Inferring the Predictability Induced by a Persistent Regressor in a Predictive Threshold Model

Supplementary Online Appendix: Further Simulation Based Properties of $W_T^{ivxc}(\hat{\lambda})$

Jesus Gonzalo & Jean-Yves Pitarakis

Tables S.1a-S.2b present 5% and 2.5% quantiles of the distributions of $W_T^{ivx}(\hat{\lambda})$ and $W_T^{ivxc}(\hat{\lambda})$ across a broad variety of scenarios on the intercepts, non-centrality parameter c and magnitudes of δ ranging between 0.54 and 0.98 with increments of 0.02. We note a very close match between the empirical quantiles of $W_T^{ivxc}(\hat{\lambda})$ and their asymptotic $\chi^2(2)$ counterparts across all magnitudes of σ_{uv} and δ. The quantiles of the standard IVX based Wald statistic $W_T^{ivx}(\hat{\lambda})$ also match closely those of the $\chi^2(2)$ except when large magnitudes of σ_{uv} (e.g. $\sigma_{uv} = -0.9$) are combined with $\delta' s$ near 1. Under the $\{\alpha_1 = \alpha_2, c = 1\}$ scenario for instance we note that the 5% critical value of $W_T^{ivx}(\hat{\lambda})$ ranges between 5.82 under $\delta = 0.54$ and 8.14 under $\delta = 0.98$ whereas that of $W_T^{ivxc}(\hat{\lambda})$ remains stably near its asymptotic counterpart of 5.99 across all magnitudes of δ.

Table S.3 is an additional power exercise using 2.5% critical values for $W_T^{ivxc}(\hat{\lambda})$ and is broadly in line with the simulations presented in our main text. As here δ is allowed to vary between 0.54 and 0.98 we can clearly track its influence on finite sample power. We note a clear monotonic increase of empirical power with δ but with a clear stabilisation when δ crosses the vicinity of 0.8. Under most scenarios power reaches magnitudes close to 100% under $T = 400$.

Tables S.4a-S.5b are designed to evaluate the impact of conditional heteroskedasticity (ruled out by our assumptions) on the size properties of our IVX based Wald statistics. We continue to allow δ to range between 0.54 and 0.98 with increments of 0.2. The error process driving the $y's$ is now parameterised as $u_t\sqrt{h_t}$ with h_t following the GARCH(1,1) process $h_t = \omega_0 + \omega_1 h_{t-1} + \omega_2 u_{t-1}^2$. We set $\omega_1 = 0.80$, $\omega_2 = 0.15$ and $\omega_0 = 1$ while maintaining all other parameters as in our initial DGP. Tables S.4a-S.4b present empirical size estimates under $T = 200$ and a 5% nominal size while Tables S.5a and S.5b repeat the same exercise under $T = 400$. The general message that comes across these simulations is that conditional heteroskedasticity appears to have little impact if at all on the limiting null distribution of the IVX based Wald statistics. Focusing on the case $T = 200$ we note that the empirical sizes of our $W_T^{ivxc}(\hat{\lambda})$ statistic match very closlely their nominal counterpart of 5% across all magnitudes of δ and σ_{uv}. It is also interesting to compare the outcomes of this exercise with say Table 5 in the main text that operated under conditional homoskedasticity. Under $\{T = 200, \sigma_{uv} = -0.9, \alpha_1 = \alpha_2, c = 1, \delta = 0.94\}$ for instance the size associated with $W_T^{ivxc}(\hat{\lambda})$ was 4.25% under homoskedasticity and 4.75% under heteroskedasticity (for a 5% nominal size).

Table S.6 aims to illustrate some of the differences between the $SupB^{ivx}$ statistic developed in GP2012 (and its $SupB^{ivxc}$ variant) and the $W_T^{ivx}(\hat{\lambda})$ developed here and more importantly highlight their very distinct roles. In its top panel for instance we consider DGPs with $\alpha_1 \neq \alpha_2$ but $\beta_1 = \beta_2 = 0$ and document the suitability of $W_T^{ivxc}(\hat{\lambda})$ in this context as $SupB^{ivxc}$ is shown to lead to 100% rejections of the null of no regime specific predictability.

Table S.1a: **5% Empirical Quantiles of** $W_T^{ivx}(\hat{\lambda})$ **and** $W_T^{ivxc}(\hat{\lambda})$

	$\alpha_1 = \alpha_2$, $c = 1$, $T = 1000$																						
δ	0.54	0.56	0.58	0.60	0.62	0.64	0.66	0.68	0.70	0.72	0.74	0.76	0.78	0.80	0.82	0.84	0.86	0.88	0.90	0.92	0.94	0.96	0.98
$\sigma_{uv} = 0.0$																							
$W_T^{ivx}(\hat{\lambda})$	5.97	6.03	6.12	6.11	6.18	6.16	6.12	6.13	6.09	6.15	6.07	6.14	6.11	6.04	6.05	6.03	6.21	6.20	6.23	6.17	6.27	6.22	6.20
$W_T^{ivxc}(\hat{\lambda})$	5.97	6.03	6.12	6.10	6.17	6.16	6.11	6.13	6.09	6.14	6.07	6.12	6.11	6.04	6.03	6.03	6.21	6.19	6.21	6.16	6.22	6.17	6.19
$\sigma_{uv} = -0.3$																							
$W_T^{ivx}(\hat{\lambda})$	5.78	5.82	5.90	5.89	5.93	6.09	6.22	6.27	6.40	6.40	6.46	6.48	6.45	6.33	6.38	6.46	6.45	6.48	6.49	6.47	6.51	6.50	6.47
$W_T^{ivxc}(\hat{\lambda})$	5.76	5.81	5.87	5.87	5.90	6.07	6.18	6.22	6.31	6.34	6.30	6.32	6.32	6.21	6.24	6.30	6.30	6.32	6.33	6.25	6.32	6.27	6.21
$\sigma_{uv} = -0.6$																							
$W_T^{ivx}(\hat{\lambda})$	5.99	6.10	6.08	5.98	6.10	6.12	6.17	6.27	6.24	6.30	6.32	6.42	6.40	6.40	6.45	6.49	6.52	6.53	6.66	6.80	6.88	7.02	7.11
$W_T^{ivxc}(\hat{\lambda})$	5.90	5.98	5.95	5.94	6.03	5.90	5.97	6.07	5.98	5.96	5.99	5.98	5.97	5.96	5.85	5.84	5.72	5.78	5.78	5.75	5.77	5.73	5.72
$\sigma_{uv} = -0.9$																							
$W_T^{ivx}(\hat{\lambda})$	5.82	5.84	5.96	5.99	6.04	6.13	6.21	6.37	6.58	6.65	6.79	6.75	6.94	7.15	7.26	7.31	7.44	7.47	7.51	7.75	7.91	8.06	8.14
$W_T^{ivxc}(\hat{\lambda})$	5.63	5.62	5.67	5.66	5.75	5.85	5.93	6.00	6.01	5.96	5.92	5.95	5.92	6.00	5.94	5.85	5.87	5.81	5.69	5.64	5.65	5.68	5.63
	$\alpha_1 \neq \alpha_2$, $c = 1$, $T = 1000$																						
$\sigma_{uv} = 0.0$																							
$W_T^{ivx}(\hat{\lambda})$	5.60	5.53	5.56	5.54	5.57	5.58	5.59	5.63	5.70	5.65	5.71	5.76	5.88	5.93	6.01	6.00	5.96	5.91	5.86	5.88	5.89	5.86	5.88
$W_T^{ivxc}(\hat{\lambda})$	5.60	5.53	5.56	5.54	5.56	5.58	5.59	5.63	5.70	5.65	5.71	5.75	5.88	5.91	5.93	6.00	5.93	5.88	5.85	5.87	5.89	5.84	5.87
$\sigma_{uv} = -0.3$																							
$W_T^{ivx}(\hat{\lambda})$	6.26	6.23	6.24	6.37	6.35	6.35	6.43	6.48	6.46	6.47	6.49	6.51	6.45	6.57	6.59	6.68	6.59	6.54	6.63	6.65	6.62	6.69	6.77
$W_T^{ivxc}(\hat{\lambda})$	6.26	6.18	6.23	6.34	6.34	6.25	6.35	6.40	6.41	6.36	6.33	6.34	6.39	6.33	6.33	6.31	6.27	6.38	6.31	6.21	6.24	6.20	6.21
$\sigma_{uv} = -0.6$																							
$W_T^{ivx}(\hat{\lambda})$	5.72	5.73	5.76	5.88	5.96	5.97	5.98	5.98	5.84	5.84	5.96	6.06	6.16	6.20	6.22	6.17	6.22	6.22	6.33	6.41	6.56	6.65	6.72
$W_T^{ivxc}(\hat{\lambda})$	5.68	5.70	5.66	5.77	5.80	5.91	5.73	5.63	5.66	5.49	5.54	5.60	5.56	5.58	5.53	5.52	5.50	5.50	5.53	5.54	5.56	5.58	5.58
$\sigma_{uv} = -0.9$																							
$W_T^{ivx}(\hat{\lambda})$	6.23	6.25	6.32	6.44	6.45	6.61	6.76	6.85	6.82	6.88	7.02	7.10	7.20	7.44	7.51	7.71	7.89	7.88	7.99	7.98	8.09	8.20	8.18
$W_T^{ivxc}(\hat{\lambda})$	5.99	6.07	6.07	6.16	6.13	6.22	6.22	6.12	6.16	6.23	6.16	6.04	6.00	5.97	5.90	5.86	5.76	5.75	5.79	5.77	5.74	5.67	5.71
$\chi^2(2)$	5.99	5.99	5.99	5.99	5.99	5.99	5.99	5.99	5.99	5.99	5.99	5.99	5.99	5.99	5.99	5.99	5.99	5.99	5.99	5.99	5.99	5.99	5.99

Table S.1b: **5% Empirical Quantiles of $W_T^{ivx}(\hat{\lambda})$ and $W_T^{ivxc}(\hat{\lambda})$ (contd.)**

	$\alpha_1 = \alpha_2$, $c = 10$, $T = 1000$																						
δ	0.54	0.56	0.58	0.60	0.62	0.64	0.66	0.68	0.70	0.72	0.74	0.76	0.78	0.80	0.82	0.84	0.86	0.88	0.90	0.92	0.94	0.96	0.98
$\sigma_{uv} = 0.0$																							
$W_T^{ivx}(\hat{\lambda})$	5.86	5.94	6.01	5.99	5.98	5.98	6.03	6.02	6.07	6.13	6.15	6.12	6.13	6.21	6.31	6.19	6.22	6.14	6.06	6.04	6.02	6.08	6.14
$W_T^{ivxc}(\hat{\lambda})$	5.86	5.94	6.01	5.99	5.98	5.98	6.03	6.01	6.06	6.13	6.15	6.12	6.13	6.21	6.31	6.19	6.22	6.14	6.06	6.04	6.01	6.08	6.14
$\sigma_{uv} = -0.3$																							
$W_T^{ivx}(\hat{\lambda})$	5.94	5.91	5.83	5.76	5.85	5.91	5.92	5.96	5.94	6.00	6.12	6.13	6.11	6.09	6.14	6.20	6.26	6.28	6.27	6.17	6.22	6.26	6.28
$W_T^{ivxc}(\hat{\lambda})$	5.94	5.91	5.83	5.76	5.82	5.91	5.92	5.91	5.94	5.99	6.10	6.09	6.07	6.06	6.12	6.19	6.21	6.25	6.18	6.12	6.15	6.21	6.20
$\sigma_{uv} = -0.6$																							
$W_T^{ivx}(\hat{\lambda})$	5.86	5.87	5.80	5.95	6.05	6.00	6.01	6.05	6.09	6.11	6.16	6.20	6.18	6.13	6.08	6.05	6.06	6.02	6.03	6.07	6.00	5.97	5.94
$W_T^{ivxc}(\hat{\lambda})$	5.81	5.78	5.78	5.94	5.97	5.93	5.99	6.01	6.08	6.08	6.13	6.12	6.11	6.07	6.01	5.95	5.96	5.94	5.94	5.96	5.91	5.86	5.79
$\sigma_{uv} = -0.9$																							
$W_T^{ivx}(\hat{\lambda})$	6.18	6.09	6.18	6.24	6.33	6.35	6.40	6.38	6.36	6.31	6.40	6.44	6.46	6.45	6.57	6.56	6.66	6.73	6.69	6.75	6.73	6.79	6.83
$W_T^{ivxc}(\hat{\lambda})$	6.14	6.03	6.13	6.19	6.27	6.30	6.37	6.34	6.27	6.26	6.33	6.31	6.33	6.34	6.33	6.38	6.37	6.37	6.36	6.30	6.33	6.25	6.23
	$\alpha_1 \neq \alpha_2$, $c = 10$, $T = 1000$																						
$\sigma_{uv} = 0.0$																							
$W_T^{ivx}(\hat{\lambda})$	5.93	5.90	5.89	5.87	5.85	5.81	5.79	5.74	5.71	5.71	5.69	5.71	5.71	5.68	5.67	5.62	5.60	5.64	5.69	5.69	5.73	5.79	5.83
$W_T^{ivxc}(\hat{\lambda})$	5.93	5.90	5.89	5.87	5.85	5.81	5.79	5.74	5.71	5.71	5.69	5.70	5.71	5.68	5.67	5.62	5.60	5.64	5.69	5.69	5.73	5.78	5.83
$\sigma_{uv} = -0.3$																							
$W_T^{ivx}(\hat{\lambda})$	5.98	5.94	6.02	6.11	6.04	6.09	6.06	6.01	6.11	6.15	6.13	6.15	6.24	6.23	6.20	6.15	6.26	6.30	6.30	6.31	6.27	6.26	6.21
$W_T^{ivxc}(\hat{\lambda})$	5.98	5.93	6.02	6.11	6.04	6.08	6.06	6.00	6.10	6.15	6.12	6.12	6.19	6.19	6.20	6.13	6.22	6.23	6.25	6.24	6.20	6.18	6.15
$\sigma_{uv} = -0.6$																							
$W_T^{ivx}(\hat{\lambda})$	6.12	6.22	6.08	6.00	5.96	5.93	5.95	5.97	5.91	6.02	6.04	6.08	6.02	6.00	5.99	5.96	5.96	6.02	5.97	6.00	6.04	6.09	6.14
$W_T^{ivxc}(\hat{\lambda})$	6.11	6.20	6.05	5.99	5.95	5.92	5.92	5.92	5.90	5.97	6.03	6.05	6.02	5.92	5.90	5.84	5.87	5.87	5.86	5.86	5.88	5.97	5.95
$\sigma_{uv} = -0.9$																							
$W_T^{ivx}(\hat{\lambda})$	6.07	6.07	6.10	6.05	6.21	6.21	6.23	6.34	6.35	6.32	6.31	6.31	6.36	6.43	6.54	6.64	6.74	6.71	6.79	6.75	6.68	6.74	6.87
$W_T^{ivxc}(\hat{\lambda})$	6.02	6.05	6.02	6.00	6.14	6.19	6.16	6.29	6.27	6.24	6.16	6.18	6.17	6.30	6.35	6.39	6.36	6.40	6.41	6.36	6.33	6.30	6.22
$\chi^2(2)$	5.99	5.99	5.99	5.99	5.99	5.99	5.99	5.99	5.99	5.99	5.99	5.99	5.99	5.99	5.99	5.99	5.99	5.99	5.99	5.99	5.99	5.99	5.99

Table S.2a: **2.5% Empirical Quantiles of $W_T^{ivx}(\hat{\lambda})$ and $W_T^{ivxc}(\hat{\lambda})$**

	$\alpha_1 = \alpha_2$, $c = 1$, $T = 1000$																						
δ	0.54	0.56	0.58	0.60	0.62	0.64	0.66	0.68	0.70	0.72	0.74	0.76	0.78	0.80	0.82	0.84	0.86	0.88	0.90	0.92	0.94	0.96	0.98
$\sigma_{uv} = 0.0$																							
$W_T^{ivx}(\hat{\lambda})$	7.23	7.16	7.14	7.10	7.15	7.16	7.19	7.22	7.22	7.19	7.09	7.13	7.22	7.31	7.23	7.31	7.25	7.16	7.25	7.16	7.12	7.14	7.17
$W_T^{ivxc}(\hat{\lambda})$	7.23	7.16	7.14	7.09	7.15	7.16	7.18	7.22	7.20	7.17	7.09	7.12	7.22	7.31	7.23	7.29	7.24	7.15	7.23	7.13	7.09	7.14	7.15
$\sigma_{uv} = -0.3$																							
$W_T^{ivx}(\hat{\lambda})$	7.23	7.34	7.36	7.46	7.50	7.59	7.48	7.60	7.51	7.67	7.60	7.75	7.78	7.84	7.82	7.80	7.82	7.77	7.81	7.93	7.97	7.92	7.95
$W_T^{ivxc}(\hat{\lambda})$	7.22	7.32	7.32	7.43	7.43	7.49	7.45	7.51	7.44	7.60	7.54	7.69	7.68	7.53	7.45	7.57	7.65	7.68	7.65	7.67	7.60	7.63	7.57
$\sigma_{uv} = -0.6$																							
$W_T^{ivx}(\hat{\lambda})$	7.35	7.48	7.54	7.67	7.55	7.63	7.65	7.55	7.66	7.60	7.65	7.77	8.04	8.05	8.02	8.17	8.33	8.26	8.37	8.49	8.54	8.67	8.58
$W_T^{ivxc}(\hat{\lambda})$	7.31	7.30	7.42	7.48	7.46	7.37	7.39	7.32	7.19	7.19	7.14	7.24	7.23	7.22	7.23	7.28	7.35	7.39	7.26	7.20	7.14	7.14	7.10
$\sigma_{uv} = -0.9$																							
$W_T^{ivx}(\hat{\lambda})$	7.02	7.00	7.17	7.36	7.42	7.53	7.57	7.81	8.12	8.35	8.50	8.69	8.82	9.05	9.15	9.34	9.43	9.52	9.63	9.66	9.99	9.78	10.16
$W_T^{ivxc}(\hat{\lambda})$	6.78	6.74	6.78	6.90	7.07	6.96	7.21	7.33	7.35	7.33	7.37	7.46	7.54	7.60	7.50	7.49	7.52	7.35	7.20	7.18	7.24	7.20	7.23
	$\alpha_1 \neq \alpha_2$, $c = 1$, $T = 1000$																						
$\sigma_{uv} = 0.0$																							
$W_T^{ivx}(\hat{\lambda})$	6.72	6.80	6.77	6.87	6.93	6.83	6.74	6.95	7.04	7.08	7.10	7.09	7.05	6.94	6.85	6.85	6.79	6.87	6.82	6.92	6.89	6.91	6.94
$W_T^{ivxc}(\hat{\lambda})$	6.72	6.80	6.77	6.87	6.91	6.83	6.74	6.95	7.04	7.08	7.10	7.07	7.02	6.92	6.85	6.81	6.79	6.84	6.82	6.92	6.88	6.90	6.93
$\sigma_{uv} = -0.3$																							
$W_T^{ivx}(\hat{\lambda})$	8.03	7.94	7.79	7.88	7.88	7.82	7.65	7.73	7.86	7.91	7.92	8.01	8.17	8.14	8.08	8.02	8.17	8.13	8.13	8.08	8.11	8.14	8.29
$W_T^{ivxc}(\hat{\lambda})$	8.02	7.93	7.78	7.87	7.81	7.80	7.65	7.70	7.80	7.73	7.84	7.89	8.10	8.05	7.94	7.93	7.82	7.80	7.67	7.59	7.48	7.58	7.51
$\sigma_{uv} = -0.6$																							
$W_T^{ivx}(\hat{\lambda})$	6.99	7.03	7.08	7.21	7.39	7.30	7.44	7.31	7.47	7.59	7.60	7.55	7.54	7.57	7.61	7.65	7.66	7.73	7.81	7.96	8.04	8.16	8.23
$W_T^{ivxc}(\hat{\lambda})$	6.97	6.88	6.93	7.18	7.23	7.11	7.20	7.17	7.18	7.26	7.13	7.16	6.94	6.86	7.02	7.00	7.02	7.01	7.02	7.05	7.09	7.09	7.13
$\sigma_{uv} = -0.9$																							
$W_T^{ivx}(\hat{\lambda})$	7.24	7.38	7.31	7.49	7.64	7.84	7.88	7.88	8.11	8.31	8.45	8.60	8.78	8.85	8.96	9.09	9.15	9.27	9.39	9.49	9.57	9.69	9.81
$W_T^{ivxc}(\hat{\lambda})$	7.02	7.15	7.11	7.27	7.29	7.38	7.42	7.47	7.50	7.48	7.44	7.35	7.40	7.31	7.31	7.43	7.23	7.06	7.13	7.08	7.04	7.05	7.00
$\chi^2(2)$	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38

Table S.2b: **2.5% Empirical Quantiles of $W_T^{ivx}(\hat{\lambda})$ and $W_T^{ivxc}(\hat{\lambda})$ (contd.)**

δ	0.54	0.56	0.58	0.60	0.62	0.64	0.66	0.68	0.70	0.72	0.74	0.76	0.78	0.80	0.82	0.84	0.86	0.88	0.90	0.92	0.94	0.96	0.98
$\alpha_1 = \alpha_2$, $c = 10$, $T = 1000$																							
$\sigma_{uv} = 0.0$																							
$W_T^{ivx}(\hat{\lambda})$	6.98	6.97	6.96	7.15	7.40	7.51	7.55	7.56	7.59	7.58	7.50	7.42	7.46	7.40	7.39	7.52	7.51	7.60	7.55	7.60	7.62	7.65	7.66
$W_T^{ivxc}(\hat{\lambda})$	6.98	6.97	6.96	7.15	7.40	7.51	7.55	7.56	7.59	7.58	7.50	7.42	7.46	7.40	7.39	7.52	7.51	7.60	7.55	7.59	7.62	7.65	7.66
$\sigma_{uv} = -0.3$																							
$W_T^{ivx}(\hat{\lambda})$	7.36	7.59	7.39	7.33	7.36	7.38	7.23	7.26	7.32	7.38	7.34	7.51	7.54	7.54	7.69	7.59	7.63	7.69	7.71	7.69	7.72	7.68	7.65
$W_T^{ivxc}(\hat{\lambda})$	7.35	7.59	7.38	7.29	7.35	7.37	7.21	7.19	7.31	7.37	7.32	7.48	7.51	7.53	7.63	7.57	7.58	7.67	7.70	7.67	7.66	7.65	7.60
$\sigma_{uv} = -0.6$																							
$W_T^{ivx}(\hat{\lambda})$	7.59	7.40	7.47	7.36	7.34	7.41	7.38	7.40	7.30	7.21	7.25	7.28	7.37	7.52	7.47	7.47	7.42	7.33	7.29	7.29	7.24	7.31	7.34
$W_T^{ivxc}(\hat{\lambda})$	7.46	7.37	7.46	7.35	7.33	7.29	7.32	7.34	7.25	7.10	7.19	7.23	7.28	7.31	7.29	7.32	7.26	7.09	7.05	7.05	7.10	7.11	7.12
$\sigma_{uv} = -0.9$																							
$W_T^{ivx}(\hat{\lambda})$	7.60	7.74	7.68	7.71	7.85	8.05	8.00	7.96	7.98	8.16	8.30	8.14	8.06	8.10	8.01	8.14	8.36	8.22	8.21	8.26	8.35	8.49	8.60
$W_T^{ivxc}(\hat{\lambda})$	7.56	7.59	7.59	7.62	7.74	7.78	7.89	7.83	7.76	7.96	7.98	7.96	7.88	7.83	7.85	7.88	7.89	7.93	7.85	7.86	7.84	7.87	7.88
$\alpha_1 \neq \alpha_2$, $c = 10$, $T = 1000$																							
$\sigma_{uv} = 0.0$																							
$W_T^{ivx}(\hat{\lambda})$	7.13	7.21	7.25	7.14	7.09	7.03	7.03	7.11	7.11	6.95	6.98	6.95	7.04	7.00	7.04	7.04	7.14	7.18	7.20	7.20	7.20	7.17	7.22
$W_T^{ivxc}(\hat{\lambda})$	7.13	7.21	7.25	7.14	7.09	7.03	7.03	7.11	7.11	6.95	6.98	6.95	7.04	7.00	7.04	7.04	7.14	7.18	7.20	7.20	7.20	7.17	7.22
$\sigma_{uv} = -0.3$																							
$W_T^{ivx}(\hat{\lambda})$	7.15	7.18	7.11	7.16	7.09	7.18	7.30	7.32	7.22	7.50	7.50	7.50	7.57	7.69	7.77	7.72	7.67	7.78	7.76	7.81	7.74	7.70	7.65
$W_T^{ivxc}(\hat{\lambda})$	7.15	7.18	7.11	7.16	7.09	7.16	7.30	7.32	7.22	7.50	7.50	7.49	7.55	7.68	7.72	7.69	7.65	7.76	7.66	7.72	7.67	7.70	7.63
$\sigma_{uv} = -0.6$																							
$W_T^{ivx}(\hat{\lambda})$	7.74	7.62	7.60	7.53	7.62	7.65	7.79	7.73	7.70	7.89	7.85	7.74	7.73	7.85	7.90	7.85	7.82	7.93	7.87	7.95	7.85	7.79	7.84
$W_T^{ivxc}(\hat{\lambda})$	7.72	7.60	7.58	7.50	7.62	7.64	7.72	7.69	7.65	7.74	7.80	7.73	7.72	7.77	7.77	7.70	7.74	7.74	7.71	7.70	7.66	7.66	7.64
$\sigma_{uv} = -0.9$																							
$W_T^{ivx}(\hat{\lambda})$	7.55	7.60	7.72	7.70	7.62	7.52	7.52	7.60	7.57	7.62	7.90	7.91	7.89	7.94	8.00	7.94	7.98	7.96	8.07	8.18	8.22	8.36	8.43
$W_T^{ivxc}(\hat{\lambda})$	7.49	7.54	7.59	7.57	7.52	7.41	7.43	7.51	7.46	7.58	7.72	7.81	7.75	7.82	7.81	7.75	7.68	7.74	7.73	7.71	7.65	7.69	7.69
$\chi^2(2)$	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38

Table S.3: **Empirical Power of $W_T^{ivxc}(\hat{\lambda})$ (2.5% Nominal)**

$W_T^{ivxc}(\hat{\lambda})$						δ													δ					
$\beta_1 = 0$	0.54	0.58	0.62	0.66	0.7	0.74	0.78	0.82	0.86	0.90	0.94	0.98	0.54	0.58	0.62	0.66	0.70	0.74	0.78	0.82	0.86	0.90	0.94	0.98
$\beta_2 = 0.025$					$\alpha_1 = \alpha_2$, $c = 1$, $\sigma_{uv} = -0.6$													$\alpha_1 \neq \alpha_2$, $c = 1$, $\sigma_{uv} = -0.6$						
T=200	8.2	10.4	12.0	13.6	15.8	17.5	19.8	21.6	22.5	23.4	24.6	25.0	12.1	14.5	16.9	19.5	21.9	24.4	26.6	28.3	30.3	31.9	32.9	33.7
T=400	28.6	35.1	42.1	48.6	56.5	61.9	65.7	69.7	72.5	75.3	76.6	78.2	36.8	44.3	51.4	58.6	64.8	69.2	74.1	77.2	80.3	82.2	83.1	84.1
T=1000	85.3	90.3	93.0	94.9	96.0	96.7	98.0	98.8	99.6	99.7	100.0	100.0	87.0	90.9	93.4	95.2	96.2	97.1	98.0	98.8	99.3	99.5	99.8	99.9
$\beta_2 = 0.05$																								
T=200	35.3	41.0	47.1	53.2	58.2	61.9	64.8	67.2	69.7	71.5	73.3	74.2	42.8	48.3	54.7	60.7	65.0	69.2	72.2	74.9	76.4	78.1	78.9	79.8
T=400	78.8	84.9	88.3	91.4	94.0	95.9	97.0	98.1	98.6	99.0	99.1	99.5	83.3	87.3	90.4	92.0	94.0	95.6	96.6	98.0	98.7	99.2	99.3	99.5
T=1000	96.7	97.2	97.6	98.4	98.7	99.1	99.5	99.7	99.9	100.0	100.0	100.0	96.1	96.7	97.5	98.1	98.6	99.2	99.6	99.8	100.0	100.0	100.0	100.0
$\beta_2 = 0.025$					$\alpha_1 = \alpha_2$, $c = 10$, $\sigma_{uv} = -0.6$													$\alpha_1 \neq \alpha_2$, $c = 10$, $\sigma_{uv} = -0.6$						
T=200	5.7	6.3	6.7	7.2	7.2	7.5	7.7	7.7	7.7	7.8	7.9	8.3	7.2	8.0	8.7	9.3	9.6	10.0	10.5	10.8	11.5	12.0	12.3	12.2
T=400	14.8	17.3	19.5	21.6	24.0	25.5	27.0	28.5	29.9	31.0	31.5	32.1	23.4	27.5	31.0	34.3	38.1	41.7	44.4	46.2	48.2	49.4	50.0	50.8
T=1000	75.8	83.9	91.2	94.4	96.4	97.6	98.2	98.7	98.9	99.2	99.3	99.3	91.6	95.5	98.1	99.3	99.5	99.7	99.8	99.9	99.9	99.9	99.9	99.9
$\beta_2 = 0.05$																								
T=200	18.0	19.8	21.9	23.2	23.2	24.6	25.7	27.3	28.0	28.9	29.5	29.8	27.2	31.1	34.0	37.4	40.1	42.4	44.0	44.9	46.1	47.8	48.4	48.8
T=400	63.8	70.4	75.6	78.9	82.7	84.9	86.6	88.6	89.8	91.1	91.8	92.3	81.6	86.7	90.8	93.2	94.9	95.9	96.5	97.2	97.4	97.7	97.8	97.9
T=1000	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	99.6	99.8	99.8	99.9	99.9	99.9	99.9	100.0	100.0	100.0	100.0	100.0
$\beta_2 = 0.025$					$\alpha_1 = \alpha_2$, $c = 1$, $\sigma_{uv} = -0.9$													$\alpha_1 \neq \alpha_2$, $c = 1$, $\sigma_{uv} = -0.9$						
T=200	8.8	10.8	12.3	13.8	15.7	17.1	18.5	18.9	19.3	20.1	20.8	21.3	13.1	15.4	17.7	20.2	22.0	24.4	26.0	28.2	29.3	30.3	31.1	31.3
T=400	30.8	38.2	46.2	53.4	61.0	67.3	71.9	75.4	77.6	79.2	79.8	80.2	39.9	48.8	57.2	64.9	71.7	77.1	80.1	82.9	84.7	85.4	86.0	86.1
T=1000	87.3	92.0	94.8	96.1	96.9	97.4	98.3	99.0	99.5	99.7	99.8	99.8	88.8	92.5	94.4	96.0	97.3	98.2	98.9	99.2	99.6	99.8	99.9	99.9
$\beta_2 = 0.05$																								
T=200	36.6	43.6	49.9	56.7	62.7	67.2	71.0	74.3	76.6	77.7	78.6	78.8	46.1	53.1	59.1	65.2	70.5	74.1	77.5	80.2	82.3	83.0	84.0	84.3
T=400	83.2	88.3	90.9	93.2	95.5	97.0	97.5	98.3	98.7	99.1	99.2	99.2	86.0	89.2	91.7	93.5	94.9	96.2	97.4	98.0	98.7	98.8	99.0	99.1
T=1000	97.0	97.7	98.4	98.8	99.0	99.5	99.6	99.8	99.9	100.0	100.0	100.0	96.2	96.8	97.1	97.8	98.7	99.0	99.5	99.8	100.0	100.0	100.0	100.0
$\beta_2 = 0.025$					$\alpha_1 = \alpha_2$, $c = 10$, $\sigma_{uv} = -0.9$													$\alpha_1 \neq \alpha_2$, $c = 10$, $\sigma_{uv} = -0.9$						
T=200	5.9	6.5	6.5	6.6	6.6	7.4	7.9	7.8	8.1	8.2	8.4	8.5	6.8	8.0	8.7	9.3	9.8	10.2	10.7	10.7	11.0	11.4	11.8	11.6
T=400	14.3	16.7	19.6	22.4	24.7	27.0	28.7	30.1	31.6	32.6	33.3	34.1	21.8	26.3	31.2	35.5	40.3	43.2	46.0	48.4	50.1	51.9	53.2	53.9
T=1000	81.1	89.6	95.1	97.8	99.1	99.7	99.9	100.0	100.0	100.0	100.0	100.0	93.5	97.3	99.2	99.9	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
$\beta_2 = 0.05$																								
T=200	18.0	21.0	22.4	24.5	24.5	26.7	28.7	29.3	30.7	31.3	32.1	32.6	26.4	29.5	32.9	35.5	38.9	41.6	43.8	46.4	48.1	49.3	50.0	50.7
T=400	67.9	74.1	80.3	85.4	89.3	92.4	94.4	95.3	96.0	96.4	96.6	96.9	84.4	89.9	94.0	96.5	97.9	98.7	99.1	99.2	99.4	99.5	99.5	99.6
T=1000	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0

Table S.4a: **Empirical Size of $W_T^{ivx}(\hat{\lambda})$ and $W_T^{ivxc}(\hat{\lambda})$ under Conditional Heteroskedasticity (T=200)**

5%	$\alpha_1 = \alpha_2,\ c = 1,\ T = 200$																						
δ	0.54	0.56	0.58	0.60	0.62	0.64	0.66	0.68	0.70	0.72	0.74	0.76	0.78	0.80	0.82	0.84	0.86	0.88	0.90	0.92	0.94	0.96	0.98
$\sigma_{uv} = 0.0$																							
$W_T^{ivx}(\hat{\lambda})$	4.40	4.45	4.35	4.55	4.80	5.05	5.15	5.25	5.20	5.20	5.20	5.10	5.30	5.30	5.50	5.55	5.40	5.25	5.40	5.25	5.35	5.25	5.25
$W_T^{ivxc}(\hat{\lambda})$	4.40	4.45	4.35	4.55	4.70	5.00	5.10	5.25	5.10	5.20	5.15	5.00	5.15	5.30	5.45	5.35	5.25	5.10	5.25	5.15	5.10	4.95	5.00
$\sigma_{uv} = -0.3$																							
$W_T^{ivx}(\hat{\lambda})$	4.80	5.05	5.05	5.30	5.40	5.55	5.55	6.00	6.15	6.25	6.55	6.80	6.65	6.60	6.60	6.35	6.55	6.70	6.75	6.70	6.65	6.75	6.80
$W_T^{ivxc}(\hat{\lambda})$	4.65	4.90	4.95	5.10	5.20	5.25	5.25	5.70	5.75	5.75	6.05	6.40	6.30	6.15	5.95	5.95	5.85	5.80	5.75	5.95	5.85	5.80	5.75
$\sigma_{uv} = -0.6$																							
$W_T^{ivx}(\hat{\lambda})$	5.15	5.55	5.45	5.80	6.05	6.20	6.55	6.65	7.10	7.25	7.70	7.60	8.05	8.15	8.15	8.35	8.45	8.40	8.60	8.60	8.60	8.65	8.85
$W_T^{ivxc}(\hat{\lambda})$	4.90	5.30	5.25	5.25	5.50	5.50	5.55	5.70	5.75	5.70	5.75	5.90	6.05	6.15	6.10	6.00	5.85	5.35	5.30	5.35	5.35	5.50	5.55
$\sigma_{uv} = -0.9$																							
$W_T^{ivx}(\hat{\lambda})$	5.15	5.35	5.85	6.20	6.75	7.20	7.65	7.90	8.05	8.40	8.95	9.40	9.50	9.85	10.20	10.80	11.10	11.45	11.95	11.95	12.05	12.00	12.30
$W_T^{ivxc}(\hat{\lambda})$	4.35	4.65	4.75	4.90	4.85	5.35	5.20	5.25	5.35	5.50	5.55	5.35	5.25	5.25	5.15	4.95	5.05	4.85	4.90	4.90	4.75	4.65	4.55
	$\alpha_1 \neq \alpha_2,\ c = 1,\ T = 200$																						
δ	0.54	0.56	0.58	0.60	0.62	0.64	0.66	0.68	0.70	0.72	0.74	0.76	0.78	0.80	0.82	0.84	0.86	0.88	0.90	0.92	0.94	0.96	0.98
$\sigma_{uv} = 0.0$																							
$W_T^{ivx}(\hat{\lambda})$	3.95	4.35	4.30	4.50	4.75	4.80	5.05	5.20	5.70	5.90	5.95	5.90	5.90	5.75	5.70	5.80	5.50	5.60	5.75	5.80	5.85	5.90	5.95
$W_T^{ivxc}(\hat{\lambda})$	3.90	4.35	4.30	4.50	4.65	4.70	4.95	5.15	5.65	5.85	5.80	5.70	5.60	5.55	5.50	5.50	5.30	5.40	5.60	5.60	5.60	5.55	5.60
$\sigma_{uv} = -0.3$																							
$W_T^{ivx}(\hat{\lambda})$	4.10	4.10	4.25	4.30	4.35	4.45	4.45	4.50	4.40	4.55	4.60	4.90	5.10	5.20	5.35	5.40	5.55	5.60	5.60	5.75	5.85	5.85	5.90
$W_T^{ivxc}(\hat{\lambda})$	4.00	4.05	4.10	4.20	4.30	4.20	4.20	4.15	4.15	4.40	4.40	4.45	4.60	4.55	4.75	4.80	4.95	4.90	4.95	5.00	5.00	5.00	5.05
$\sigma_{uv} = -0.6$																							
$W_T^{ivx}(\hat{\lambda})$	4.55	4.70	4.70	4.90	5.05	5.20	5.25	5.40	5.45	5.65	5.60	5.95	5.85	6.30	6.55	6.65	6.90	7.35	7.40	7.30	7.30	7.20	7.15
$W_T^{ivxc}(\hat{\lambda})$	4.30	4.30	4.40	4.40	4.45	4.50	4.50	4.55	4.35	4.15	4.45	4.55	4.50	4.65	4.80	4.80	4.70	4.60	4.55	4.45	4.40	4.25	4.10
$\sigma_{uv} = -0.9$																							
$W_T^{ivx}(\hat{\lambda})$	5.30	5.50	5.85	6.50	6.60	7.05	7.20	7.60	7.90	8.25	8.40	8.80	8.95	9.00	9.40	10.30	10.80	10.80	11.15	11.50	11.45	11.75	11.85
$W_T^{ivxc}(\hat{\lambda})$	4.35	4.45	4.60	5.10	5.15	5.25	5.35	5.40	5.50	5.65	5.55	5.35	5.25	5.30	5.25	5.15	5.35	5.35	5.30	5.10	5.20	5.10	5.05

Table S.4b: **Empirical Size of $W_T^{ivx}(\hat{\lambda})$ and $W_T^{ivxc}(\hat{\lambda})$ under Conditional Heteroskedasticity (T=200), contd.**

	$\alpha_1 = \alpha_2$, $c = 10$, $T = 200$																						
δ	0.54	0.56	0.58	0.60	0.62	0.64	0.66	0.68	0.70	0.72	0.74	0.76	0.78	0.80	0.82	0.84	0.86	0.88	0.90	0.92	0.94	0.96	0.98
$\sigma_{uv} = -0.0$																							
$W_T^{ivx}(\hat{\lambda})$	5.10	5.20	5.60	5.70	6.00	6.25	6.30	6.40	6.45	6.25	6.15	6.25	6.40	6.30	6.20	6.10	6.05	6.00	6.00	5.85	5.90	5.90	5.80
$W_T^{ivxc}(\hat{\lambda})$	5.10	5.20	5.60	5.70	6.00	6.25	6.25	6.40	6.40	6.20	6.15	6.25	6.40	6.30	6.20	6.10	6.00	6.00	6.00	5.85	5.90	5.90	5.80
$\sigma_{uv} = -0.3$																							
$W_T^{ivx}(\hat{\lambda})$	5.60	5.65	5.90	6.00	6.10	6.15	6.05	6.15	6.20	6.35	6.35	6.60	6.45	6.60	6.60	6.55	6.65	6.60	6.50	6.40	6.30	6.35	6.40
$W_T^{ivxc}(\hat{\lambda})$	5.55	5.65	5.90	6.00	6.10	6.10	6.05	6.10	6.15	6.25	6.35	6.55	6.45	6.50	6.45	6.55	6.55	6.55	6.35	6.25	6.20	6.30	6.30
$\sigma_{uv} = -0.6$																							
$W_T^{ivx}(\hat{\lambda})$	5.95	5.75	5.85	5.95	5.90	6.05	6.15	6.40	6.60	6.75	6.90	7.10	7.20	7.25	7.25	7.20	7.10	7.40	7.50	7.60	7.65	7.75	7.85
$W_T^{ivxc}(\hat{\lambda})$	5.70	5.70	5.60	5.60	5.60	5.85	5.95	6.10	6.30	6.45	6.75	6.95	6.75	6.85	6.90	6.85	6.95	6.95	7.00	7.00	7.10	7.25	7.25
$\sigma_{uv} = -0.9$																							
$W_T^{ivx}(\hat{\lambda})$	7.00	7.15	7.35	7.55	7.25	7.15	7.10	7.20	7.45	7.60	7.60	7.55	7.85	7.60	7.95	7.90	7.90	8.05	8.00	8.05	8.25	8.30	8.20
$W_T^{ivxc}(\hat{\lambda})$	6.55	6.75	6.80	7.00	6.70	6.65	6.60	6.40	6.60	6.70	6.65	6.60	6.70	6.50	6.50	6.65	6.70	6.75	6.70	6.65	6.75	6.75	6.85
	$\alpha_1 \neq \alpha_2$, $c = 10$, $T = 200$																						
δ	0.54	0.56	0.58	0.60	0.62	0.64	0.66	0.68	0.70	0.72	0.74	0.76	0.78	0.80	0.82	0.84	0.86	0.88	0.90	0.92	0.94	0.96	0.98
$\sigma_{uv} = -0.0$																							
$W_T^{ivx}(\hat{\lambda})$	5.55	5.40	5.35	5.30	5.30	5.35	5.20	5.15	5.30	5.45	5.50	5.40	5.45	5.55	5.55	5.90	5.95	5.95	5.90	5.75	5.75	5.75	5.70
$W_T^{ivxc}(\hat{\lambda})$	5.55	5.40	5.35	5.30	5.30	5.35	5.20	5.15	5.30	5.45	5.50	5.40	5.45	5.55	5.50	5.90	5.90	5.90	5.90	5.75	5.70	5.70	5.65
$\sigma_{uv} = -0.3$																							
$W_T^{ivx}(\hat{\lambda})$	4.70	4.65	4.80	4.90	4.80	4.95	5.25	5.30	5.25	5.30	5.20	5.20	5.25	5.30	5.25	5.35	5.45	5.55	5.60	5.60	5.70	5.65	5.70
$W_T^{ivxc}(\hat{\lambda})$	4.70	4.60	4.80	4.90	4.80	4.85	5.25	5.15	5.20	5.25	5.05	5.15	5.15	5.15	5.10	5.20	5.20	5.35	5.35	5.45	5.55	5.45	5.50
$\sigma_{uv} = -0.6$																							
$W_T^{ivx}(\hat{\lambda})$	5.55	5.30	5.50	5.70	5.80	5.70	5.85	5.95	5.95	6.10	6.05	6.00	6.30	6.50	6.60	6.80	6.90	7.10	7.25	7.20	7.20	7.35	7.45
$W_T^{ivxc}(\hat{\lambda})$	5.35	5.25	5.40	5.65	5.60	5.55	5.75	5.65	5.65	5.75	5.70	5.85	5.85	6.10	6.20	6.40	6.35	6.45	6.60	6.75	6.80	6.85	6.85
$\sigma_{uv} = -0.9$																							
$W_T^{ivx}(\hat{\lambda})$	6.10	6.15	6.20	6.35	6.55	6.65	7.00	6.95	6.90	7.05	7.25	7.00	6.85	6.70	6.55	6.65	6.65	6.70	6.80	6.80	7.00	7.10	7.30
$W_T^{ivxc}(\hat{\lambda})$	5.80	5.90	6.00	6.15	6.30	6.25	6.45	6.45	6.30	6.20	6.00	6.10	6.20	6.00	5.85	5.90	5.85	5.70	5.75	5.70	5.50	5.65	5.55

Table S.5a: **Empirical Size of $W_T^{ivx}(\hat{\lambda})$ and $W_T^{ivxc}(\hat{\lambda})$ under Conditional Heteroskedasticity (T=400)**

	$\alpha_1 = \alpha_2$, $c = 1$, $T = 400$																						
δ	0.54	0.56	0.58	0.6	0.62	0.64	0.66	0.68	0.7	0.72	0.74	0.76	0.78	0.8	0.82	0.84	0.86	0.88	0.9	0.92	0.94	0.96	0.98
$\sigma_{uv} = 0$																							
$W_T^{ivx}(\hat{\lambda})$	4.50	4.75	4.75	4.75	4.65	4.65	4.85	5.05	4.85	4.95	4.85	5.00	4.65	4.60	4.65	4.55	4.55	4.35	4.40	4.45	4.50	4.40	4.45
$W_T^{ivxc}(\hat{\lambda})$	4.50	4.75	4.70	4.75	4.65	4.65	4.85	5.00	4.85	4.95	4.80	4.95	4.65	4.55	4.55	4.50	4.55	4.35	4.35	4.40	4.35	4.30	4.30
$\sigma_{uv} = -0.3$																							
$W_T^{ivx}(\hat{\lambda})$	4.00	4.30	4.50	4.65	4.90	4.95	4.80	4.90	4.80	4.95	5.05	5.30	5.15	5.40	5.55	5.55	5.55	5.55	5.40	5.65	6.00	5.90	5.95
$W_T^{ivxc}(\hat{\lambda})$	4.00	4.20	4.30	4.60	4.80	4.70	4.65	4.50	4.50	4.45	4.55	4.65	4.65	4.70	4.75	4.60	4.65	4.65	4.60	4.50	4.50	4.70	4.70
$\sigma_{uv} = -0.6$																							
$W_T^{ivx}(\hat{\lambda})$	4.35	4.30	4.35	4.60	4.75	4.80	4.80	5.15	5.50	5.50	5.25	5.55	5.80	5.75	5.70	5.75	5.80	6.10	6.25	6.40	6.50	6.55	6.65
$W_T^{ivxc}(\hat{\lambda})$	3.95	3.95	3.80	4.05	4.05	4.20	4.15	4.40	4.55	4.50	4.50	4.45	4.45	4.30	4.40	4.20	4.15	4.20	4.00	4.15	4.45	4.25	4.20
$\sigma_{uv} = -0.9$																							
$W_T^{ivx}(\hat{\lambda})$	5.35	5.30	5.65	6.20	6.25	6.15	6.15	6.55	6.80	7.10	7.40	7.75	8.10	8.35	8.60	9.05	9.25	9.55	9.70	9.75	9.90	10.25	10.60
$W_T^{ivxc}(\hat{\lambda})$	4.85	4.70	4.75	5.00	5.00	4.90	4.95	4.90	4.75	4.95	5.10	5.25	5.30	5.15	4.85	4.70	4.65	4.55	4.40	4.45	4.40	4.45	4.45
	$\alpha_1 \neq \alpha_2$, $c = 1$, $T = 400$																						
δ	0.54	0.56	0.58	0.6	0.62	0.64	0.66	0.68	0.7	0.72	0.74	0.76	0.78	0.8	0.82	0.84	0.86	0.88	0.9	0.92	0.94	0.96	0.98
$\sigma_{uv} = 0$																							
$W_T^{ivx}(\hat{\lambda})$	4.05	4.35	4.65	4.90	4.70	5.00	5.20	5.25	5.30	5.30	5.50	5.30	5.35	5.25	5.30	5.40	5.45	5.40	5.30	5.40	5.60	5.45	5.45
$W_T^{ivxc}(\hat{\lambda})$	4.05	4.35	4.65	4.90	4.70	4.95	5.20	5.25	5.30	5.20	5.35	5.25	5.15	5.25	5.30	5.40	5.45	5.35	5.30	5.40	5.60	5.45	5.30
$\sigma_{uv} = -0.3$																							
$W_T^{ivx}(\hat{\lambda})$	5.15	5.15	5.05	5.05	4.85	4.95	4.85	4.80	4.95	4.85	5.20	5.35	5.40	5.70	5.45	5.40	5.60	5.85	6.20	6.30	6.30	6.30	6.45
$W_T^{ivxc}(\hat{\lambda})$	5.15	5.15	5.05	4.95	4.70	4.75	4.70	4.70	4.70	4.75	4.90	5.00	5.10	5.20	5.15	5.15	5.00	5.15	5.20	5.45	5.30	5.40	5.25
$\sigma_{uv} = -0.6$																							
$W_T^{ivx}(\hat{\lambda})$	5.20	5.40	5.75	5.65	5.70	5.75	5.65	5.95	6.05	6.25	6.55	6.45	6.45	6.40	6.45	6.60	6.65	6.90	7.05	7.25	7.40	7.40	7.65
$W_T^{ivxc}(\hat{\lambda})$	4.70	4.80	5.00	5.10	5.30	5.15	5.05	5.10	5.15	5.35	5.60	5.60	5.25	5.20	5.20	5.00	5.10	5.10	5.20	5.20	5.05	5.15	4.95
$\sigma_{uv} = -0.9$																							
$W_T^{ivx}(\hat{\lambda})$	5.05	4.80	5.15	5.35	5.70	5.75	5.90	6.25	6.50	7.10	7.40	7.55	7.70	8.40	8.75	9.05	9.60	10.05	10.65	11.15	11.85	12.05	12.15
$W_T^{ivxc}(\hat{\lambda})$	4.25	4.20	4.15	4.20	4.30	4.60	4.30	4.40	4.40	4.65	4.55	4.50	4.35	4.35	4.45	4.35	4.25	4.35	4.55	4.55	4.50	4.50	4.55

Table S.5b: **Empirical Size of $W_T^{ivx}(\hat{\lambda})$ and $W_T^{ivxc}(\hat{\lambda})$ under Conditional Heteroskedasticity (T=400), contd.**

	$\alpha_1 = \alpha_2$, $c = 10$, $T = 400$																						
δ	0.54	0.56	0.58	0.6	0.62	0.64	0.66	0.68	0.7	0.72	0.74	0.76	0.78	0.8	0.82	0.84	0.86	0.88	0.9	0.92	0.94	0.96	0.98
$\sigma_{uv} = 0$																							
$W_T^{ivx}(\hat{\lambda})$	4.25	4.20	4.15	4.10	4.25	4.30	4.45	4.80	4.65	4.55	4.70	4.85	5.15	5.30	5.35	5.35	5.50	5.60	5.70	5.95	6.00	5.85	5.90
$W_T^{ivxc}(\hat{\lambda})$	4.25	4.20	4.15	4.10	4.25	4.30	4.45	4.80	4.60	4.55	4.70	4.80	5.15	5.30	5.35	5.35	5.50	5.60	5.70	5.95	6.00	5.85	5.90
$\sigma_{uv} = -0.3$																							
$W_T^{ivx}(\hat{\lambda})$	4.55	4.60	4.75	4.85	4.90	5.10	5.25	5.55	5.50	5.50	5.30	5.45	5.40	5.55	5.70	5.75	5.60	5.70	5.65	5.55	5.60	5.70	5.75
$W_T^{ivxc}(\hat{\lambda})$	4.50	4.45	4.75	4.80	4.85	5.05	5.25	5.45	5.50	5.50	5.30	5.45	5.30	5.50	5.65	5.65	5.50	5.65	5.60	5.55	5.55	5.60	5.70
$\sigma_{uv} = -0.6$																							
$W_T^{ivx}(\hat{\lambda})$	4.65	4.50	4.75	4.80	4.65	4.70	4.65	4.75	4.80	4.90	4.95	5.05	5.30	5.45	5.55	5.50	5.60	5.65	5.70	6.00	6.00	5.85	5.95
$W_T^{ivxc}(\hat{\lambda})$	4.55	4.45	4.60	4.65	4.55	4.65	4.55	4.60	4.70	4.90	4.85	4.85	4.90	5.10	5.30	5.30	5.20	5.20	5.50	5.75	5.65	5.45	5.45
$\sigma_{uv} = -0.9$																							
$W_T^{ivx}(\hat{\lambda})$	5.80	6.00	5.80	6.05	6.10	6.15	6.25	6.25	6.70	6.85	6.80	6.95	6.85	6.90	7.05	7.10	7.15	7.20	7.25	7.15	7.00	7.20	7.35
$W_T^{ivxc}(\hat{\lambda})$	5.55	5.65	5.70	5.85	5.85	5.95	5.95	6.10	6.35	6.50	6.25	6.40	6.40	6.40	6.40	6.45	6.25	6.30	6.30	6.15	6.10	5.90	5.80
	$\alpha_1 \neq \alpha_2$, $c = 10$, $T = 400$																						
δ	0.54	0.56	0.58	0.6	0.62	0.64	0.66	0.68	0.7	0.72	0.74	0.76	0.78	0.8	0.82	0.84	0.86	0.88	0.9	0.92	0.94	0.96	0.98
$\sigma_{uv} = 0$																							
$W_T^{ivx}(\hat{\lambda})$	4.55	4.75	4.75	4.70	4.90	4.90	5.10	5.30	5.30	5.35	5.15	5.15	5.20	5.15	5.25	5.50	5.60	5.65	5.55	5.55	5.45	5.40	5.30
$W_T^{ivxc}(\hat{\lambda})$	4.55	4.75	4.70	4.70	4.90	4.90	5.10	5.30	5.30	5.30	5.15	5.15	5.20	5.15	5.20	5.50	5.55	5.65	5.55	5.55	5.40	5.40	5.30
$\sigma_{uv} = -0.3$																							
$W_T^{ivx}(\hat{\lambda})$	5.80	5.70	5.75	5.20	5.15	5.05	5.30	5.40	5.30	5.40	5.30	5.15	5.15	5.00	4.85	4.80	4.90	4.85	4.80	4.75	4.80	4.70	4.60
$W_T^{ivxc}(\hat{\lambda})$	5.80	5.70	5.70	5.20	5.10	5.05	5.30	5.40	5.30	5.40	5.25	5.10	5.10	4.90	4.75	4.70	4.70	4.75	4.65	4.70	4.70	4.60	4.55
$\sigma_{uv} = -0.6$																							
$W_T^{ivx}(\hat{\lambda})$	4.65	4.40	4.65	4.70	4.60	4.75	4.85	4.80	5.00	5.15	4.95	5.10	5.35	5.45	5.50	5.60	5.70	5.75	5.70	5.70	5.85	5.80	5.90
$W_T^{ivxc}(\hat{\lambda})$	4.45	4.40	4.60	4.60	4.50	4.70	4.85	4.80	4.80	5.00	4.95	5.05	5.25	5.30	5.25	5.45	5.20	5.25	5.10	5.00	5.05	5.05	4.95
$\sigma_{uv} = -0.9$																							
$W_T^{ivx}(\hat{\lambda})$	5.35	5.25	5.15	5.15	5.40	5.65	5.75	6.00	6.25	5.95	5.80	5.90	6.00	6.25	6.40	6.40	6.70	6.60	6.75	6.80	6.90	6.95	7.25
$W_T^{ivxc}(\hat{\lambda})$	5.25	5.15	5.10	4.95	5.35	5.50	5.55	5.50	5.65	5.50	5.40	5.50	5.65	5.70	5.70	5.65	5.60	5.70	5.85	5.75	5.70	5.60	5.60

Table S.6: **Size and Power Comparisons:** $W_T^{ivxc}(\hat{\lambda})$ **and** $SupB^{ivxc}$ **(Nominal 5%),** $\sigma_{uv} = -0.9$

δ	0.82	0.86	0.90	0.94	0.82	0.86	0.90	0.94
$W_T^{ivxc}(\hat{\lambda})$	$\alpha_1 \neq \alpha_2$, $\beta_1 = \beta_2 = 0$, $c = 1$				$\alpha_1 \neq \alpha_2$, $\beta_1 = \beta_2 = 0$, $c = 10$			
T=200	4.35	4.35	4.15	4.25	4.95	4.95	4.80	5.00
T=400	4.60	4.70	4.60	4.55	5.45	5.35	5.30	5.10
T=1000	4.80	4.45	4.40	4.20	5.75	5.85	6.05	5.90
$SupB^{ivxc}$								
T=200	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
T=400	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
T=1000	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
$\beta_1 = 0$,$\beta_2 = 0.025$								
$W_T^{ivxc}(\hat{\lambda})$	$\alpha_1 = \alpha_2$,$c = 1$				$\alpha_1 = \alpha_2$, $c = 10$			
T=200	30.20	31.30	32.20	32.60	14.40	14.60	15.10	15.70
T=400	83.80	85.90	87.20	88.00	42.90	44.80	46.20	47.50
T=1000	99.20	99.60	99.80	99.90	100.00	100.00	100.00	100.00
$SupB^{ivxc}$								
T=200	39.80	40.80	41.65	41.80	7.80	8.00	8.30	8.36
T=400	92.05	92.95	93.50	93.70	34.45	35.60	36.45	37.35
T=1000	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
$\beta_1 = 0$,$\beta_2 = 0.05$								
$W_T^{ivxc}(\hat{\lambda})$	$\alpha_1 = \alpha_2$, $c = 1$				$\alpha_1 = \alpha_2$, $c = 10$			
T=200	83.80	85.00	86.40	86.60	41.80	43.10	44.40	45.50
T=400	98.80	99.20	99.50	99.70	97.40	98.00	98.30	98.40
T=1000	99.80	99.90	100.00	100.00	100.00	100.00	100.00	100.00
$SupB^{ivxc}$								
T=200	92.95	93.55	93.95	94.05	31.20	31.80	32.25	32.70
T=400	100.00	100.00	100.00	100.00	97.95	98.05	98.30	98.40
T=1000	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00