the main text, "$\simeq$" means equal except for terms that go to 0 as n increases.)

Fix a state (B,S) satisfying that $B=S\ge 1$. We have that for all $\varepsilon>0$ there is some n such that $\left|\tilde V_{s,n}(B,S) - V_{s,n}(B,S)\right| < \varepsilon$. To see this, recall that by Result 2 there is immediate trade when the market is balanced. Also, by Result 4, $V_b \simeq 1-V_s$. Hence, we have

$$\begin{aligned} V_s(B,S) &\simeq \tfrac{1}{S}\,V_s(B,S) + \tfrac{S-1}{S}\,V_s(B-1,S-1)\\ &\Rightarrow\; V_s(B,S)\simeq V_s(B-1,S-1)\simeq \ldots \simeq V_s(1,1)\,. \end{aligned}$$

Proceeding similarly, we have that it is also the case that $W = \tilde{V}_s \simeq V_s(1,1)$ because, as n increases, it is increasingly unlikely that an arrival happens before the market clears. Hence, we have $V_s(1,1) \simeq V_s^{\mathrm{m}}(1,1) \simeq \tilde{V}_s(1,1)$, and so $V_s \simeq \tilde{V}_s = W_s$ when $B = S$.

For each n, let $D_{s,n}$ denote the maximum distance between $V_{s,n}$ and $\tilde{V}_{s,n}$ among all states where $B \le S$, and let $D_{b,n}$ denote the maximum distance between $V_{b,n}$ and $\tilde{V}_{b,n}$ among all states where $B > S$. Let $D_n \equiv \max\{D_{b,n}, D_{s,n}\}$.

Case 1: Assume first $\limsup_{n\to\infty} D_n = 0$ for all sequences $(k_n)_n$ tending to $+\infty$ and corresponding equilibria. In this case, we have that $W \simeq 1 - V_b$ for all $B \le S$ and $W \simeq 1 - V_b$ for all $B > S$. Furthermore, in this case we can rewrite (B.4) as

$$W(1,1) \simeq \tfrac{r}{\gamma+r}\,\xi(1,1) + \tfrac{\gamma}{\gamma+r}\,W^{\mathrm{a}}(1,1)\,.$$

Hence, W satsifies equations (4.3)–(4.5) replacing all V_s's by the corresponding W's. As it is argued in the main text, this implies that W satisfies equation (4.2).

Case 2: Now assume, for the sake of contradiction, that $\limsup_{n\to\infty} D_n > 0$, and without loss of generality and for simplicity (considering a subsequence if necessary), assume that $\lim_{n\to\infty} D_n > 0$. Assume also, taking a subsequence if necessary, there is a state (B,S) such that $B \le S$ (the other case is analogous) and $D_n = |V_{s,n}(B,S) - \tilde{V}_{s,n}(B,S)|$ for all n.[21] We can then write, for each n,

$$D_n \le \tfrac{\frac{1}{S}\,k_n\,\ell}{k_n\,\ell+\gamma+r}\,|\tilde{V}^{\mathrm{m}}_{s,n} - V^{\mathrm{m}}_{s,n}| + \tfrac{\frac{S-1}{S}\,k_n\,\ell}{k_n\,\ell+\gamma+r}\,D_n + \tfrac{\gamma}{k_n\,\ell+\gamma+r}\,D_n\,.$$

We first rule out that $B = S$. Indeed, assuming for the sake of contradiction that $B = S$ we have that, as we argued, $V_s \simeq \tilde{V}_s$. In this case, we have that $\lim_{n\to\infty} D_n = 0$, but we assumed that $\lim_{n\to\infty} D_n > 0$.

We assume then, from now on, that $B < S$. Therefore, $\tilde{V}^{\mathrm{m}}_{s,n} = \tilde{V}_{s,n}$, and we can write

$$D_n \le \tfrac{\frac{1}{S}\,k_n\,\ell}{\frac{1}{S}\,k_n\,\ell+r}\,|\tilde{V}_{s,n} - V^{\mathrm{m}}_{s,n}|\,.$$

There are three cases:

1. Assume first that there is a subsequence indexed by $(n_i)_i$ such that, for each i, there is trade delay at state (B,S) in the n_i-th equilibrium. In this case, $V^{\mathrm{m}}_{s,n_i} = V_{s,n_i}$. Nevertheless, this implies

$$D_{n_i} \le \tfrac{k_{n_i}\,\ell+\gamma}{k_{n_i}\,\ell+\gamma+r}\,D_{n_i} \;\;\Rightarrow\;\; D_{n_i} = 0\,.$$

   This contradicts that $\lim_{n\to\infty} D_n > 0$.

[21] Given that the number of states is finite, it is always possible to find a constant subsequence.

2. Assume now that $B = 0$ (and so $\ell(B,S) = 0$). In this case we have

$$D_n \leq \tfrac{\gamma}{\gamma+r} D_n \;\;\Rightarrow\;\; D_n = 0\,.$$

This is, again, a contradiction.

3. We then have that, without loss of generality, we can assume $S > B > 0$ and that there is trade for sure in state (B,S) in the n-th equilibrium for all n. Assume, taking a subsequence if necessary, that for each state $(B',S') \in \mathscr{B}^* \times \mathscr{S}^*$ the agreement rate at such a state, equal to $\alpha_n(B',S')\,k_n\,\ell(B',S')$ tends to some value $\delta(B',S') \in [0,+\infty]$ as n increases (with the convention that $\alpha_n(B',S') = 0$ when $B' = 0$ or $S' = 0$). We let $m \leq \bar{S}$ denote the minimal natural number such that $\delta(B-m,S-m) \neq \infty$. Note that $m > 0$ since $S > B > 0$ and, for all n, there is trade for sure in state (B,S) in the n-th equilibrium (and so $\alpha_n(B,S) = 1$). Then, as $n \to \infty$,

$$\tilde{V}_{s,n}(B,S) \simeq \tilde{V}_{s,n}(B-m,S-m)$$

where, with some abuse of notation, in this formula and in the rest of the proof "$\simeq$" means equal except for terms that go to 0 as $n \to \infty$ (note that, differently from our main text, now the sequence of equilibria is fixed). Similarly, by Result 4, we have that, $V_{s,n} \simeq 1 - V_{b,n}$, and therefore

$$V_{s,n}(B,S) \simeq \tfrac{1}{S}\,V_{s,n}(B,S) + \tfrac{S-1}{S}\,V_{s,n}(B-1,S-1)\,.$$

Thus, $V_{s,n}(B,S) \simeq V_{s,n}(B-1,S-1)$ and, proceeding iteratively, $V_{s,n}(B,S) \simeq V_{s,n}(B-m,S-m)$. If $B-m > 0$ then we have

$$\begin{aligned} D_n &\simeq \left|V_{s,n}(B-m,S-m) - \tilde{V}_{s,n}(B-m,S-m)\right| \\ &\precsim \underbrace{\frac{\frac{1}{S-m}\,\delta}{\delta+\gamma+r}\,D_n + \frac{\frac{S-m-1}{S-m}\,\delta}{\delta+\gamma+r}\,D_n + \frac{\gamma}{\delta+\gamma+r}\,D_n}_{=(*)} \simeq \frac{\delta+\gamma}{\delta+\gamma+r} D_n\,, \end{aligned}$$

where δ and γ are evaluated at $(B-m,S-m)$, and where "$\precsim$" means that the left-hand side is lower than the right-hand side plus terms that go to 0 as n increases.[22] Thus, $D_n \simeq 0$, which is a contradiction. Therefore, it is the case that $B-m = 0$, so we have

$$D_n \precsim \tfrac{\gamma}{\gamma+r} D_n \;\;\Rightarrow\;\; D_n \simeq 0$$

where γ is evaluated at $(0,S-B)$, but this is again a contradiction.

□

[22]Formally, the equation should be interpreted as meaning that, for all $\varepsilon > 0$, there is an $\bar{n}$ such that if $n > \bar{n}$ then $D_n \leq (*) + \varepsilon$ in all equilibria.

Proof Corollary 4.1

Proof. We first show that equation (4.7) holds. We do this for some fixed $S_t > B_t$ (the other case is proven analogously). Note that, for each n, we have

$$V_{s,t} \simeq \tilde{\mathbb{E}}_t\Big[\int_t^{t+\Delta} e^{-r\,t}\left(\mathbb{I}_{B_t>S_t} + \xi(1,1)\,\mathbb{I}_{B_t=S_t}\right) r\,\mathrm{d}t\Big] + e^{-r\,\Delta}\,\tilde{\mathbb{E}}_t[V_{s,t+\Delta}]\ .$$

The probability that the balancedness of the market changes in $[t, t+\Delta]$ is $1-e^{-\bar{\gamma}_b\,\Delta}$, where $\bar{\gamma}_b$ is the highest γ_b among all states. Hence, the first term on the right-hand side of the previous expression is no higher than

$$(1-e^{-\bar{\gamma}_b\,\Delta})\,(1-e^{-r\,\Delta})\ .$$

This term decreases as Δ^2 as $\Delta \to 0$. As a result, equation (4.7) holds.

We now prove the inequality in the statement of the corollary. As in the proof of Proposition 4.1 (see its first paragraph), we fix a sequence $(k_n)_n$ tending to $+\infty$ and, for each n, an equilibrium for the model with $\lambda = k_n\,\ell$. Also, s in the proof of Proposition 4.1, assume without loss of generality that $\alpha_n(B', S')\,k_n\,\ell(B', S')$ tends to some value $\delta(B', S') \in [0, +\infty]$ as n increases. Fix some time t and state (B_t, S_t). Assume without loss of generality that $B_t < S_t$. Let $m \in \{0, ..., B_t\}$ be as defined in the proof of Proposition 4.1 as the lowest value such that $\delta(B_t, S_t) = +\infty$. Recall that

$$V(B_t, S_t) \simeq V(B_t-m, S_t-m)\ .$$

If $m = B_t$ then we have, the limit equilibrium and risk-neutral measures coincide in state $(0, S_t - B_t)$. This implies that we can write

$$\tilde{\mathbb{E}}_t[V_{s,t+\Delta}] \simeq \left(1-\gamma_t\,\Delta\right) V_{s,t} + \gamma_t\,\Delta\,V^{\mathrm{a}}_{s,t} + o(\Delta) \simeq \mathbb{E}_t[V_{s,t+\Delta}]$$

where all terms on the term between the equalities are evaluated at $(0, S_t - B_t)$. Hence, the result holds when $m = B_t$. If, instead, $m < B_t$ then there is delay in state (B_t-m, S_t-m) in the limit equilibrium measure; that is,

$$\delta(B_t-m, S_t-m) \equiv \lim_{n\to\infty} \alpha_n(B_t-m, S_t-m)\,k_n\,\ell(B_t-m, S_t-m) < +\infty\ .$$

This implies that

$$\begin{aligned}
\tilde{\mathbb{E}}_t[V_{s,t+\Delta}] &\simeq \left(1-\left(\tfrac{S_t-m-1}{S_t-m}\,\delta_t+\gamma_t\right)\Delta\right) V_{s,t}+\tfrac{S_t-m-1}{S_t-m}\,\delta_t\,\Delta\,V_{s,t}(B_t-m-1, S_t-m-1)+\gamma_t\,\Delta\,V^{\mathrm{a}}_{s,t}+o(\Delta)\\
\mathbb{E}_t[V_{s,t+\Delta}] &\simeq \left(1-(\delta_t+\gamma_t)\,\Delta\right) V_{s,t}+\delta_t\,\Delta\,V_{s,t}(B_t-m-1, S_t-m-1)+\gamma_t\,\Delta\,V^{\mathrm{a}}_{s,t}+o(\Delta)
\end{aligned}$$

where all variables without an explicit state dependence are evaluated at (B_t-m, S_t-m). Given that for all n high enough there is delay in state (B_t-m, S_t-m), Result 3 implies that $V_{s,t}(B_t-m-1, S_t-m-1) \geq V_{s,t}$. Hence, our result holds. □

Proof of Proposition 4.2

Proof. We begin the proof with a result analogous to Proposition 4.2 in a simplified setting.

Lemma B.1. *Assume, with some abuse of notation, that $\gamma_\theta(B,S) = \gamma_\theta(S-B)$ for all states (B,S) and types $\theta \in \{b,s\}$. Then, there exists some $\bar{k}$ such that if $k > \bar{k}$ then there is no equilibrium with trade delay. There exists an increasing function $p : \mathbb{Z} \to [0,1]$ such that $V_s(B,S) \simeq p(S-B)$ for all states (S,B).*

Proof. We assume, for the sake of contradiction, that there is a sequence $(k_n)_n$, a corresponding sequence of equilibria and a sequence of states (B_n, S_n) such that, in the n-th equilibrium, equilibrium offers are rejected with positive probability. For each state (B,S), we use $V_{b,n}$ and $V_{s,n}$ to denote the continuation values of buyers and sellers in the n-th equilibrium, and α_n to denote the probability of acceptance of an equilibrium offer (so $\alpha_n(B_n, S_n) < 1$ for all n).

Taking a subsequence if necessary, assume that, for each state (B,S) the continuation values $V_{b,n}(B,S)$ and $V_{s,n}(B,S)$ converge to some values $V_b(B,S)$ and $V_s(B,S)$, and the matching rates $\alpha_n\, k_n\, \ell$ converge to some value $\delta \equiv \delta(B,S) \in [0,+\infty]$. We further assume, taking again a subsequence if necessary, that for each state (B,S) with $B,S > 0$, either $\alpha_n < 1$ for all n, or $\alpha_n = 1$ for all n.

We first focus on characterizing the limit continuation value of seller, V_s, for states (B,S) is such that $0 \le B \le S$. The equations are given by:

1. Consider first the case where (B,S) is such that $S > B \ge 1$ and $\alpha_n < 1$ for all n. Using equation (3.1) we have that the limit continuation value for a seller, V_s, satisfies

$$(V_s - V_s(B-1, S-1))\,\delta = -\tfrac{B\,S}{S-B}\, r \;.$$

   It is then clear that there is no state where $\delta = 0$, that is, where trade occurs at a rate that becomes arbitrarily small as k increases. (The logic for this result is analogous to that of Result 1.) Using this, we can use again equation (3.1) to obtain

$$V_s = \tfrac{\gamma}{\gamma+r}\, V_s^{\mathrm{a}} + \tfrac{r}{\gamma+r}\, \tfrac{B\,(S-1)}{S-B} = V_s(B-1, S-1) - \tfrac{r}{\delta}\, \tfrac{B\,S}{S-B} \;. \tag{B.5}$$

   Note that the second equality implies that, as indicated in Result 3, traders on the long side of the market gain from other's transactions in states where is trade delay, $V_s(B-1, S-1) > V_s$.

2. Consider now the case where (B,S) is such that $B,S > 0$ and $\alpha_n = 1$ for all n. In this case, if $S > 1$, we have

$$V_s = V_s(B-1, S-1) \;.$$

   Note that, by Result 2, this is the case for states with $B = S$.

3. Finally, for states where $B = 0$ we have

$$V_s = \tfrac{\gamma}{\gamma+r}\, V_s^{\mathrm{a}}\ .$$

Let, for each state (B,S) with $0 \le B < S$, $\Delta \equiv V_s(0,S{-}B) - V_s$, and $\Delta = 0$ for each state (B,S) with $B = S$. Since, when $B \ge 1$, we have $V_s(B{-}1,S{-}1) \ge V_s$, it is the case that $\Delta \ge 0$ for all states. Let (B,S) be a state which maximizes $\Delta(B',S')$ among all states with $B' \le S'$ and assume, for the sake of contradiction, that $\Delta \equiv \Delta(B,S) > 0$ (so necessarily $0 < B < S$). If there are multiple states with this property, assume that (B,S) is such that S is minimal among all of them. Assume first that (B,S) is such that $\alpha_n = 1$ for all n. In this case, since $V_s(B{-}1,S{-}1) = V_s$ (by part 2 in the previous argument), we have

$$\Delta = V_s(0,S{-}B) - V_s(B{-}1,S{-}1) = \Delta(B{-}1,S{-}1)\ .$$

This contradicts the assumption that (B,S) is a state with a minimal number of sellers among those which maximize Δ. Then, it is necessarily the case that (B,S) is such that $\alpha_n < 1$ for all n. In this case, we have that, using equation (B.5),

$$\begin{aligned}\Delta &= \tfrac{\gamma_b}{\gamma+r}\,\overbrace{(V_s(1,S{-}B) - V_s(B{+}1,S))}^{\le\Delta(B+1,S)} + \tfrac{\gamma_s}{\gamma+r}\,\overbrace{(V_s(0,S{-}B{+}1) - V_s(B,S{+}1))}^{=\Delta(B,S+1)} + \tfrac{r}{\gamma+r}\,\tfrac{B\,(S-1)}{S-B} && \text{(B.6)}\\ &< \tfrac{\gamma}{\gamma+r}\,\Delta\ , && \text{(B.7)}\end{aligned}$$

where the inequality holds because $V_s(1,S{-}B) \le V_s(0,S{-}B{-}1)$ and $\frac{B\,(S-1)}{S-B} > 0$. This is a clear contradiction. Therefore, we have that, for all states (B,S), $V_s = V_s(0,S{-}B)$ and that $\alpha_n(B,S) = 1$ if n is high enough.

We now prove that p exists satisfying the conditions in the statement. We use $p(\cdot)$ to denote the solution of equations

$$\begin{aligned} p(N) &= \tfrac{\gamma_b(N)}{\gamma(N)+r}\, p(N{-}1) + \tfrac{\gamma_s(N)}{\gamma(N)+r}\, p(N{+}1) && \text{if } N > 0\ , && \text{(B.8)}\\ p(N) &= \tfrac{r}{\gamma(N)+r} + \tfrac{\gamma_b(N)}{\gamma(N)+r}\, p(N{-}1) + \tfrac{\gamma_s(N)}{\gamma(N)+r}\, p(N{+}1) && \text{if } N < 0\ , && \text{(B.9)}\\ p(0) &= \tfrac{r}{\gamma(0)+r}\,\xi(1,1) + \tfrac{\gamma_b(0)}{\gamma(0)+r}\, p(-1) + \tfrac{\gamma_s(0)}{\gamma(0)+r}\, p(1)\ . && && \text{(B.10)}\end{aligned}$$

The function $p(\cdot)$ can be proven to be unique using standard fixed-point arguments similar to those in Section B.1. These equations approximate equations (4.3)–(4.5) replacing $V_s(B,S)$ by $p(S-B)$, so it is clear that $V_s(B,S) \simeq p(S-B)$ for all states (B,S). Furthermore, for each $\bar{N} \ge 0$, one can rewrite equation (4.9) for all $N \ge \bar{N}$ as

$$p(N) = \mathbb{E}[e^{-r\,\bar{\tau}}\,|N_0{=}N]\, p(\bar{N})$$

where $\bar{\tau}$ is the stochastic time it takes the net supply to reach $\bar{N}$ for the first time. It is then clear that $p(\cdot)$ is decreasing on $\mathbb{Z}_+$. A similar argument shows that it is also the case that $p(\cdot)$ is decreasing on $\mathbb{Z}_-$. $\square$

(Continuation of the proof of Proposition 4.2)

The proof of Proposition 4.2 analogous to the Lemma B.1 until equation (B.6). Now, in equation (B.6), the arrival rate of θ-traders into the market are $\gamma_\theta \equiv \gamma_\theta(B,S)$ in state (B,S), and $\gamma_\theta(0,S-B)$ in state $(0,S-B)$, which are potentially different. Then, equation (B.6) is now replaced by

$$\begin{aligned}
\Delta &= \tfrac{\gamma_b}{\gamma+r}\,V_s(B+1,S) + \tfrac{\gamma_s}{\gamma+r}\,V_s(B,S+1) + \tfrac{r}{\gamma+r}\,\tfrac{B\,(S-1)}{S-B} \\
&\quad - \big(\tfrac{\gamma_b(0,S-B)}{\gamma(0,S-B)+r}\,V_s(1,S-B) + \tfrac{\gamma_s(0,S-B)}{\gamma(0,S-B)+r}\,V_s(0,S-B+1)\big) \\
&\geq \tfrac{\gamma}{\gamma+r}\,\Delta \\
&\quad + \underbrace{\big(\tfrac{\gamma_b}{\gamma+r} - \tfrac{\gamma_b(0,S-B)}{\gamma(0,S-B)+r}\big)V_s(0,S-B-1) + \big(\tfrac{\gamma_s}{\gamma+r} - \tfrac{\gamma_s(0,S-B)}{\gamma(0,S-B)+r}\big)V_s(0,S-B+1) + \tfrac{r}{\gamma+r}\,\tfrac{B\,(S-1)}{S-B}}_{=(*)}\ .
\end{aligned}$$

Hence, a sufficient condition for the statement to hold is that the term $(*)$ in the previous equation is positive. Using Condition 1 we have that, for each $\theta \in \{s,b\}$,

$$\tfrac{\gamma_\theta}{\gamma+r} \geq \tfrac{\gamma_\theta(0,S-B)}{\gamma(0,S-B)+r} - \tfrac{r}{\gamma+r}\,\tfrac{B}{2}\ .$$

So

$$(*) \geq -\tfrac{r}{\gamma+r}\,B + \tfrac{r}{\gamma+r}\,\tfrac{B\,(S-1)}{S-B} > 0\ .$$

Thus, Condition 1 is sufficient to guarantee that, if k is high enough, there is no equilibrium with trade delay. □

Proof of Corollary 4.2

Proof. Consider an increase on the discount rate from r_1 to r_2, with $r_1 < r_2$, and let $p^{r_i}(\cdot)$ denote the market price function for each r_i, $i = 1,2$. Assume that $p^{r_1}(0) \geq p^{r_2}(0)$ (the reverse case is analogous). In this case, for all $N > 0$ the price $p^{r_1}(N) > p^{r_2}(N)$. Indeed, using τ_0 to denote the (stochastic) time it takes for the market to become balanced (which is independent of r) and using equation (4.9), we can write

$$p^{r_1}(N) = \mathbb{E}\big[\,e^{-r_1\tau_0}\,\big|\,N_0 = N\big]\,p^{r_1}(0) > \mathbb{E}\big[\,e^{-r_2\tau_0}\,\big|\,N_0 = N\big]\,p^{r_2}(0) = p^{r_2}(N)\ . \tag{B.11}$$

Let $\bar{N}$ be the maximum value satisfying $p^{r_1}(\bar{N}) < p^{r_2}(\bar{N})$. Notice that equation (4.9) can be rewritten, for any $N \leq \bar{N} < 0$ and $i \in \{1,2\}$, as

$$p^{r_i}(N) = 1 - \mathbb{E}\big[\,e^{-r_i\bar{\tau}}\,\big|\,N_0 = N\big]\,(1 - p^{r_i}(\bar{N}))$$

where $\bar{\tau}$ is the first time where $N_t = \bar{N}$. It is then clear, using equation (B.11) and $p^{r_1}(\bar{N}) < p^{r_2}(\bar{N})$, that for all $N \leq \bar{N}$ we have $p^{r_1}(N) < p^{r_2}(N)$. Thus, in fact, $\bar{N}$ is such that

$$p^{r_1}(N) \geq p^{r_2}(N) \text{ for all } N > \bar{N} \text{ and } p^{r_1}(N) < p^{r_2}(N) \text{ for all } N \leq \bar{N}\ .$$

This property (and the fact that the ergodic distribution of N is independent of the discount rate) ensures that the distribution of $p^{r_2}(N)$ is a spread of $p^{r_1}(N)$.

Assume now that $\gamma_b(0,0)=\gamma_b(1,1)$ and $\gamma_s(0,0)=\gamma_s(1,1)$. In this case, the limit ergodic distributions of N under both the equilibrium measure and the risk-neutral measure coincide. Let F be such a distribution. Then, the expected price under such a distribution is

$$\mathbb{E}[p(\tilde{N})|F] = \sum_{\tilde{N}\in\mathbb{Z}} F(\{\tilde{N}\})\, p(N) \ .$$

It is also the case that

$$\begin{aligned}\lim_{t\to\infty} \mathbb{E}[p_t] = \mathbb{E}[p(\tilde{N})|F] &= \lim_{t\to\infty} \mathbb{E}\Big[\int_t^{\infty} e^{-r\,(s-t)}\, (\mathbb{I}_{N_s<0}+\xi(1,1)\,\mathbb{I}_{N_s=0})\, r\, \mathrm{d}s \Big] \\ &= \mathbb{E}\big[I_{\tilde{N}<0}+\xi(1,1)\,\mathbb{I}_{\tilde{N}=0} \big| F\big] = F(-\mathbb{N}) + \xi(1,1)\, F(\{0\}) \ ,\end{aligned}$$

where $-\mathbb{N}$ is the set of strictly negative integers. This proves that the ergodic mean of the market price is independent of r. □

Proof of Propositon 4.3

Proof. Similar to the proof of Proposition 4.1, fix a sequence $(k_n, M_n)_n$ tending to $(+\infty,+\infty)$ and, for each n, an equilibrium of the model where $k = k_n$ and $(\gamma_{b,n},\gamma_{s,n}) = M_n\,(\tilde{\gamma}_b,\tilde{\gamma}_s)$. For each state (B,S) with $B,S>0$, we use $\tilde{\delta}_n(B,S)$ the trade rate at such state in the n-th equilibrium under the risk-neutral measure as in equation (4.1). Taking a subsequence if necessary, assume that $(\tilde{\delta}_n(B,S))_n$ tends to some limit $\tilde{\delta}(B,S)\in\mathbb{R}_+\cup\{+\infty\}$ for all states (B,S). Define $\varepsilon_n^{\mathrm{a}} \equiv 1-V_n^{\mathrm{a}}$ and $\varepsilon_n^{\mathrm{t}} \equiv 1-V_n(B-1,S-1)$. Then, from equation (3.6) we obtain that, when $B<S$ and $\tilde{\delta}_n(B,S)>0$,

$$V_{s,n}-V_{s,n}(B-1,S-1) = -\frac{\frac{(B-1)\,S}{S-1}\,\tilde{\delta}_n\,\varepsilon_n^{\mathrm{a}} + B\,(\gamma_n\,\varepsilon_n^{\mathrm{t}}+r)}{\frac{S-B}{S-1}\,\tilde{\delta}_n} \in [-1,1] \ . \tag{B.12}$$

By Result 4 we have that $\varepsilon_n^{\mathrm{a}}\to 0$ and $\varepsilon_n^{\mathrm{t}}\to 0$. If, for example, $B<S$ then the previous equation implies $\tilde{\delta}_n \geq \frac{B\,(S-1)}{S-B}\, r$ for each n.

As in the main text, we use τ_0 to denote the stopping time until the market is balanced. Furthermore, for each state (B,S), we denote

$$\phi_n(B,S) \equiv 1-\tilde{\mathbb{E}}\big[e^{-r\,\tau_0}\big|(B_0,S_0)=(B,S)\big] \in [0,1]$$

for each n, where the expectation is computed using the risk-neutral measure of the n-th equilibrium. Note that if $B=S$ then $\phi_n(B,S)=0$. Also, it satisfies the equation

$$\phi_n(B,S) = \tfrac{r}{\tilde{\delta}_n+\gamma+r} + \tfrac{\tilde{\delta}_n}{\tilde{\delta}_n+\gamma_n+r}\,\phi_n(B-1,S-1) + \tfrac{\gamma_{b,n}}{\tilde{\delta}_n+\gamma_n+r}\,\phi_n(B+1,S) + \tfrac{\gamma_{s,n}}{\tilde{\delta}_n+\gamma_n+r}\,\phi_n(B,S+1) \tag{B.13}$$

where, if $B = 0$ or $S = 0$, $\tilde{\delta}_n$ should be replaced with 0. Take a subsequence of our original sequence such that $(\phi_n(B,S))_n$ is converging to some $\phi(B,S)$ for all states of the world. Assume, for the sake of contradiction, that $\bar{\phi} \equiv \max_{(B',S')} \phi(B',S') > 0$. Assume $\phi(B',S') = \bar{\phi}$ for some state (B',S') with $B' < S'$ (the other case is analogous). Let (B,S) such that $B < S$ satisfying that, for all other states (B',S') with $B' < S'$ and $\phi(B',S') = \bar{\phi}$, (i) $S' - B' \geq S - B'$, and (ii) if $S' - B' = S - B$ then $S' < S$. Thus, (B,S) has a minimal excess supply among all states with maximal ϕ and, among those with lowest excess supply, has minimal number of sellers. If $B = 0$ then we can write (B.13) as

$$\underbrace{\phi_n(B,S)}_{\to\bar{\phi}} = \underbrace{\tfrac{r}{\gamma_n}\,(1-\phi(B,S))}_{\to 0} + \underbrace{\tfrac{\gamma_{b,n}}{\gamma}}_{>0}\,\underbrace{\phi_n(B+1,S)}_{\to<\bar{\phi}} + \tfrac{\gamma_{s,n}}{\gamma_n}\,\underbrace{\phi_n(B,S+1)}_{\to\leq\bar{\phi}}\,,$$

where we used that by assumption $\gamma_b > 0$ (since $B < S$) and $S-(B+1) < S-B$. This is a clear contradiction. Assume, on the contrary, that $B > 0$. In this case, since $(S-1)-(B-1) = S-B$ we have $\phi(B-1,S-1) < \bar{\phi}$. We can write (B.13) as

$$\underbrace{\phi(B,S)}_{\to\bar{\phi}} = \underbrace{\tfrac{r}{\tilde{\delta}_n+\gamma_n}\,(1-\phi(B,S))}_{\to 0} + \tfrac{\tilde{\delta}_n}{\tilde{\delta}_n+\gamma_n}\,\underbrace{\phi_n(B-1,S-1)}_{\to<\bar{\phi}} + \tfrac{\gamma_{b,n}}{\tilde{\delta}_n+\gamma_n}\,\underbrace{\phi_n(B+1,S)}_{\to<\bar{\phi}} + \tfrac{\gamma_{s,n}}{\tilde{\delta}_n+\gamma_n}\,\underbrace{\phi_n(B,S+1)}_{\to\leq\bar{\phi}}\,.$$

It is then clear that we reach, again, a contradiction.

The previous argument implies that $\phi(B,S) = 0$ for all states (B,S); that is, the discounting until the market balances (under the risk-neutral measure) is 1. Using equation (4.6), we have then that $\lim_{n\to\infty} V_{s,n}(B,S) \to \lim_{n\to\infty} V_{s,n}(1,1)$ for all states (B,S). Hence, by equation (B.12), $\lim_{n\to\infty} \frac{r}{\tilde{\delta}_n} = 0$ for all states. This implies that delay vanishes as $n \to 0$. □

Proof of Corollary 4.3

Proof. As in the proof of Proposition 4.3, fix a sequence $(k_n, M_n)_n$ tending to $(+\infty,+\infty)$ and, for each n, an equilibrium of the model where $k = k_n$ and $(\gamma_{b,n},\gamma_{s,n}) = M_n\,(\gamma_b,\gamma_s)$. Note that all equilibria in the model with parameters $(\lambda, M_n\gamma_b, M_n\gamma_s, r)$ coincide with equilibria of the model with parameters $(\lambda/M_n, \gamma_b, \gamma_s, r/M_n)$. Furthermore, by Proposition 4.3, the trade rate under the risk-neutral measure $\tilde{\delta}_n$ is such that $\lim_{n\to\infty} \frac{r}{\tilde{\delta}_n} = 0$ for all states. Hence, as n increases, the discounted time it takes for the distribution of (S_t, B_t) to approximate the ergodic distribution shrinks to 0. It is then clear that the result holds. □

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