Spurious Regressions in Financial Economics?

Wayne E. Ferson
University of Washington and NBER

Sergei Sarkissian
McGill University

Timothy Simin
University of Washington

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ABSTRACT

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Ferson and Simin are from the University of Washington School of Business Administration, Department of Finance and Business Economics, Box 353200, Seattle, Washington 98195-3200. Ferson may be reached at ph: (206) 543-1843, fax 685-9392, http://faculty.washington.edu/apmodels. Sarkissian may be reached at Faculty of Management, 1001 Sherbrooke Street West, McGill University, Montreal, PQ, Canada H3A 1G5, ph. (514) 398-4876, fax 398-3876, ss@management.mcgill.ca. Simin may be reached at tsimin@u.washington.edu, (206) 543-4773. We are grateful to Eugene Fama for suggesting the question that motivates this research. Ferson acknowledges financial support from the Pigott-Paccar professorship at the University of Washington.

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ABSTRACT

We study biases associated with regression models in which persistent lagged variables predict stock returns, either linearly or in interaction with contemporaneous values of a market index return. We focus on the issue of spurious regression, related to the classic studies of Yule (1926) and Granger and Newbold (1974). We find that spurious regression is a concern in regressions of stock returns on persistent lagged instruments, especially when the predictable component of returns is large. In regressions where the lagged instruments interact with a market index return, the spurious regression problem is not as severe. Without persistent time-variation in the expected market return and beta, spurious regression bias is not an important issue. However, when a common persistent factor drives expected market returns and betas, spurious regression becomes a concern. Large sample sizes are no defense against the spurious regression bias.

Researchers have long found the stock market to be a fascinating context for empirical modeling. Predictive models for common stock returns, in particular, hold an obvious appeal. The conditional asset pricing literature in finance focuses on models in which stock returns are assumed to be predictable based on lagged instrumental variables. Examples of such variables include the levels of short-term interest rates, dividend-to-price ratios for stock market indexes, and yield spreads between low-grade and high-grade corporate bonds or between long- and short-term Treasury bonds.[1] Many of these variables behave as highly persistent time series.

This paper studies the sampling properties of stock return regressions with persistent lagged regressors. In particular, we focus on spurious regression phenomena analogous to that in Yule (1926) and Granger and Newbold (1974). These studies warned that spurious regression relations may be found between the levels of independent, trending time series. For example, given two independent near random walks, it is likely that an OLS regression of one on the other would produce a "significant" slope coefficient, based on the usual statistics. In the asset pricing context that motivates this paper, researchers use persistent time-series such as the instruments described above to model time-variation in assets' expected returns and betas. Unlike the regressions in levels studied by Yule (1926) and Granger and Newbold (1974), in our regressions the dependent variables are asset rates of return, which are not highly persistent. However, the returns are considered to be the sum of an unobserved expected return, plus an unpredictable noise. If the "true" expected returns are persistent, but an unrelated instrument is chosen, there is a risk of finding a spurious regression. However,

[1] One representative study for each type of variable follows, listed in the order the variables are mentioned: Fama and Schwert (1977), Fama and French (1988), Keim and Stambaugh (1986) and Fama and French (1989).

because the unpredictable noise represents a substantial portion of the variance of stock returns, the extent of the spurious regression problem is not a priori obvious. It is therefore important to assess to what extent regression models for stock returns with persistent regressors, as have appeared in numerous recent studies, may be susceptible to spurious regression effects.[2]

In asset pricing model applications, such as the Capital Asset Pricing Model [Sharpe (1964)], Intertemporal Asset Pricing model [Merton (1973), Breeden (1979)] or Arbitrage Pricing Model [Ross (1976)], expected stock returns are determined by the sum of one or more "betas," each multiplied by a return index or "factor." In some applications the betas are fixed parameters and the factor premiums are modelled by a regression on lagged instrumental variables. Examples include Hansen and Hodrick (1983), Gibbons and Ferson (1985), Ferson and Harvey (1991) and many others; see Ferson (1995) for a review. Such models provide motivation in an asset pricing context, for our study of spurious regression with persistent, lagged regressors.

In other asset pricing examples, the betas are time-varying. Using lagged instruments to model the predictability in betas results in regression models with interaction terms. The lagged instruments interact with contemporaneous values of the asset-pricing factors. Such regressions, following Maddala (1977), are studied by Shanken (1990) and applied to the

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[2] Previous studies have addressed other statistical problems in predictive regressions for stock returns. Stambaugh (1998), Nelson and Kim (1993), Goetzmann and Jorion (1993) and Bekaert, Hodrick and Marshall (1997) study small samples biases due to dependent stochastic regressors. Kim, Nelson and Startz (1993) study structural change induced misspecification. Summers (1988) and Campbell and Shiller (1988) consider dependent regressors with unit roots. Kandel and Stambaugh (1990), Fama and French (1988) and Hodrick (1992) focus on autocorrelation in regressions for long-horizon stock returns. Boudhouk et al (1994) provide a review of related literature.

problem of mutual fund performance measurement by Ferson and Schadt (1996). The list of recent empirical studies using similar regression models in an asset pricing context is long and growing.[3] These studies provide additional motivation for our investigation of regression models with interaction terms.

In the context of regressions with interaction terms, a concern is that the underlying "betas" in the model may be persistent time-series. A spurious relation may be found when independent persistent instruments are used to model the betas. However, because the persistent beta is multiplied by a market index or factor return, the sampling properties of the regression depend on the relation between the beta and the underlying index.

We find that if the true expected return is not predictable over time, there is no serious problem with spurious regression bias in predictive regressions, even if the measured regressor is highly persistent. However, accounting for spurious regression bias, we can reject the hypothesis that the expected return of the Standard and Poors 500 index (SP500) is unpredictable.

Assuming that there is persistence in expected returns, our simulations suggest that spurious regression can be a serious concern for regressions of stock returns on persistent lagged instruments, given parameter values within the range observed in actual monthly data. As expected, the bias is greater when the component of returns that is predictable is larger and

[3] A partial list of additional studies includes Kothari and Shanken (1995), who study dynamic portfolio strategies; Eckbo and Smith (1998), who study insider trading; Ferson and Harvey (1993, 1997), who study international equity returns; Jagannathan and Wang (1996) and Jagannathan et al. (1998), who study the pricing of labor income risk; Fama and French (1997), who study the cost of equity capital; Christopherson et al. (1998), Zhang (1998) and Busse (1999), who study investment fund performance. Similar regression models are used by commercial services such as BARRA, for portfolio risk analysis [BARRA (1997)]. Additional applications of such regression models in economics and other areas of the social and natural sciences could be added to this list.

when the instruments are more highly persistent.

When the regressions include interaction terms between a lagged instrument and the contemporaneous value of a market index return, our results suggest that the spurious regression affect is not as serious as in the simple regressions on lagged instruments. When time-variation in the expected market return and beta is not persistent, spurious regression bias is not an important issue. However, when a common persistent factor drives expected market returns and betas, spurious regression becomes a concern. Large sample sizes are no defense against the spurious regression bias.

The paper is organized as follows. Section 2 describes the data. Section 3 begins with the case of a lagged predictor variable in a stock return regression. Section 4 reviews regressions with interaction terms. In Section 5 we describe the results of a simulation study of the spurious regression issue in the context of models with lagged interaction terms. Section 6 offers concluding remarks.

2. The Data

We use data on security returns and a set of lagged instruments, representative of the conditional asset pricing literature. Table 1 reports summary statistics for the monthly data, covering various subperiods of 1926 through 1997. The sample periods depend on the variable, and the notes to the table provide the details. We report the means, medians, standard deviations and the first order sample autocorrelations. Note that the first order autocorrelations frequently suggest a high degree of persistence. For example, the short term Treasury bill yields, levels of industrial output and money stock, and the dividend yield of the Standard and Poors 500, all have first order autocorrelations in excess of 0.95. We use the first order autocorrelation as a key parameter of our simulations, to measure the degree of persistence in the regressors.

Also included in Table 1 are summaries of regressions for the monthly return of the Standard and Poors 500 stock index, measured in excess of the one-month Treasury bill return from Ibbotson Assocates, on the lagged instruments. These are simple regressions using one instrument at a time, similar to Fama and French (1989). The R-squares range from less than 1% to more than 4%. We evaluate the statistical significance of these R-squares below. We use "*true* R^2" as an another parameter of our simulations. The true R^2 is the ratio of predictable variance, based on lagged variables, to the total variance of the return. Finally, we report the first order autocorrelations of the residuals of the regressions. Granger and Newbold (1974) focus on the Durbin Watson statistic (a transformation of the residual autocorrelation) as an indicator of the likely extent of spurious regression. Here the residual first order autocorrelations are generally on the order of 0.1 or smaller. This reflects the fact that the dependent variable is a stock return, containing a large noise component.

3. Regressions with Lagged Predictor Variables

We start with a model in which the future stock return is regressed over time by the analyst on a lagged predictor variable:

$$r_{t+1} = \alpha + \delta Z_t + v_{t+1}. \qquad (1)$$

We consider a setting similar to Granger and Newbold (1974), where the data are generated by two independent time-series whose persistence we control through the parameter, ρ:

$$(Z_t^*, Z_t)' = \begin{Bmatrix} \rho & 0 \\ 0 & \rho \end{Bmatrix} (Z_{t-1}^*, Z_{t-1})' + (\varepsilon_t^*, \varepsilon_t)'. \quad (2)$$

The errors $(\varepsilon_t^*, \varepsilon_t)$ are iid normal variates with mean zero and dispersion, σ. We assume that the data generating process for the stock return is defined in terms of the unobservable variable Z_t^*, as:

$$r_{t+1} = \mu + Z_t^* + u_{t+1}, \quad (3)$$

where u_{t+1} is white noise with variance, σ_u^2. We interpret Z^* as the deviations of the conditional mean return from the unconditional mean, μ, where expectations are conditioned on an unobserved "market" information set. Thus, we have a situation in which the "true" returns may be predictable, if Z_t^* could be observed. However, because the analyst uses an independent instrument Z_t, the true value of δ in the regression (1) is zero. Because the instruments may be persistent time-series, we expect to find a spurious regression problem when ρ is close to 1.0. We focus on the sampling properties of the coefficient δ in the regression (1) and of its t-ratio.[4]

Our setting is related to Phillips (1986) and Stambaugh (1998). Phillips derives the asymptotic distributions of the OLS estimators, for the case where $\rho=1$, $u_{t+1} \equiv 0$ and $\{\varepsilon_t^*, \varepsilon_t\}$ are general independent mean zero processes. We allow a nonzero $\text{var}(u_{t+1})$ to accommodate the large noise component of stock market returns. We allow $\rho<1$, to include stationary

[4] We report results for the OLS t-ratio. In these experiments similar results are obtained when the t-ratios are formed as in Hansen (1982) and White (1980).

regressors.

Stambaugh (1998) studies a special case of our setup, where the errors $\{\varepsilon_t^*,\varepsilon_t\}$ are perfectly correlated, or equivalently, the analyst observes and uses the correct regressor. He studies finite sample biases that arise due to lagged stochastic regressors. The bias is related to the well-known small sample bias of the autocorrelation coefficient [e.g. Kendall (1958)]. As in Stambaugh (1998), the analyst in our setting uses a regressor that is stochastic. However, because the observed regressor, Z_t, is independent of the true regressor Z_t^*, the finite sample bias derived by Stambaugh would be zero in our case. Yet, we find that there remains a finite sample bias, in the absence of the bias studied by Stambaugh, because of the spurious regression phenomenon.

3.1 When Expected Returns are Not Persistent

In order to illustrate the importance of persistence in the underlying "true" expected return we modify equation (2) as follows:

$$(Z_t^*,Z_t)' = \begin{Bmatrix} 0 & 0 \\ 0 & \rho \end{Bmatrix} (Z_{t-1}^*,Z_{t-1})' + (\varepsilon_t^*,\varepsilon_t)'. \qquad (4)$$

In equation (4) the true expected return Z_t^* has an autocorrelation of zero. The true return is pure noise around a conditional mean that is time-varying, but independently distributed over time. The measured instrument has autocorrelation, ρ.

We use equation (4) to show that if there is no persistence in the expected return, the spurious regression phenomenon is not a serious concern, even when the measured regressor is highly persistent. Table 2 shows the results of a simulation exercise using several values of ρ and T, where T is the number of time-series observations in the artificial sample. In each case 1,000 trials of the simulation are used. The data are generated according to equations (3) and (4), where μ is the average monthly return of the SP500, but the analyst estimates the regression (1). Thus, the true value of the slope δ is zero. We record the estimates of the slope, its t-ratio and the first order autocorrelation of the residual at each trial and summarize their empirical distributions. The experiment is run for three sample sizes, T=60, T=350 and T=2,000.

The value of the true R-squared, $R^2 \equiv Var(Z^*) /[Var(Z^*)+ \sigma_u^2]$, is the fraction of the variance of the return that could be predicted if the conditional mean Z^* could be measured. Thus, the R^2 measures the importance of the conditional mean return, relative to the noise, for the variance of the stock return. In Table 2, we show only R^2=0.01, but the results are similar for R^2 as large as 0.4. The artificial data are calibrated to the SP500 stock index by setting $Var(Z^*)$ equal to 0.10 times the sample variance of the monthly returns of the SP500. This approximately corresponds to the sample R-squares obtained in monthly data with a list of standard instruments. Results using a scale factor of 0.30 were similar. The value of σ_u^2 is determined from the definition of R^2 given above.

Table 2 shows that the t-statistics for the slope, the coefficients of determination and sample residual autocorrelations are generally well behaved in the tails and are not highly sensitive to the value of ρ. For example, the empirical critical values of the t-ratios, using a two-sided 5% test, range between 1.87 and 2.13 across the 27 cells of the table. Under the normal distribution the value would be 1.96.

The estimated coefficients of determination at the 2.5% tail areas may be compared with their theoretical critical values from the F or Chi-squared distributions. For T=(60, 350, 2000), the F distribution, assuming normally distributed errors, implies critical values of

(.082, .014, .0025), while the asymptotic Chi-squared approximation for TR^2 implies the values (.212, .036, .006). The empirical critical values are in the range of these figures for each sample size.

Finally, the residual autocorrelations in Table 2 may be evaluated relative to standard approximations. Under the null hypothesis that the true autocorrelation is zero, the expected sample autocorrelation is approximately $-1/\sqrt{T}$ and its standard error is $1/\sqrt{T}$. Using these approximations, the values of the sample autocorrelation three standard errors above the mean are (.26, .106, .062) for T=(60, 350, 2000). These are close to the critical values reported in Table 2. We conclude that if the true expected return is not persistent, there is no serious problem with spurious regression bias in the predictive regressions, even if the measured regressor is highly persistent.

3.2 Testing for Persistent Expected Returns

Using the regressions in Table 1 and the simulations of Table 2, we can evaluate the null hypothesis that expected returns vary over time as an independent white noise process. Table 2 suggests that with 350 observations, a coefficient of determination larger than 1.63% is significant against this null at the 2.5% level. In Table 1, six out of 14 examples comfortably exceed this level. Thus, the hypothesis that the expected returns are white noise can be rejected. This motivates proceeding under the assumption that expected returns have some degree of persistence over time.

3.3 Spurious Regression with Persistent Expected Returns

Table 3 assesses regressions where the true expected returns are persistent over time.

The simulation is similar to Table 2, but the true expected return now has autocorrelation ρ, according to equation (2). For a given value of ρ the value of σ^2 is determined as $\sigma^2=(1-\rho^2)Var(Z^*)$. The A panels of Table 3 report empirical critical values, above which 2.5% of the statistics lie in the 1,000 simulation trials.

Variables like short term interest rates and dividend yields can have first order sample correlations in excess of 0.95, as we saw in Table 1. When $\rho=.95$ and R^2 is 0.01, the smallest critical values for the t-ratios of the slope coefficient, δ, are 2.12 (T=60, panel A), 2.14 (T=350, panel B) and 2.18 (T=2,000, panel C). For smaller values of ρ and $R^2=0.01$ the values are closer to 1.96. Thus, when the predictable fraction of the variance in the true returns is 1% and the instruments are not extremely persistent ($\rho\leq0.95$), the t-statistics are well behaved. The hypothesis that a given instrument does not predict returns can be tested, in this case, relying on the usual t-statistics. Also, the critical values of the coefficient of determination are close to their theoretical values, using the F distribution. For example, at T=2000 the empirical and theoretical values coincide, at 0.0025.

When there is more predictability in the true return the critical t-ratios are larger, indicating a finite sample bias. Even when the true R^2 is between 1% and 10%, typical of the sample values found in monthly data, we find substantial biases when the regressors are persistent, with $\rho>0.95$. These biases do not diminish with larger sample sizes. Such a result is typical of a spurious regression problem, as we explain below.

With moderate values of R^2 and $\rho\geq0.95$, spurious regression becomes a potentially serious concern. Consider the plausible scenario with a sample of T=350 observations where $\rho=0.95$ and $R^2=0.20$. In view of the spurious regression phenomenon, an analyst who was not sure that the "true" instrument is being used and who wanted to conduct a 5%, two tailed t-test for the significance of the measured instrument, would have to use a t-ratio of 4.23. The coefficient of determination would have to exceed 5.9%. These are substantially more stringent than the usual rules of thumb for significance. It is interesting to find that the spurious regression problem occurs well outside the classical setting in which the levels of economic time series are the regressands. Even stock return regressions, when persistent regressors are used, can be affected.

Finally, given $\rho \geq 0.95$ and large values of R^2, the biases in the t-ratios can be truly dramatic. Critical values in excess of 5.0, and extreme values as large as 7.9 are shown in the table.

Table 3 also reports the empirical critical values for the estimated coefficients of determination and the residual autocorrelation of the regressions. There are a number of interesting patterns in these results. Some of the patterns may be understood in view of previous theoretical work on spurious regression. Phillips (1986) derives the asymptotic distributions of the OLS estimators of equation (1), given the special case where $\rho=1$, $u_{t+1} \equiv 0$, while $\{\varepsilon_t^*, \varepsilon_t\}$ are general independent mean zero processes. He shows that the t-ratio for δ diverges for large T, while $t(\delta)/\sqrt{T}$, δ and the coefficient of determination converge to well-defined random variables. The sample autocorrelation of the residual converges in probability to 1.0.[5]

These theoretical results refer to infinite sample sizes, but some patterns consistent with these results may be observed in Table 3, when T=2,000. The asymptotic results lead us to expect explosive behavior of the t-ratio for δ, when $\rho=1$. In Table 3 we find that when $\rho \geq$

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[5] Marmoul (1998) extends these results to multiple regression with partially integrated processes, and provides references to more recent theoretical literature. Phillips (1998) reviews analytical tools for the asymptotic analysis of spurious regression problems.

0.98, the empirical critical values of the t-ratios are larger at T=2000 than at T=60 or T=350. A large sample size is no cure for the spurious regression bias. Thus, the results from the asymptotic theory with unit roots provide useful intuition for values of ρ that are plausibly consistent with the data in Table 1.

At extremely high values of ρ we approach a discontinuity. When ρ=1, both Z_t and Z_t^* are random walks, their unconditional variances are not defined and our "true" R^2 is not defined. In the limiting case, as T gets large, we expect the regression slopes and estimated coefficients of determination to approach well-defined random variables. For T=2000 and large values of ρ, the empirical critical values of the slope coefficients and their standard errors in Table 3 grow larger. Close to the ρ=1 limit the standard errors appear to explode faster than the slopes.

Phillips (1986) shows that the sample autocorrelation in the regression studied by Granger and Newbold (1979) converges in limit to to 1.0. In Table 3 we find large autocorrelation coefficients at T=2000 for the larger values of R^2, but none of the critical values are larger than 0.5. Since u_{t+1}=0 in the cases studied by Phillips, we expect to see explosive autocorrelations only when R^2 is large. When R^2 is small the white noise component of the returns serves to dampen the residual autocorrelation of the regressions.

3.4 Summary

We conclude this section with a summary of our observations about the stock return regressions. In view of the simulations in Table 2 and the data in Table 1, we conclude that a hypothesis that *expected* returns of the S&P500 are white noise can be rejected. This motivates the assumption that the expected returns are persistent. Given persistent expected returns, we find that spurious regression can be a serious concern well outside the classic setting of Yule (1926) and Granger and Newbold (1974). When stock returns are the dependent variable, the returns are much less persistent than the levels of most economic time series. Yet, when the *expected* returns are persistent, there is a risk of spurious regression bias. Furthermore, spurious regression bias is not avoided with large sample sizes. A researcher can use our tables to assess the significance of a regression with a persistent lagged regressor, in view of the possibility of a spurious regression. Our simulations also suggest that results from the asymptotic theory with unit roots are useful for understanding regressions when the degree of persistence is similar to many of the lagged instruments used in the finance literature.

4. Regressions with Interaction Terms

We now consider regression models with a market index return, $r_{m,t+1}$, and interaction terms in a lagged predictor variable. The regression model is:

$$r_{t+1} = \alpha_0 + \alpha_1 Z_t^* + \beta_t\, r_{m,t+1} + u_{t+1},$$

where $E(u_{t+1})=E(u_{t+1}[Z^*_t, r_{m,t+1}])=0$. Following Maddala (1977) the time-dependence of the beta coefficient β_t is modeled as a linear function of the lagged variable: $\beta_t = b_0 + b_1 Z_t^*$. This results in a regression with an interaction term:

$$r_{t+1} = \alpha_0 + \alpha_1 Z_t^* + b_0\, r_{m,t+1} + b_1\, r_{m,t+1} Z_t^* + u_{t+1}, \tag{5}$$

where b_1 is the coefficient on the interaction term. One motivation for the regression (5) is, for example the CAPM. If the returns r_{t+1} and $r_{m,t+1}$ are measured in excess of a reference asset return and $r_{m,t+1}$ is the market index excess return, the CAPM implies that $\alpha_0 + \alpha_1 Z_t^* = 0$ for all t, and therefore $(\alpha_0, \alpha_1)=0$. We are interested in drawing inferences about the values of these coefficients and the coefficients, b_0 and b_1, that determine the conditional beta.

4.1 Scale Effects

In most finance applications the scale of the lagged regressor is not specified by theory, so we would not want it to affect the results. Consider first an example where the analyst measures a non-homogeneous scale transformation of the true lagged predictor, $Z_t = c_0 + c_1 Z_t^*$. This is similar to examples studied by Bernhardt and Jung (1979) and Agren and Jonsson (1992), who show that models with interaction terms are not generally invariant to such transformations.

The analyst estimates the model:

$$r_{t+1} = A_0 + A_1 Z_t + B_0 r_{m,t+1} + B_1 r_{m,t+1} Z_t + v_{t+1}. \qquad (6)$$

In the case of the CAPM, the model produces estimates of a time-varying beta, $B_0 + B_1 Z_t$, and a time-varying "alpha," $A_0 + A_1 Z_t$ [see Christopherson, Ferson and Glassman (1998)]. By substitution we obtain the coefficients as: $A_1 = \alpha_1/c_1$, $A_0 = \alpha_0 - \alpha_1 c_0/c_1$, $B_1 = b_1/c_1$, $B_0 = b_0 - c_0 b_1/c_1$.

When the true alpha is zero $(\alpha_0, \alpha_1)=0$, then $A_0=0$ and $A_1=0$. Thus, tests of the hypothesis that alpha is zero may be conducted without concern for the scale of the lagged instrument.

Bernhardt and Jung (1979) emphasize the importance of including the linear term in the lagged variables, A_1. Without the Z_t term, the higher order coefficients B_0 and B_1, and their t-ratios, can be biased. This bias is relevant under the alternative hypothesis that alpha is not zero. Note that even the signs of the coefficients A_0 and B_0 can be affected by a transformation of the instrument. Thus, the interpretation of a nonzero conditional alpha is problematic when the scale of the instrument is arbitrary. The conditional betas will also be affected by the scale of the instrument.

Note that if $E(Z_t)=E(Z_t^*)=0$, such that $c_0=0$, then the signs of the coefficients are preserved, for positive transformations, $c_1>0$. The T-statistics for all of the coefficients in the transformed model are well-specified. Thus, if the instruments are mean zero, the t-ratios may be used to evaluate the interaction terms, even when the scale of the instruments is arbitrary. In view of these scale effects we advocate using mean-zero instruments in asset pricing applications of regression models with interaction terms in the lagged instruments.

5. Regressions with Interaction Terms: Simulation Evidence

To study spurious regression in models with lagged interaction terms, the data are generated according to:

$$r_{t+1} = \beta_t \, r_{m,t+1} + u_{t+1}, \qquad (7)$$

$$\beta_t = 1 + Z_t^*,$$

$$r_{m,t+1} = \mu_{sp} + k \, Z^*_t + w_{t+1},$$

where $r_{m,t+1}$ is interpreted as a market index return. The market return is generated by analogy to the predicted return in the previous tables. The β_t is a time-varying beta coefficient. As Z_t^* has mean equal to zero, the expected value of the beta is 1.0 and we control the persistence of the beta via the parameter, ρ, according to equation (2). The parameters k, μ_{sp}, and the variance of the market-wide shock w_{t+1} and asset-specific shock, u_{t+1} are described below. There is no intercept, or "alpha," in the data generating process, consistent with asset pricing theory. Because of the interaction terms, the data will generally be conditionally heteroskedastic.

The analyst in the simulations estimates the regression model (6). He uses the lagged instrument, Z_t, which is independent of Z_t^*. The true values of the coefficients in the regression (6) are A_0=0, A_1=0, B_0=1 and B_1=0. We form t-ratios for the coefficients using the White (1980)-Hansen (1982) consistent standard errors.

We consider two cases in how we model the market index return. In the first case, the beta and the conditional mean of the market return follow white noise processes. In the second case, there is a common persistent factor driving the movements in both. Common factors in time-varying betas and expected market premiums are important in asset pricing studies such as Chan and Chen (1988), Ferson and Korajczyk (1995) and Jagannathan and Wang (1996). However, Ferson and Harvey (1997) argue that a regression model like (6) should be robust to the process describing the conditional mean of the market index. Our simulations shed light on the sensitivity of the sampling properties of the regression to the persistence in the conditional mean of the market.

5.1 Simulation Parameters

We model the market excess return according to equation (7). The unconditional expected return, $\mu_{sp} = 0.06$, is the monthly average return of the Standard and Poor's 500 stock index in excess of the one-month Treasury bill, from Table 1. The variance of the error is $\sigma_w^2 = \sigma_{sp}^2 - k^2 \text{Var}(Z^*)$, where $\sigma_{sp} = 0.057$ matches the S&P500 return from Table 1. We fix $\text{Var}(Z^*) = 0.087$, the estimated average monthly variance of the market betas on 57 randomly selected stocks from CRSP over the period 1963-1995.[6]

In the models with interaction terms there are at least two ways to think about the true value of the R^2. The *predictive* R^2 is similar to what we used in the previous tables. It measures the proportion of the variance of the return that could be predicted if Z^* was observed: $R_p^2 = \text{Var}\{E(r_{t+1}|Z_t^*)\}/\text{Var}(r_{t+1})$. We chose the scale parameter, k, in the simulations to match a given value of the predictive R-squared, which implies $k^2 = \sigma_{sp}^2 R_p^2 / \text{Var}(Z^*)$.

Since the regression model (6) includes the contemporaneous returns on the market index, the R^2's that would be observed when the regressions include the market index are likely to be higher than those of pure predictive regressions. Hence, we define the *contemporaneous* R^2 as $R_c^2 = \text{Var}\{\beta_t r_{m,t+1})\}/\text{Var}(r_{t+1})$. This is the R^2 that could in principle be observed by regressing the asset return on the contemporaneous market index return, if the true value of the time-varying beta was known. In our experiments the two versions of R^2 are monotonically related and we report both in the tables.

[6] We calibrate the variance of the betas to actual monthly data by randomly selecting 57 stocks with complete CRSP data for January, 1926 through December, 1997. We estimate simple regression betas for each stock's excess return against the SP500 excess return, using a series of rolling 5-year windows. This produces a series of 805 beta estimates for each firm. We calculate the sample variance of the series for each firm, and average the variance across the 57 firms.

We generate artificial returns for hypothetical stocks using equation (7).[7] The conditional market beta of the stock, given Z^*, is $1+Z^*$. We determine the variance of the error term σ_u^2 in (7), and the variance of the shocks to the persistent components, σ_ε^2 in equation (2), to match the given values of ρ and R^2. For given values of ρ we set $\sigma_{\varepsilon 2} = \sigma^2(Z^*)(1-\rho^2)$. We find σ_u^2 so that the predictive R-squared of the return r_{t+1} is equal to the predictive R^2 of the market return, $r_{m,t+1}$.

5.2 No Persistence in the True Returns

Table 4 reports the results for a special case of the model where we set $\rho = 0$ in the data generating process for the market return and true beta, so that Z^* is white noise and $\sigma^2(\varepsilon^*) = \mathrm{Var}(Z^*)$. This case is analogous to the experiment in Table 2, as the predictable (but unobserved by the analyst) component of the stock return follows a white noise process. We allow a range of values for the autocorrelation, ρ, of the measured instrument, Z. For a given value of ρ, we choose $\sigma^2(\varepsilon) = \mathrm{Var}(Z^*)(1-\rho^2)$, so the measured instrument and the unobserved beta have the same variance.

Table 4 records the upper 2.5% tail critical values for the t-ratios of the coefficients in regression (6) as well as the mean values of the estimates of B_0, taken across the 1,000

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[7] Substituting the three equations of (7) together we can express the data generating process for the return r_{t+1} as:

$$r_{t+1} = (1+Z_t^*)(\mu_{sp} + k\, Z^*_t + w_{t+1}) + u_{t+1}.$$

Because of the interaction between the two Z^*_t terms, we transform the data generated from this expression to obtain the desired true parameter values in the regression model (6), which are $A_0=A_1=B_1=0$, $B_0=1$. The transformed return is $a + br_{t+1}$, where the constants are $b = [1+\mu_{sp}\, k\, \mathrm{Var}(Z^*)/\mathrm{Var}(r_m)]^{-1}$, $a = \mu_{sp} - b\{\mu_{sp} + k\, \mathrm{Var}(Z^*)\}$. This transformation makes the means of r_{t+1} and r_{mt+1} equal to eachother.

simulation trials. The mean values are always close to the true value of 1.0, and closer for the larger sample sizes. The critical values for all of the coefficients are well behaved. When T = 2000, the t-ratios are close to their asymptotic values across all values of ρ and R^2. Thus, like in the simpler predictive regressions, when the true data are not persistent the use of even a highly persistent independent regressor does not create a spurious regression bias.

5.3 Dependent Persistent Beta and Expected Market Returns

Table 5 presents the results from the experiment where the measured instrument and the true beta have the same degree of persistence. Here we fix the Var(Z) = Var(Z^*) and choose, for a given value of ρ, $\sigma^2(\varepsilon) = \sigma^2(\varepsilon^*) = Var(Z^*)(1-\rho^2)$. For values of $\rho<0.95$ and all of the R^2 values, the regressions seem generally well-specified at the smaller sample sizes. Thus, unlike the predictive regressions in Table 3, the regressions with interaction terms do not appear to have spurious regression problems for these parameter values. The most noticeable differences between tables 4 and 5 appear when T = 2000, and for the larger values of ρ and R^2. Here we find that spurious regression bias can become important, although not as severe as in the case of the pure predictive regressions of Table 3.

6. Conclusions

We study statistical issues associated with regression models in which persistent lagged variables predict stock returns, either linearly or in interaction with contemporaneous values of a market index return. We focus on the issue of spurious regression, related to the classic studies of Yule (1926) and Granger and Newbold (1974). Unlike the regressions in those papers, our regression models include asset rates of return, which are not highly persistent, as the dependent variables. However, the returns are the sum of an unobserved expected return plus an unpredictable noise term. If the "true" expected returns are persistent, but an unrelated instrument is chosen, there is a risk of finding a spurious regression.

We first provide evidence rejecting the hypothesis that the expected return of the Standard and Poors 500 index is unpredictable, accounting for spurious regression bias. This justifies our assumption that there is some persistence in the data generating process. We find that spurious regression is a concern in regressions of stock returns on persistent lagged instruments, especially when the component of returns that is actually predictable is large. It is interesting to find that spurious regression can be a concern well outside the classical setting in which trending levels of economic time series are the variables.

We also study the spurious regression problem in the context of regressions with interaction terms. The concern in this case is that the underlying "betas" and expected factor premiums in the model may be persistent time-series. A spurious relation may be found when persistent instruments are used in the regressions. In this case, however, the persistent beta is multiplied by a market index return, which modifies the results. Our simulation evidence suggests that there is no need for concern when the betas and the expected market premium are not persistent, even if a persistent regressor is used. When the betas and expected market premiums are persistent, the spurious regression affects are less severe than in the case of pure predictive regressions.

References

Agren, Anders and Bo Jonsson, 1992, A note on the use of logged variables in interaction terms: Why units matter, *Oxford Bulletin of Economics and Statistics* 54 v.4, 609-613.

BARRA, 1997, newsletter.

Bekaert, G., R. Hodrick and D. Marshall, 1997, On biases in tests of the expectations hypothesis of the term structure, *Journal of Financial Economics* 44, 309-348.

Bernhardt, Irwin and Bong S. Jung, 1979, The interpretation of least squares regressions with interaction or polynomial terms, *Review of Economics and Statistics* 79, v.61 (3), 481-483.

Breeden, Douglas T., 1979, An intertermporal asset pricing model with stochastic consumption and investment opportunities, *Journal of Financial Economics* 7, 265-296.

Breen, William, Lawrence R. Glosten and Ravi Jagannathan, Economic significance of predictable variations in stock index returns, *Journal of Finance* 44, 1177-1190.

Boudhoukh, J. and Matthew Richardson, 1994, The statistics of long-horizon regressions revisited,

Busse, Jeffrey, 1999, Volatility timing by mutual funds, working paper, Emory University (forthcoming, Review of Financial Studies).

Campbell, John Y., 1987, Stock returns and the term structure, *Journal of Financial Economics* 18, 373-400.

Campbell, John Y., and Robert Shiller, 1988, The dividend ratio and small sample bias, *Economics letters* 29, 325-331.

Chan, C.K., and Nai-fu Chen, 1988, Journal of Finance

Christopherson, Jon A., W. Ferson and D. Glassman, 1998, Conditioning manager alpha on economic information: Another look at the persistence of performance, *Review of Financial Studies* 11, 111-142.

Eckbo, Espen and David C. Smith, 1998, The conditional performance of insider trades, *Journal of Finance*

Fama, Eugene F., 1990, Stock returns, expected returns, and real activity, *Journal of Finance* 45, 1089-1108.

Fama, Eugene F., and Kenneth R. French, 1997, Industry cost of equity, *Journal of Financial Economics* 43:2, 153-194.

Fama, Eugene F. and G. William Schwert, 1977, Asset returns and inflation, *Journal of Financial Economics* 5, 115-146.

Ferson, Wayne E., 1989, Changes in expected security returns, risk and the level of interest rates, *Journal of Finance* 44, 1191-1218.

Ferson, Wayne E. and Campbell R. Harvey, 1997, The fundamental determinants of international equity returns: A perspective on country risk and asset pricing, *Journal of Banking and Finance*, 21, 1625-1665.

_________________, 1993, The risk and predictability of international equity returns, *Review of Financial Studies* 6, 527-566.

__________________, 1991, The variation of economic risk premiums, *Journal of Political Economy* 99, 385-415.

Ferson, W., and R. Korajczyk, Do arbitrage pricing models explain the predictability of stock returns? *Journal of Business* 68, 309-349.

Ferson, W., and R. Schadt, 1996, Measuring fund strategy and performance in changing economic conditions. *Journal of Finance* 51, 425-462.

Gibbons, M. R. and W. Ferson, 1985, Testing asset pricing models with changing expectations and an unobservable market portfolio, *Journal of Financial Economics* 14, 217-236.

Goetzmann, W., and P. Jorion, 1993, Testing the predictive power of dividend yields, *Journal of Finance* 48, 663-

Granger, C.W.J. and P. Newbold, 1974, Spurious regressions in Economics, *Journal of Econometrics* 4, 111-120.

Hansen, Lars P., 1982, Large sample properties of the generalized method of moments estimators, *Econometrica* 50, 1029-1054.

Hansen, Lars P. and Robert J. Hodrick, 1983, Risk averse speculation in the forward foreign exchange market: An econometric analysis of linear models, in: J. Frenkel, ed., Exchange rates and international macroeconomics (University of Chicago Press, Chicago, IL).

Harvey, Campbell R., 1991, The world price of covariance risk, *Journal of Finance* 46, 111-157.

Hodrick, Robert J., 1992, Dividend yields and expected stock returns: Alternative procedures for estimation and inference, *Review of Financial Studies* 5, 357-386.

Jagannathan, Ravi, and Zhenwu Wang, 1996, The conditional CAPM and the cross-section of expected returns, *Journal of Finance* 51, 3--54.

Jagannathan, Ravi, Keiichi Kubota, and Y. Takehara, 1998, The relation between labor-income risk and average returns: Empirical evidence for the Japanese stock market, *Journal of Business* 71, 319-47.

Kandel, Shmuel and Robert F. Stambaugh, 1990, Expectations and volatility of consumption and asset returns, *Review of Financial Studies* 3, 207-232.

Kendall, M.G., 1954, A note on the bias in the estimation of autocorrelation, *Biometrica* 41, 403-404.

Keim, Donald B. and Robert F. Stambaugh, 1986, Predicting returns in the bond and stock markets, *Journal of Financial Economics* 17, 357-390.

Nelson, C. and Myung J. Kim, 1993, Predictable stock returns: The role of small sample bias, *Journal of Finance* 48, 641-

Kothari, S.P., and Jay Shanken, 1997, Book-to-market time series analysis, *Journal of Financial Economics* 44, 169-203.

Merton, Robert C., 1973, An intertemporal Capital Asset Pricing Model, *Econometrica* 41, 867-87.

Nelson, C.R and M.J. Kim, 1993, Predictable stock returns: the role of small sample bias, *Journal of Finance* 48, 641-661.

Phillips, P.C.B., 1998, New tools for understanding spurious regressions, *Econometrica* 66, 1299-1326.

Phillips, P.C.B., 1986, Understanding spurious regressions in Econometrics, *Journal of Econometrics* 33, 311-340.

Pontiff, Jeffrey, and Lawrence Schall, 1998, Book-to-market as a predictor of market returns, *Journal of Financial Economics* 49, 141-60.

Ross, S., 1976, The arbitrage theory of capital market asset pricing, *Journal of Economic Theory* 13, 341-60.

Shanken, Jay, 1990, Intertemporal asset pricing: An empirical investigation, *Journal of Econometrics* 45, 99-120.

Stambaugh, R.S., 1998, Predictive regressions, working paper, Wharton School, University of Pennsylvania.

Summers, L., 1988,

Maddala, G.S, 1977, *Econometrics*, McGraw-Hill, New York.

Sharpe, William. F., 1964, Capital asset prices: A theory of market equilibrium under conditions of risk, *Journal of Finance* 19, 425-42.

Zhang, L., 1998, working paper, New York University (forthcoming Journal of Finance).

White, H., 1980, A heteroskedasticity-consistent covariance matrix estimator and a direct test for heteroskedasticity, *Econometrica* 48, 817-38.

Table 1

Common Instrumental Variables: Summary Statistics and Market Regression Results

Panel A reports the number of observations (N), mean, median, standard deviation, and first order auto-correlation (ρ_1) for the instruments listed in the left column and defined in the footnote. Panel B reports the coefficient of determination (R^2), slope coefficient (β), t-statistic, and residual first order auto-correlation (ρ^e_1) from the regression of the S&P 500 excess returns ($r_{S\&P,t}$) on each of the lagged instruments (Z_{t-1}). In each case, the return is measured for a one-month period, t-1 to t, and the lagged instrument is public information at time t-1.

	Panel A: Summary Statistics					Panel B: $r_{S\&P,t} = \alpha + \beta Z_{t-1} + e_t$			
	N	Mean	Median	Std.Dev.	ρ_1	R^2	β	t-stat.	ρ^e_1
S&P 500	863	0.00710	0.01001	0.05702	0.09276	0.00861	0.09276	2.73375	0.00169
T-bill 1-month	575	0.00407	0.00387	0.00238	0.96950	0.02225	-2.55176	-3.61078	0.00661
T-bill 3-month	574	0.00440	0.00422	0.00249	0.98327	0.01734	-2.14774	-3.17737	0.00992
Tb3m	767	0.00333	0.00294	0.00275	0.99175	0.00597	-1.28898	-2.14322	-0.03261
Div. yield	851	0.00356	0.00333	0.00115	0.96952	0.01131	5.30385	3.11635	0.08571
Junk	610	0.00914	0.00756	0.00428	0.97411	0.00259	0.48290	1.25659	0.01242
Term	535	0.00112	0.00110	0.00103	0.92837	0.02347	6.09881	3.57880	-0.00067
Hb3	573	0.00055	0.00036	0.00094	0.30580	0.02501	6.87576	3.82703	0.00768
Two – one	573	0.00040	0.00028	0.00058	0.43458	0.02027	0.95985	3.43735	0.01577
Six – one	465	0.00085	0.00060	0.00237	0.17321	0.04234	3.59504	4.52466	-0.05692
Lag(two) – one	572	0.00040	0.00025	0.00099	0.15878	0.00176	1.71815	1.00172	0.01603
PPI	594	0.17796	0.10833	0.21610	0.98584	0.00490	-0.01321	-1.70760	0.01870
Ind. Prod.	594	0.16157	0.20538	0.25887	0.96463	0.01587	-0.01984	-3.09008	0.00804
M1	455	2.03854	1.29167	2.71085	0.99058	0.00075	-0.00042	-0.58442	0.01691

S&P 500 is the monthly return on the S&P 500 index for the period 1/26 – 12/97 found in the CRSP database in excess of the U.S. T-bill total return data found in the CTI CRSP file. The **T-bill 1-month** and the **T-bill 3-month** represent the yield to maturity found in the Fama T-bill Yield File for the periods 1/50 – 1/97. **Tb3m** is the 3-Month T-Bill Rate (Secondary market) average of the daily closing bid price from the Federal Reserve Economic Database (FRED) divided by 12 for the period 1/34 – 3/98. The **Div. yield** is calculated by summing up 12 lags of ($I_{NYSE,t}$ * (1+$R_{NYSE,t-1}$)) – $I_{NYSE,t-1}$) and then dividing by 12*($I_{NYSE, t-12}$), where I_{NYSE} is the level of the value weighted index excluding dividends and R_{NYSE} is the return on the value weighted NYSE index including dividends found in the CRSP indices files. **Hb3** is the difference between the returns of the three and one-month T-bills found in the Fama T-bill Yield File for the period 2/50 – 11/97. **Junk** is the difference between the CITIBASE series FYBAAC Bond Yield: Moody's BAA corporate and FYAAAC Bond Yield: Moody's AAA corporate (%/annum) for the period 1/47 – 6/97. **Term** is the spread between the lagged 10-year T-bond constant maturity rate from the FRED file GS10 and the current 3-month T-bill (**Tb3m**) over the period 5/53 – 3/98 divided by 12. **Two – one**, **Six – one**, and **Lag(two) – one** are computed as the spreads on the returns of the two and one-month bills, six and one-month bills, and the lagged value of the two-month and current one-month bills; these returns may be found in the Fama T-bill Yield File. The final three series are the monthly first difference of the twelve-month moving average of the level of the series: **PPI** is the producer price index for finished goods (1982 = 100) from the CITIBASE file PWF, **Ind. Prod.** is the total index of industrial production (1992 = 100) from the CITIBASE file IP, and **M1** is the seasonally adjusted money stock in billions of dollars from the FRED.

Table 2

The Monte Carlo Simulation Results from the Regression with Lagged Predictor Variables: Zero Autocorrelation of the True Predictor

The table reports the 97.5 percentile of the Monte Carlo distribution of 1000 OLS *t*-statistics, estimated coefficients of determination, and first-order residual autocorrelations from the regression

$$r_{t+1} = \alpha + \delta Z_t + v_{t+1} ,$$

where r_{t+1} is the excess return, Z_t is the predictor variable, and t =1, ..., T. Panel A depicts the results for T = 60, panel B for T = 350, and panel C for T = 2000. The parameter ρ is the autocorrelation coefficient of the predictors, Z_t^* and Z_t. The true coefficient of determination from the true regression of excess returns r_{t+1} on the instrument Z_t^* is set to 0.01.

ρ	**0**	**0.1**	**0.5**	**0.9**	**0.95**	**0.98**	**0.99**	**0.995**	**0.999**
					Panel A:60 observations				
t-statistics	1.9970	2.0291	1.9113	2.0272	1.9912	2.0183	2.0857	2.1210	2.1197
Estimated R^2	0.0846	0.0867	0.0802	0.0779	0.0742	0.0819	0.0878	0.0886	0.0870
Res. autocorr.	0.2335	0.2310	0.2322	0.2284	0.2281	0.2220	0.2213	0.2208	0.2203
					Panel B:350 observations				
t-statistics	1.8900	1.8788	1.9208	1.9518	1.9972	2.0883	2.1347	2.0751	1.9665
Estimated R^2	0.0123	0.0130	0.0141	0.0163	0.0140	0.0150	0.0146	0.0143	0.0147
Res. autocorr.	0.0985	0.0985	0.0986	0.0969	0.0967	0.0954	0.0948	0.0946	0.0941
					Panel C:2000 observations				
t-statistics	1.8917	1.9120	1.8548	1.9565	1.8872	1.9134	1.8735	1.9723	2.1312
Estimated R^2	0.0055	0.0053	0.0056	0.0049	0.0049	0.005	0.0049	0.0051	0.0053
Res. autocorr.	0.0628	0.0626	0.0625	0.0621	0.0619	0.0622	0.0620	0.0627	0.0624

Table 3

The Monte Carlo Simulation Results from the Regression with Lagged Predictor Variables: Non-zero Autocorrelation of the True Predictor

The table reports the 97.5 percentile of the Monte Carlo distribution of 1000 OLS *t*-statistics, estimated coefficients of determination, and first-order residual autocorrelations from the regression

$$r_{t+1} = \alpha + \delta Z_t + v_{t+1},$$

where r_{t+1} is the excess return, Z_t is the predictor variable, and t=1,...,T. Panel A depicts the results for T=60, panel B for T=350, and panel C for T=2000. The parameter ρ is the autocorrelation coefficient of the predictors, Z_t^* and, Z_t. The $\boldsymbol{R^2}$ is the true coefficient of determination from the true regression of excess returns r_{t+1} on the instrument Z_t^*.

Panel A: 60 observations

R^2/ρ	0	0.1	0.5	0.9	0.95	0.98	0.99	0.995	0.999
					t-statistics				
0.01	2.0631	2.1042	1.8958	2.1131	2.1214	2.2299	2.2782	2.2564	2.3012
0.05	1.9915	1.9812	1.9292	2.2764	2.3342	2.3711	2.3418	2.3156	2.2566
0.1	1.9698	2.0675	1.9834	2.4488	2.5791	2.5305	2.5220	2.4115	2.2457
0.2	1.9920	1.9708	2.1183	2.8243	3.0402	2.9745	2.7686	2.6206	2.2691
0.3	1.9490	2.0099	2.1725	3.1339	3.5106	3.3506	3.1058	2.8195	2.3643
0.4	1.9376	1.9929	2.1787	3.5038	3.9306	3.6441	3.3595	3.0881	2.4688
					Estimated R^2				
0.01	0.0914	0.0930	0.0894	0.0863	0.0858	0.0878	0.0908	0.0945	0.0929
0.05	0.0896	0.0925	0.0910	0.0935	0.1003	0.0962	0.0979	0.0941	0.0954
0.1	0.0881	0.0914	0.0963	0.1120	0.1207	0.1115	0.1092	0.1032	0.0970
0.2	0.0865	0.0854	0.0914	0.1456	0.1555	0.1440	0.1320	0.1175	0.0986
0.3	0.0844	0.0819	0.0970	0.1899	0.1944	0.1764	0.1598	0.1324	0.1038
0.4	0.0835	0.0823	0.1086	0.2062	0.2306	0.2100	0.1834	0.1527	0.1114
					Residual autocorrelations				
0.01	0.2347	0.2370	0.2465	0.2384	0.2405	0.2294	0.2286	0.2302	0.2276
0.05	0.2311	0.2366	0.2548	0.2491	0.2417	0.2374	0.2313	0.2325	0.2303
0.1	0.2392	0.2477	0.2741	0.2855	0.2639	0.2452	0.2380	0.2366	0.2290
0.2	0.2303	0.2439	0.3211	0.3606	0.3132	0.2760	0.2470	0.2441	0.2360
0.3	0.2273	0.2580	0.3651	0.4283	0.3870	0.3120	0.2731	0.2481	0.2396
0.4	0.2329	0.2683	0.4122	0.5005	0.4618	0.3646	0.3116	0.2634	0.2373

Table 3 (continued)

Panel B: 350 observations

R^2/ρ	0	0.1	0.5	0.9	0.95	0.98	0.99	0.995	0.999
					t-statistics				
0.01	2.0215	2.0681	2.0641	2.0687	2.1412	2.3083	2.2555	2.1626	2.0378
0.05	2.0654	2.0796	2.1197	2.3647	2.7086	3.2411	3.1504	2.9351	2.4014
0.1	2.0345	2.0996	2.1141	2.6445	3.3180	3.9839	4.0877	3.7138	2.8633
0.2	2.0417	2.0867	2.1392	3.1438	4.2308	5.1891	5.4759	4.9388	3.7818
0.3	2.0810	2.1221	2.1135	3.6334	4.8761	6.4734	6.7806	6.2375	4.6327
0.4	2.0557	2.0757	2.1249	4.0320	5.6083	7.5881	7.9335	7.4188	5.4829
					Estimated R^2				
0.01	0.0153	0.0154	0.0163	0.0175	0.0175	0.0184	0.0188	0.0172	0.0145
0.05	0.0160	0.0158	0.0157	0.0207	0.0257	0.0337	0.0372	0.0320	0.0205
0.1	0.0161	0.0160	0.0163	0.0263	0.0353	0.0538	0.0596	0.0504	0.0285
0.2	0.0158	0.0156	0.0173	0.0398	0.0596	0.0946	0.1060	0.0944	0.0494
0.3	0.0155	0.0156	0.0173	0.0519	0.0821	0.1306	0.1479	0.1378	0.0742
0.4	0.0156	0.0158	0.0167	0.0628	0.1079	0.1690	0.1954	0.1776	0.1034
					Residual autocorrelations				
0.01	0.0986	0.0996	0.1009	0.1030	0.1043	0.1038	0.1007	0.0981	0.0959
0.05	0.1027	0.1074	0.1248	0.1429	0.1431	0.1380	0.1280	0.1163	0.0999
0.1	0.1011	0.1123	0.1502	0.1957	0.1960	0.1867	0.1647	0.1420	0.1082
0.2	0.1010	0.1204	0.1967	0.2893	0.3035	0.2930	0.2556	0.2011	0.1239
0.3	0.1020	0.1295	0.2490	0.3896	0.4044	0.3973	0.3597	0.2816	0.1432
0.4	0.0990	0.1383	0.2986	0.4816	0.4989	0.4981	0.4525	0.3703	0.1795

Panel C: 2000 observations

R^2/ρ	0	0.1	0.5	0.9	0.95	0.98	0.99	0.995	0.999
					t-statistics				
0.01	1.9072	1.8452	2.0660	2.0573	2.1827	2.3351	2.6828	3.0526	3.0514
0.05	1.8731	1.8460	2.0726	2.2828	2.6446	3.2869	4.3022	5.0190	5.6070
0.1	1.8541	1.8766	2.0695	2.5237	3.1747	4.2948	5.5784	6.9865	7.8237
0.2	1.8616	1.8101	2.1345	3.0330	4.0491	5.7825	7.4672	10.0768	11.2785
0.3	1.8444	1.8586	2.1643	3.4695	4.6154	6.7369	9.1374	12.3064	14.3450
0.4	1.9296	1.8916	2.2450	4.0345	5.2866	7.8130	10.5385	14.4897	17.4039
					Estimated R^2				
0.01	0.0023	0.0023	0.0025	0.0028	0.0030	0.0036	0.0043	0.0053	0.0060
0.05	0.0024	0.0024	0.0025	0.0036	0.0045	0.0078	0.0118	0.0190	0.0194
0.1	0.0025	0.0024	0.0026	0.0045	0.0062	0.0118	0.0219	0.0345	0.0365
0.2	0.0026	0.0025	0.0027	0.0060	0.0101	0.0204	0.0386	0.0665	0.0756
0.3	0.0024	0.0025	0.0029	0.0085	0.0135	0.0280	0.0565	0.0958	0.1167
0.4	0.0023	0.0024	0.0030	0.0102	0.0172	0.0374	0.0740	0.1248	0.1615
					Residual autocorrelations				
0.01	0.0413	0.0424	0.0473	0.0506	0.0511	0.0512	0.0506	0.0486	0.0438
0.05	0.0428	0.0479	0.0685	0.0888	0.0906	0.0969	0.0951	0.0946	0.0630
0.1	0.0438	0.0537	0.0926	0.1347	0.1450	0.1514	0.1583	0.1580	0.1012
0.2	0.0436	0.0637	0.1439	0.2306	0.2483	0.2679	0.2836	0.2834	0.1856
0.3	0.0433	0.0734	0.1933	0.3247	0.3497	0.3782	0.3937	0.3952	0.2807
0.4	0.0421	0.0832	0.2437	0.4167	0.4489	0.4823	0.4990	0.4996	0.3769

Table 4

The Monte Carlo Simulations Results from the Regression with Interactive Terms: Zero Autocorrelation of the True Predictor

The table reports the results of 1000 Monte Carlo simulations of the model

$$r_{t+1} = \alpha_0 + \alpha_1 Z_t + \beta_0 r_{m,t+1} + \beta_1 Z_t r_{m,t+1} + v_{t+1},$$

where r_{t+1} is the excess return, $r_{m,t+1}$ is the market return , and Z_t is the predictor variable, and t =1, ..., T. The market return is defined as follows: $r_{m,t+1} = \mu + kZ_t^* + e_{t+1}$, where μ is the mean monthly return on the S&P 500 index, Z_t^* is the true predictor, $k = \sqrt{\boldsymbol{R_p^2}\sigma_{SP}^2 / \sigma_{Z^*}^2}$ is the scalar, σ_{SP}^2 and σ_{Z}^2 are the monthly variances of the S&P 500 index and the true instrument respectively, and $\sigma_e^2 = \sigma_{SP}^2 - k^2\sigma_{Z^*}^2$. Panel A depicts the results for T=60, Panel B for T=350 and Panel C for T=2000. The parameter ρ is the autocorrelation coefficient of the measured predictor, Z_t. The true predictor, Z_t^*, is a white noise process, independent from Z_t. $\boldsymbol{R_p^2}$ and $\boldsymbol{R_c^2}$ are the predictive and contemporaneous coefficients of determination.

Panel A: 60 observations

R_p^2/ρ	0	0.1	0.5	0.9	0.95	0.98	0.99	0.995	0.999	R_c^2
					t-statistics: a0					
0.01	1.9914	1.9647	1.9861	2.1086	2.1658	2.1692	2.1353	2.1217	2.1017	**0.28**
0.05	1.9854	1.9744	1.9783	2.0790	2.1591	2.2060	2.1770	2.1436	2.1089	**0.31**
0.1	1.9681	1.9796	1.9757	2.0647	2.1357	2.2253	2.2066	2.1975	2.1915	**0.34**
0.2	1.9317	1.9468	1.9520	2.0376	2.0976	2.2416	2.2243	2.1849	2.1652	**0.42**
0.3	1.8984	1.9120	1.9093	2.0226	2.1241	2.2315	2.2062	2.1573	2.1590	**0.49**
0.4	1.8632	1.8770	1.8849	1.9875	2.1350	2.1642	2.1823	2.1404	2.1338	**0.56**
					t-statistics: a1					
0.01	2.1836	2.1394	2.0540	2.1373	2.1090	2.1007	2.1592	2.1080	2.1090	**0.28**
0.05	2.1865	2.1778	2.0812	2.1505	2.1433	2.1011	2.1449	2.1301	2.0532	**0.31**
0.1	2.2093	2.2371	2.0868	2.1684	2.1332	2.1718	2.1335	2.1167	2.0746	**0.34**
0.2	2.2523	2.3069	2.0873	2.1642	2.1249	2.1169	2.1167	2.1155	2.0550	**0.42**
0.3	2.2792	2.3140	2.0733	2.1665	2.1657	2.1177	2.1695	2.1470	2.1228	**0.49**
0.4	2.2474	2.2794	2.1055	2.1620	2.1504	2.1282	2.1777	2.1827	2.1985	**0.56**
					Mean b0					
0.01	1.0133	1.0136	1.0148	1.0147	1.0154	1.0170	1.0172	1.0170	1.0166	**0.28**
0.05	1.0102	1.0106	1.0121	1.0122	1.0126	1.0130	1.0124	1.0120	1.0115	**0.31**
0.1	1.0080	1.0085	1.0101	1.0103	1.0105	1.0101	1.0090	1.0084	1.0079	**0.34**
0.2	1.0050	1.0055	1.0071	1.0075	1.0071	1.0056	1.0041	1.0035	1.0030	**0.42**
0.3	1.0028	1.0033	1.0047	1.0052	1.0042	1.0020	1.0003	0.9997	0.9994	**0.49**
0.4	1.0011	1.0015	1.0028	1.0032	1.0015	0.9988	0.9972	0.9967	0.9965	**0.56**
					t-statistics: b0					
0.01	2.2182	2.2186	2.2280	2.3064	2.2672	2.2726	2.3153	2.3180	2.3430	**0.28**
0.05	2.2372	2.2803	2.2226	2.2296	2.2508	2.2265	2.3396	2.2753	2.2790	**0.31**
0.1	2.2708	2.2814	2.2341	2.2014	2.2393	2.2969	2.3390	2.2806	2.3017	**0.34**
0.2	2.2017	2.1973	2.2117	2.1140	2.2776	2.3254	2.3484	2.3353	2.3299	**0.42**
0.3	2.1985	2.1811	2.2339	2.1144	2.3036	2.3195	2.3403	2.2942	2.2947	**0.49**
0.4	2.1682	2.1840	2.2172	2.2054	2.3656	2.3907	2.3134	2.3168	2.2591	**0.56**
					t-statistics: b1					
0.01	2.2952	2.3105	2.3419	2.4089	2.4732	2.3639	2.4752	2.5308	2.4976	**0.28**
0.05	2.4987	2.4534	2.4134	2.4487	2.4199	2.4605	2.5702	2.5661	2.5556	**0.31**
0.1	2.4570	2.4419	2.4154	2.4055	2.4804	2.5007	2.6306	2.6637	2.6234	**0.34**
0.2	2.3916	2.3917	2.4706	2.3788	2.5647	2.6020	2.6511	2.5150	2.5473	**0.42**
0.3	2.4216	2.4329	2.4658	2.4084	2.5046	2.5627	2.5834	2.5853	2.5692	**0.49**
0.4	2.3830	2.3284	2.3578	2.4624	2.4005	2.5105	2.5889	2.6501	2.6838	**0.56**

Panel B: 350 observations

R_p^2 /ρ	0	0.1	0.5	0.9	0.95	0.98	0.99	0.995	0.999	R_c^2
					t-statistics: a0					
0.01	2.0159	2.0190	2.0209	1.9751	1.9832	2.0658	2.0404	2.0387	2.0804	**0.28**
0.05	2.0072	1.9962	2.0019	1.9613	1.9633	2.0521	2.0129	2.0750	2.1028	**0.31**
0.1	1.9619	1.9541	1.9608	1.9486	1.9684	2.0308	1.9854	2.0488	2.0442	**0.34**
0.2	1.9244	1.8871	1.8878	1.9429	1.9755	2.0038	1.9789	2.0168	2.0050	**0.42**
0.3	1.9121	1.9154	1.9223	1.9256	1.9792	1.9571	2.0057	1.9926	2.0767	**0.49**
0.4	1.8760	1.8711	1.8747	1.9073	1.9491	1.9166	1.9540	1.9969	1.9835	**0.56**
					t-statistics: a1					
0.01	1.9898	1.9988	2.0624	2.1830	2.1918	2.1453	2.2715	2.2798	2.2828	**0.28**
0.05	2.0063	2.0414	2.0734	2.1972	2.2113	2.1623	2.2341	2.2758	2.2778	**0.31**
0.1	2.0135	2.0262	2.0781	2.1850	2.1886	2.1460	2.2095	2.2559	2.2751	**0.34**
0.2	1.9914	2.0031	2.0999	2.2040	2.2105	2.1248	2.1934	2.2608	2.2755	**0.42**
0.3	2.0027	2.0085	2.0737	2.1395	2.2118	2.1516	2.2031	2.2431	2.2618	**0.49**
0.4	2.0228	2.0184	2.1004	2.1462	2.1921	2.1838	2.2131	2.2343	2.2517	**0.56**
					Mean b0					
0.01	0.9970	0.9970	0.9972	0.9968	0.9963	0.9949	0.9929	0.9908	0.9910	**0.28**
0.05	0.9975	0.9976	0.9978	0.9974	0.9970	0.9958	0.9940	0.9923	0.9925	**0.31**
0.1	0.9980	0.9980	0.9982	0.9978	0.9975	0.9965	0.9949	0.9934	0.9937	**0.34**
0.2	0.9986	0.9986	0.9987	0.9985	0.9983	0.9976	0.9963	0.9951	0.9955	**0.42**
0.3	0.9990	0.9990	0.9991	0.9989	0.9989	0.9984	0.9974	0.9965	0.9969	**0.49**
0.4	0.9994	0.9994	0.9994	0.9993	0.9993	0.9991	0.9984	0.9977	0.9981	**0.56**
					t-statistics: b0					
0.01	1.8762	1.8716	1.8982	1.8515	1.8262	1.9285	2.0065	1.9920	2.0038	**0.28**
0.05	1.8215	1.8369	1.8515	1.8005	1.7754	1.9079	1.9625	1.9165	2.0106	**0.31**
0.1	1.8857	1.8672	1.8213	1.8128	1.7527	1.9516	1.9530	1.9562	1.9673	**0.34**
0.2	1.8540	1.8548	1.8882	1.8539	1.8212	1.8535	1.9199	1.9800	1.9123	**0.42**
0.3	1.9328	1.9210	1.9111	1.8844	1.8458	1.8747	1.9251	1.9509	1.9856	**0.49**
0.4	1.9139	1.9020	1.8905	1.8838	1.8757	1.9298	1.9443	1.9112	1.9279	**0.56**
					t-statistics: b1					
0.01	1.9892	2.0027	2.0271	2.0917	2.0984	2.1552	2.2159	2.2546	2.2133	**0.28**
0.05	1.9976	1.9983	2.0477	2.1007	2.0212	2.1453	2.2019	2.2134	2.2031	**0.31**
0.1	1.9696	1.9730	2.0639	2.0983	2.0006	2.1197	2.1917	2.1918	2.1969	**0.34**
0.2	2.0293	2.0035	2.1082	2.1546	1.9266	2.1147	2.1998	2.1644	2.0935	**0.42**
0.3	2.0278	1.9870	2.1566	2.0448	1.9848	2.1526	2.1805	2.1975	2.0829	**0.49**
0.4	2.0254	2.0283	2.0943	2.0792	1.9124	2.1981	2.1396	2.1633	2.0999	**0.56**

Panel C: 2000 observations

R_p^2 /ρ	0	0.1	0.5	0.9	0.95	0.98	0.99	0.995	0.999	R_c^2
					t-statistics: a0					
0.01	1.8352	1.8299	1.8325	1.8594	1.8887	1.8764	1.8695	1.8941	2.0360	**0.28**
0.05	1.8539	1.8452	1.8129	1.8457	1.8679	1.8871	1.8431	1.8911	2.0133	**0.31**
0.1	1.8324	1.8284	1.8175	1.8326	1.8695	1.8829	1.8399	1.8863	1.9745	**0.34**
0.2	1.8218	1.8181	1.8090	1.8141	1.8307	1.8922	1.8430	1.8839	2.0091	**0.42**
0.3	1.8314	1.8328	1.8309	1.7997	1.8437	1.8966	1.8413	1.8721	1.9618	**0.49**
0.4	1.8449	1.8431	1.8323	1.8000	1.8635	1.8981	1.8468	1.8701	1.9934	**0.56**
					t-statistics: a1					
0.01	2.0023	2.0007	2.1202	1.8188	1.8776	2.0087	2.0247	1.9744	1.8724	**0.28**
0.05	1.9761	1.9724	2.1005	1.8189	1.8656	2.0102	2.0252	1.9628	1.8333	**0.31**
0.1	1.9511	1.9730	2.0631	1.8426	1.8637	2.0159	2.0457	1.9729	1.8584	**0.34**
0.2	1.9795	1.9996	2.0578	1.8399	1.8707	2.0189	2.0389	1.9729	1.8939	**0.42**
0.3	1.9612	1.9740	2.0305	1.8616	1.8705	2.0113	2.0401	1.9680	1.9451	**0.49**
0.4	1.9587	1.9454	1.9781	1.8589	1.8685	1.9991	2.0414	1.9146	1.9586	**0.56**
					Mean b0					
0.01	1.0008	1.0008	1.0008	1.0009	1.0009	1.0011	1.0014	1.0020	1.0034	**0.28**
0.05	1.0008	1.0007	1.0007	1.0008	1.0008	1.0010	1.0013	1.0019	1.0034	**0.31**
0.1	1.0007	1.0007	1.0007	1.0007	1.0007	1.0009	1.0012	1.0018	1.0034	**0.34**
0.2	1.0006	1.0006	1.0006	1.0006	1.0006	1.0008	1.0011	1.0017	1.0033	**0.42**
0.3	1.0005	1.0005	1.0005	1.0005	1.0005	1.0006	1.0009	1.0015	1.0031	**0.49**
0.4	1.0004	1.0004	1.0004	1.0004	1.0004	1.0005	1.0008	1.0013	1.0029	**0.56**
					t-statistics: b0					
0.01	1.9363	1.9416	1.9087	1.9093	1.8850	1.8938	1.8801	1.8932	1.9445	**0.28**
0.05	1.9235	1.9268	1.9130	1.8820	1.8925	1.8641	1.9213	1.8880	1.9280	**0.31**
0.1	1.8774	1.8779	1.8846	1.9003	1.9037	1.9214	1.8931	1.9010	1.9695	**0.34**
0.2	1.9692	1.9756	1.9376	1.9270	1.9205	1.9626	1.9485	1.8759	1.9818	**0.42**
0.3	1.9654	1.9528	1.9459	1.9950	1.9953	1.9808	1.9521	1.8932	1.9855	**0.49**
0.4	2.0160	2.0110	2.0209	2.0082	2.0152	1.9729	1.9679	1.8698	1.9236	**0.56**
					t-statistics: b1					
0.01	2.0652	2.1655	2.1111	1.9529	1.8593	1.8631	1.8498	1.9410	2.0426	**0.28**
0.05	2.1400	2.0982	2.1390	1.9250	1.8172	1.8839	1.8901	1.9593	1.9631	**0.31**
0.1	2.0807	2.0632	2.1094	1.9182	1.8099	1.9248	1.9233	1.9417	1.9736	**0.34**
0.2	2.0729	2.0324	2.0251	1.8957	1.8091	1.9597	1.9517	1.9723	1.9894	**0.42**
0.3	2.0910	2.0396	2.0671	1.8651	1.8167	1.9350	1.9496	1.9266	1.9983	**0.49**
0.4	2.1055	2.0785	2.0406	1.9030	1.8201	1.9164	1.9056	1.9476	1.9813	**0.56**

Table 5

The Monte Carlo Simulations Results from the Regression with Interactive Terms: Non-zero Autocorrelation of the True Predictor

The table reports the results of 1000 Monte Carlo simulations of the model

$$r_{t+1} = \alpha_0 + \alpha_1 Z_t + \beta_0 r_{m,t+1} + \beta_1 Z_t r_{m,t+1} + v_{t+1},$$

where r_{t+1} is the excess return, $r_{m,t+1}$ is the market return , and Z_t is the predictor variable, and t =1, ..., T. The market return is defined as follows: $r_{m,t+1} = \mu + kZ_t^* + e_{t+1}$, where μ is the mean monthly return on the S&P 500 index, Z_t^* is the true predictor, $k = \sqrt{\boldsymbol{R}_p^2 \sigma_{SP}^2 / \sigma_{Z^*}^2}$ is the scalar, σ_{SP}^2 and σ_Z^2 are the monthly variances of the S&P 500 index and the true instrument respectively, and $\sigma_e^2 = \sigma_{SP}^2 - k^2 \sigma_{Z^*}^2$. Panel A depicts the results for T=60, Panel B for T=350 and Panel C for T=2000. The parameter ρ is the autocorrelation coefficient of the predictors, Z_t^* and Z_t. The true predictor, Z_t^*, is independent of Z_t. $\boldsymbol{R}_p^2$ and $\boldsymbol{R}_c^2$ are the predictive and contemporaneous coefficients of determination.

Panel A: 60 observations

R_p^2/ρ	0	0.1	0.5	0.9	0.95	0.98	0.99	0.995	0.999	R_c^2
					t-statistics: a0					
0.01	1.9914	1.9757	1.9916	2.0643	2.0687	2.0909	2.0292	1.9949	2.0069	**0.28**
0.05	1.9854	1.9713	1.9388	1.9710	1.9924	2.0038	1.9610	1.8964	1.8978	**0.31**
0.1	1.9681	1.9549	1.9303	1.9753	1.9565	1.9382	1.9318	1.8427	1.8103	**0.34**
0.2	1.9317	1.9334	1.8782	1.9513	1.8933	1.8558	1.8047	1.7461	1.6776	**0.42**
0.3	1.8984	1.9022	1.8843	1.8730	1.7898	1.7469	1.6757	1.6634	1.5764	**0.49**
0.4	1.8632	1.8683	1.8186	1.7802	1.6496	1.6628	1.5642	1.5069	1.4849	**0.56**
					t-statistics: a1					
0.01	2.1836	2.1509	2.0568	2.1266	2.0939	2.1904	2.1592	2.1430	2.1039	**0.28**
0.05	2.1865	2.2123	2.0873	2.1339	2.0900	2.2312	2.2045	2.2255	2.1141	**0.31**
0.1	2.2093	2.2478	2.1017	2.1289	2.0651	2.2709	2.2283	2.2153	2.1212	**0.34**
0.2	2.2523	2.3029	2.1563	2.1053	2.0675	2.2209	2.1971	2.1939	2.1310	**0.42**
0.3	2.2792	2.3103	2.1741	2.0897	2.1184	2.2002	2.2064	2.1667	2.1386	**0.49**
0.4	2.2474	2.3356	2.1380	2.1149	2.1047	2.1971	2.1949	2.1826	2.1454	**0.56**
					Mean b0					
0.01	1.0133	1.0136	1.0139	1.0094	1.0096	1.0133	1.0143	1.0140	1.0135	**0.28**
0.05	1.0102	1.0105	1.0112	1.0073	1.0064	1.0089	1.0097	1.0093	1.0087	**0.31**
0.1	1.0080	1.0083	1.0091	1.0057	1.0040	1.0056	1.0064	1.0060	1.0053	**0.34**
0.2	1.0050	1.0052	1.0059	1.0033	1.0004	1.0008	1.0017	1.0014	1.0006	**0.42**
0.3	1.0028	1.0029	1.0034	1.0014	0.9976	0.9970	0.9979	0.9978	0.9971	**0.49**
0.4	1.0011	1.0012	1.0015	0.9998	0.9952	0.9938	0.9946	0.9947	0.9941	**0.56**
					t-statistics: b0					
0.01	2.2182	2.2467	2.2691	2.4038	2.4639	2.4188	2.3232	2.3257	2.2736	**0.28**
0.05	2.2372	2.2654	2.2809	2.3948	2.4544	2.3938	2.2815	2.3058	2.2567	**0.31**
0.1	2.2708	2.3016	2.2888	2.4207	2.4810	2.4082	2.2869	2.2579	2.2411	**0.34**
0.2	2.2017	2.2404	2.2164	2.5403	2.6106	2.5164	2.2693	2.1833	2.2459	**0.42**
0.3	2.1985	2.1647	2.3515	2.6707	2.6837	2.5803	2.3517	2.1513	2.2017	**0.49**
0.4	2.1682	2.1524	2.3004	2.7714	2.7858	2.6682	2.3645	2.1714	2.1874	**0.56**
					t-statistics: b1					
0.01	2.2952	2.3317	2.3886	2.5871	2.5614	2.5568	2.4296	2.4238	2.3891	**0.28**
0.05	2.4987	2.4686	2.3604	2.5746	2.5392	2.5259	2.4381	2.4589	2.4227	**0.31**
0.1	2.4570	2.4804	2.4340	2.5481	2.5578	2.5012	2.4375	2.4406	2.4079	**0.34**
0.2	2.3916	2.3993	2.4236	2.5596	2.5293	2.4552	2.4468	2.4152	2.3764	**0.42**
0.3	2.4216	2.4549	2.5231	2.6328	2.4991	2.4418	2.4096	2.4043	2.3646	**0.49**
0.4	2.3830	2.3962	2.5586	2.7420	2.5317	2.3777	2.3327	2.3939	2.3530	**0.56**

Panel B: 350 observations

R_p^2 /ρ	0	0.1	0.5	0.9	0.95	0.98	0.99	0.995	0.999	R_c^2
					t-statistics: a0					
0.01	2.0159	2.0200	1.9579	1.9005	1.9192	1.9376	1.8445	1.8536	1.9748	**0.28**
0.05	2.0072	1.9825	1.9115	1.9195	1.8056	1.8006	1.6005	1.6230	1.7297	**0.31**
0.1	1.9619	1.9569	1.9106	1.8284	1.7933	1.7022	1.4912	1.4537	1.5651	**0.34**
0.2	1.9244	1.9117	1.8907	1.8807	1.7678	1.5616	1.3606	1.1720	1.1528	**0.42**
0.3	1.9121	1.8895	1.9349	1.9470	1.6987	1.4672	1.1640	0.9868	0.8687	**0.49**
0.4	1.8760	1.8825	1.9643	1.8893	1.7382	1.3965	0.9701	0.7475	0.6414	**0.56**
					t-statistics: a1					
0.01	1.9898	2.0158	2.0613	2.1946	2.2208	2.1387	2.2058	2.2034	2.2203	**0.28**
0.05	2.0063	2.0235	2.0421	2.2003	2.2129	2.1905	2.2135	2.2173	2.2362	**0.31**
0.1	2.0135	2.0126	2.0334	2.2382	2.2271	2.2123	2.2323	2.2309	2.2760	**0.34**
0.2	1.9914	2.0013	2.0632	2.2273	2.2609	2.3190	2.2457	2.2538	2.2726	**0.42**
0.3	2.0027	2.0010	2.0330	2.2225	2.3165	2.3611	2.2697	2.2394	2.2427	**0.49**
0.4	2.0228	2.0173	2.0228	2.2511	2.3947	2.4193	2.3837	2.2297	2.2247	**0.56**
					Mean b0					
0.01	0.9970	0.9970	0.9973	0.9970	0.9956	0.9915	0.9876	0.9856	0.9862	**0.28**
0.05	0.9975	0.9976	0.9978	0.9970	0.9953	0.9905	0.9858	0.9830	0.9824	**0.31**
0.1	0.9980	0.9981	0.9981	0.9970	0.9952	0.9898	0.9845	0.9811	0.9795	**0.34**
0.2	0.9986	0.9987	0.9986	0.9971	0.9951	0.9890	0.9829	0.9787	0.9756	**0.42**
0.3	0.9990	0.9991	0.9989	0.9973	0.9952	0.9887	0.9819	0.9772	0.9728	**0.49**
0.4	0.9994	0.9994	0.9991	0.9975	0.9954	0.9887	0.9815	0.9764	0.9708	**0.56**
					t-statistics: b0					
0.01	1.8762	1.8797	1.9118	2.3109	2.5493	3.0807	3.4492	3.4330	2.4736	**0.28**
0.05	1.8215	1.8442	1.9258	2.3472	2.6488	3.2659	3.6014	3.4171	2.4568	**0.31**
0.1	1.8857	1.8482	1.9357	2.3740	2.8182	3.4033	3.6845	3.4940	2.4362	**0.34**
0.2	1.8540	1.8503	1.9436	2.5301	2.9837	3.6461	4.0031	3.6931	2.4214	**0.42**
0.3	1.9328	1.8857	1.9547	2.7146	3.2534	3.9286	4.3094	3.8627	2.4681	**0.49**
0.4	1.9139	1.8801	2.0291	2.9583	3.5568	4.4210	4.6807	4.2100	2.5352	**0.56**
					t-statistics: b1					
0.01	1.9892	2.0111	2.0315	2.3797	2.5540	2.7169	2.8195	2.8199	2.4669	**0.28**
0.05	1.9976	1.9723	2.0131	2.3728	2.6048	2.7145	2.9012	2.8417	2.4971	**0.31**
0.1	1.9696	1.9928	2.0437	2.3919	2.5569	2.7313	2.9957	2.8526	2.4895	**0.34**
0.2	2.0293	2.0164	2.0492	2.4027	2.7192	2.9077	3.0510	2.8984	2.4974	**0.42**
0.3	2.0278	1.9826	2.1332	2.5111	2.8293	3.0739	3.0335	2.8810	2.4915	**0.49**
0.4	2.0254	2.0335	2.1194	2.6861	2.9769	3.3752	3.1137	2.8687	2.4924	**0.56**

Panel C: 2000 observations

R_p^2 /ρ	0	0.1	0.5	0.9	0.95	0.98	0.99	0.995	0.999	R_c^2
					t-statistics: a0					
0.01	1.8692	1.8844	1.9272	1.8896	1.9346	1.9267	1.8827	1.8355	1.6932	**0.28**
0.05	1.8599	1.8753	1.9553	1.9353	1.9925	2.0357	2.0061	1.9437	1.2905	**0.31**
0.1	1.8541	1.8977	1.9628	1.9683	2.1111	2.2217	2.1214	2.0065	0.9355	**0.34**
0.2	1.8430	1.8967	1.9702	2.0906	2.2690	2.4370	2.3064	2.2473	0.4162	**0.42**
0.3	1.8456	1.8911	1.9976	2.1805	2.4090	2.7160	2.6745	2.6268	0.0362	**0.49**
0.4	1.9044	1.9405	2.0185	2.2837	2.5583	2.9662	2.8631	2.7829	-0.3079	**0.56**
					t-statistics: a1					
0.01	1.9854	1.9538	2.0964	1.8417	1.9227	2.0318	2.0367	2.0135	1.9301	**0.28**
0.05	1.9438	1.9583	2.0780	1.8844	1.9425	2.1120	2.1347	2.1067	1.9357	**0.31**
0.1	1.9336	1.9504	2.0567	1.9107	2.0159	2.1512	2.2265	2.1878	1.9343	**0.34**
0.2	1.9517	1.9645	2.0512	1.9760	2.1352	2.3264	2.3671	2.3482	2.0334	**0.42**
0.3	1.9509	1.9456	2.0420	2.0379	2.2752	2.5550	2.6526	2.5549	2.1510	**0.49**
0.4	1.9529	1.9586	2.0216	2.1023	2.3986	2.7409	2.8794	2.8719	2.4316	**0.56**
					Mean b0					
0.01	1.0007	1.0007	1.0007	1.0004	1.0002	0.9998	0.9995	0.9993	1.0012	**0.28**
0.05	1.0006	1.0006	1.0006	1.0003	1.0000	0.9992	0.9983	0.9976	0.9980	**0.31**
0.1	1.0005	1.0005	1.0005	1.0002	0.9997	0.9987	0.9975	0.9963	0.9957	**0.34**
0.2	1.0004	1.0004	1.0004	0.9999	0.9994	0.9979	0.9963	0.9945	0.9925	**0.42**
0.3	1.0003	1.0003	1.0003	0.9998	0.9991	0.9974	0.9954	0.9931	0.9904	**0.49**
0.4	1.0002	1.0002	1.0002	0.9996	0.9989	0.9970	0.9948	0.9921	0.9888	**0.56**
					t-statistics: b0					
0.01	1.9574	1.9386	1.9922	2.3337	2.6605	3.4552	4.2696	5.7033	7.0901	**0.28**
0.05	1.9414	1.9195	1.9942	2.4419	2.7149	3.6328	4.6466	5.9594	7.2925	**0.31**
0.1	1.9416	1.9174	1.9403	2.4187	2.8991	3.8385	5.0825	6.2396	7.5345	**0.34**
0.2	1.9834	1.9835	2.0073	2.6050	3.1587	4.4863	5.8346	7.0948	8.2279	**0.42**
0.3	1.9410	1.9702	2.1020	2.7654	3.4060	5.0321	6.4639	8.3523	8.8610	**0.49**
0.4	1.9957	2.0248	2.0846	3.0667	3.8507	5.6124	7.3207	9.3921	9.8032	**0.56**
					t-statistics: b1					
0.01	2.1684	2.1872	2.1321	2.2117	2.2266	2.7094	3.3065	3.9091	4.3752	**0.28**
0.05	2.1948	2.1285	2.1291	2.2175	2.3019	2.7842	3.4125	4.0819	4.4323	**0.31**
0.1	2.0888	2.0786	2.1412	2.2444	2.4311	2.8784	3.5893	4.2963	4.4725	**0.34**
0.2	2.0890	2.0487	2.1812	2.3970	2.6969	3.1846	3.8871	4.9002	4.6651	**0.42**
0.3	2.1000	2.0240	2.1382	2.4149	2.9217	3.6760	4.3224	5.3160	4.7675	**0.49**
0.4	2.1180	2.1156	2.1526	2.5746	3.3077	4.0624	4.9320	5.8716	5.0393	**0.56**