First and Last name: ....

Student ID: ....

Group Number: ....

Extraordinary Exam in Econometric Techniques (June 2019)

Read carefully each question. Answer very clearly and inside the assigned space. **The value of each question is shown in brackets.**

The exam grades will appear in "Aula global" on June 24th-25th. The day and place of the revision will be announced by each professor. Any change will be announced in advance.

Total time: 105 **minutes**. **Total Grade:** 60.

Good Luck

Question 1 [20 Points]

The company **IWANTTOBEAJEDI** manages investment funds whose quarterly returns $\{y_t\}$ is given by the following process

$$y_t = \phi_0 + \phi_1 y_{t-1} + a_t, \quad a_t \overset{iid}{\sim} N(0, \sigma_a^2 = 2), \tag{1}$$

where

$$\phi_0 = 2,$$

$$\phi_1 = 0.5.$$

(a.) Is this model causal?

Solution The corresponding lag polynomial is $(1 - 0.5L)$ with the root $L = 2$. Since $|L| = |2| > 1$ the process is causal.

(b.) Rewrite this model in its moving average form.

Solution
The MA(∞) representation is:

$$\begin{aligned} y_t &= \frac{\phi_0}{1-\phi_1} + \frac{1}{1-\phi_1 L} a_t \\ &= \frac{\phi_0}{1-\phi_1} + \sum_{j=0}^{\infty} (\phi_1)^j a_{t-j} \\ &= 4 + \sum_{j=0}^{\infty} (0.5)^j a_{t-j} \end{aligned}$$

(c.) Compute the mean and variance of y_t.

Solution
The mean and the variance for AR(1) process is

$$\begin{aligned} E(y_t) &= \frac{\phi_0}{1-\phi_1} = \frac{2}{1-0.5} = 4, \\ Var(y_t) &= \frac{\sigma_a^2}{1-\phi_1^2} = \frac{2}{1-0.5^2} = 2.667 \end{aligned}$$

(d.) Compute the autocorrelations of y_t: ρ_k, $k = 1, 2$.

Solution
The autocorrelations for AR(1) of is given as

$$\rho_k = (\phi_1)^k = \begin{cases} \phi_1 = 0.5 & \text{for } k = 1 \\ (\phi_1)^2 = 0.25 & \text{for } k = 2. \end{cases}$$

(e.) Some clients would like to know whether the mean returns of the company can be negative or not. Given that the sample mean of y_t over the last 100 periods is $\bar{y}_{100} = 1\%$, construct a 95% confidence interval for the mean value of y_t and answer the question of the clients.

Solution

The 95% confidence interval for the mean estimator of the process y_t is given as

$$\overline{y}_T \pm 1.96\sqrt{\frac{\psi^2(1)\sigma_a^2}{T}},$$

where

$$\psi^2(L) = \left(\frac{1}{1-\phi_1 L}\right)^2.$$

Hence, the 95% confidence interval is

$$1\% \pm 1.96\sqrt{\frac{4*2}{100}}$$
$$1\% \pm 0.554$$

Hence, on average the returns of the company cannot be negative.

Question 2 [20 Points]

In the laboratory of the prestigious University of **CarlosHarvard** the following data have been generated: $x_1 = 0.5377$, $x_2 = 1.8339$, $x_3 = -2.2588$ using the model:

$$x_t = u_t - 0.3u_{t-1}$$
$$u_t \overset{iid}{\sim} N(0,1), \tag{2}$$

with $u_0 = 0.8622$. The objective of this experiment is to analyze different forms of prediction. [**IT IS ADVISED TO ROUND CALCULATIONS TO THREE DECIMALS**].

(a.) Given you know the model (**??**) and assuming that you are at $T = 3$, compute the predictions for x_4, x_5 and x_6.

Solution

The best predictor for the moving average process x_t is an expectation conditional on the information available at time $T = 3$. Hence the solution is

$$\begin{aligned} x_4 &= u_4 - 0.3u_3; \text{ and } E(x_4|I_3) = -0.3u_3 = (-0.3)(x_3 + 0.3x_2 + 0.3^2x_1 + 0.3^3u_0) = 0.4911, \\ x_5 &= u_5 - 0.3u_4; \text{ and } E(x_5|I_3) = 0, \\ x_6 &= u_6 - 0.3u_5; \text{ and } E(x_6|I_3) = 0. \end{aligned}$$

(b.) Researchers at other prestigious research center of the University of **GetafeNYU** think that they can predict in a faster way by using only a mean of the process as a predictor. Thus they do not need to know the model that generates the data. Find the mean of the process and calculate predictions of x_4, x_5 and x_6 using the information available at time $T = 3$.

Solution

Prediction of $x_4 = \overline{x} = (x_1 + x_2 + x_3)/3 = 0.0376$,
Prediction of $x_5 = \overline{x} = (x_1 + x_2 + x_3)/3 = 0.0376$,
Prediction of $x_6 = \overline{x} = (x_1 + x_2 + x_3)/3 = 0.0376$.

(c.) The two research centers want to compare their predictions. To do that, they generate, using the true model, the corresponding observations: $x_4 = 0.3188$, $x_5 = -1.3077$ and $x_6 = -0.4336$. For each prediction, calculate the square of the prediction error. Which research center should win?

Solution

For the first model the squared prediction errors are

$$\begin{aligned}(e_4)^2 &= (x_4 - E(x_4|I_3))^2 = (0.3188 - 0.4911)^2 = 0.0297\\ (e_5)^2 &= (x_5 - E(x_5|I_3))^2 = (-1.3077 - 0)^2 = 1.7101\\ (e_6)^2 &= (x_6 - E(x_6|I_3))^2 = (-0.4336 - 0)^2 = 0.188.\end{aligned}$$

For the first model the squared prediction errors are

$$\begin{aligned}(e_4)^2 &= (x_4 - \overline{x})^2 = (0.3188 - 0.0376)^2 = 0.0791\\ (e_5)^2 &= (x_5 - \overline{x})^2 = (-1.3077 - 0.0376)^2 = 1.8098\\ (e_6)^2 &= (x_6 - \overline{x})^2 = (-0.4336 - 0.0376)^2 = 0.222.\end{aligned}$$

We have that the all squared errors for the **CarlosHarvard** are smaller when compared to the corresponding squared errors of **GetafeNYU** predictor. Hence **CarlosHarvard** produces better forecasts.

(d.) The team from the University of **GetafeNYU** is not happy for not being won, it says that the University of **CarlosHarvard** could make its predictions since the MA model which generates the data is invertible. Discuss in two lines this comment.

Solution

The team from **GetafeNYU** is right. The model generating the data is invertible ($0.3 < 1$) and thanks to this in section (a) researches from **CarlosHarvard** have been able to compute u_3.

Question 3 [20 Points]

Researchers at the business school of the University of **LSE-UCL** are considering the following dynamic model to study the effect of oil prices (x_t) on the price of gasoline (y_t)

$$y_t = 0.3y_{t-1} + 0.5x_{t-1} + 0.2x_{t-2} + e_t, \tag{3}$$

where the error terms e_t are i.i.d with mean zero and variance equal to 1.

(a.) Write the model (**??**) in terms of the lag operator. Is it a stable model?

Solution

$$C(L)y_t = B(L)x_t + e_t,$$

where

$$\begin{aligned} B(L) &= 0.5L + 0.2L^2, \\ C(L) &= 1 - 0.3L. \end{aligned}$$

Since the root of the equation $C(L) = 1 - 0.3L = 0$ is $|L| = \left|\frac{1}{0.3}\right| > 1$ then model is stable.

(b.) Compute the short-run multiplier m_0 and long-run multiplier m_T.

Solution

The short run multiplier m_0 is:

$$m_0 = \frac{B(0)}{C(0)} = \frac{0}{1} = 0.$$

The total multiplier is

$$m_T = \sum_{j=0}^{\infty} m_j = \frac{B(1)}{C(1)} = \frac{0.7}{0.7} = 1.$$

(c.) Compute the mean lag.

Solution

The mean lag can be computed as

$$\text{Mean Lag} = \frac{B'(1)}{B(1)} - \frac{C'(1)}{C(1)} = 1.71$$

(d.) Compute the median lag.

Solution

Lag	0	1
$\frac{\sum_{j=0}^{L} m_j}{m_T}$	$\frac{m_0}{m_T} = 0$	$\frac{m_0+m_1}{m_T} = \frac{0.5}{1} \geq 0.5$

where the first two multipliers can be found from

$$\begin{aligned} \frac{B(L)}{C(L)} &= \frac{0.5L + 0.2L^2}{1 - 0.3L} = (0.5L + 0.2L^2)(1 + 0.3L + 0.09L^2 + ...) \\ &= (0.5L + (0.2 + 0.15)L^2 + ...) = 0.5L + 0.35L^2 + ... \end{aligned}$$

Hence $m_1 = 0.5$ and $m_2 = 0.35$ and the median lag is 1.