UC3M
Mathematical Optimization for Economics
Final Exam, May 15 2025

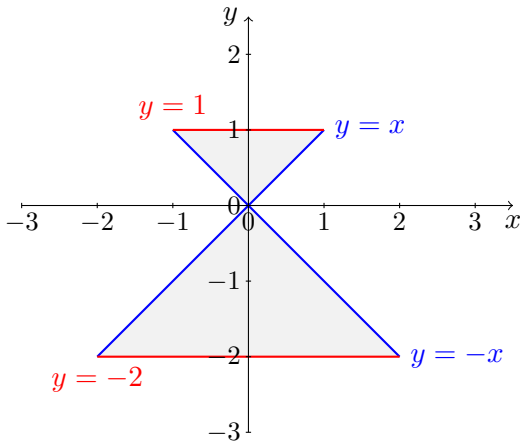
Niu: _____ Group: _____

Name: _____

Question:	1	2	3	4	Total
Points:	20	20	35	25	100
Score:					

1

On the set $A = \{(x,y) \in \mathbb{R}^2 : |x| \leq y \leq 1\} \cup \{(x,y) \in \mathbb{R}^2 : -2 \leq y \leq -|x|\}$ whose representation is given in the figure below the Pareto order is considered.



(a) (10 points) Find the maximal points, minimal points, the maximum and the minimum of A, if they exist. Justify your answers.

(b) (10 points) Consider the function $f(x,y) = x^2 + y^2$ defined on the set A. Using the level curves of the function $f(x,y)$, and the directions of maximum (minimum) growth, identify, giving a graphical reasoning, the local extrema and the global extrema of f on A.

2

Consider the function $f(x, y) = x^2 + 3y^4 - 4y^3$.

(a) (10 points) Find and classify its local extrema.

(b) (10 points) Does f attain a global minimum on the set $A = \left\{(x, y) \in R^2 \, : \, y > \frac{3}{4}\right\}$? Justify your answer.

3

Consider the following Lagrange problem:

$$\text{Optimize } \left\{ f(x,y,z) = 4x^{\frac{1}{2}}yz \right\} \text{ subject to: } 2x + y + 2z = 10,\ x > 0.$$

(a) (10 points) Write the Lagrangian of the problem and find all its critical points.

(b) (15 points) Apply the sufficient conditions to the obtained critical point(s) and classify them.

(c) (10 points) Give an estimation of the optimal value of the objective function when the constraint changes from $2x + y + 2z = 10$ to $2x + y + 2z = 9.5$.

4

Consider the program

$$\begin{aligned} \text{Minimize} \quad & -x - 2y \\ \text{s.t.} \quad & x^2 + 2y^2 \leq 3. \end{aligned}$$

(a) (15 points) Obtain the solutions of the corresponding Kuhn-Tucker equations for the program.

(b) (10 points) Justify that the program has a global solution. (Note: This part can be answered without doing the first one).
