UC3M
Mathematical Optimization for Economics
Final Exam, 31 May 2021

Niu: ______________ Group: ______

Name: ______________________________

Question:	1	2	3	4	Total
Points:	12	12	18	18	60
Score:					

Instructions:

- **DURATION OF THE EXAM: 90'**.
- Calculators are **NOT** allowed.
- **Turn off** your smart phone.
- **DO NOT UNSTAPLE** the exam.
- Please show a valid ID to the professor if required.
- Read the exam carefully. The exam has 4 questions, for a total of 60 points.
- Justify all your answers.

1

Consider the function $f(x,y) = -\ln(x^2+y^2)$ defined on the set

$$A = \{(x,y) \in \mathbb{R}^2 \,:\, x^2+y^2 \le 4, x+y \le 2, x-y \le 2\}.$$

(a) (6 points) Draw the set A and discuss whether the function f and the set A satisfy the assumptions of the Theorem of Weierstrass. Can you ensure the existence of global extremes of f in A?

(b) (6 points) Draw the level curves of f of levels -1, 0 and 1 on the plane, showing the directions in which f increases/decreases and determine (if they exist) the global extrema of f on A. In case they do not exist, justify the reason.

2

Consider the function

$$f(x, y) = e^{1+ax^2+by^2},$$

where a and b are unknown parameters, both different from 0.

(a) (6 points) Find the critical points of f.

(b) (6 points) For each of the critical points found in the item above, determine the range of values of the parameters a and b, for which the critical point considered is

- A local maximum.
- A local minimum.
- A saddle point.

3

Consider the problem of Lagrange:

$$\text{Opt. } f(x,y) = 2x^3 - y^3 \quad \text{s.t.: } x^2 + y^2 = 5$$

(a) (3 points) Obtain the Lagrange equations.

(b) (6 points) Find the critical points of the Lagrangian.

(c) (3 points) Find, if they exist, the global maximum and the global minimum.

(d) (6 points) Suppose that the constraint changes to $x^2 + y^2 = 5.1$, that is, the problem becomes now

$$\text{Opt. } f(x,y) = 2x^3 - y^3 \quad \text{s.t.: } x^2 + y^2 = 5.1.$$

Without solving the problem again, calculate approximately the maximum value of $f(x,y)$ after this change.

4

Consider the function $f(x,y) = \dfrac{x^4}{2} - y^4 + 2y^2$ defined on the set

$$A = \{(x,y) \,:\, x^2 + y^2 \leq 4\}.$$

(a) (6 points) Establish the Kuhn–Tucker necessary optimality conditions to the problem

$$\max f(x,y) \quad \text{subject to } (x,y) \in A.$$

(b) (9 points) Find all the solutions of the Kuhn–Tucker conditions established in part (a).

(c) (3 points) Find the global maximum of f on A.
