UC3M
Mathematical Optimization for Economics
Extraordinary Final Exam July 1, 2025

Niu: ______________ Group: ____

Name: ______________________________________

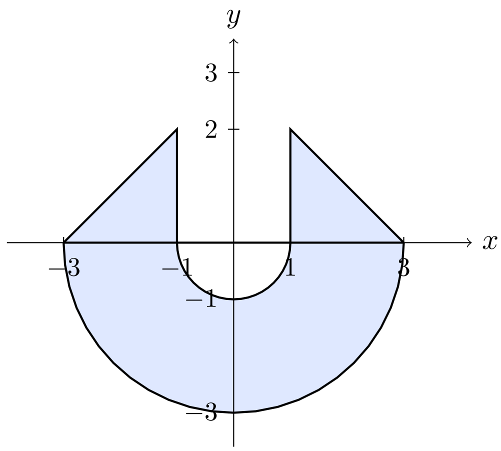
Exercise	1	2	3	4	Total
Points	20	20	30	30	100
Grade					

1

Consider the set A defined as

$$\left\{(x,y) \in \mathbb{R}^2 : y \leq 0,\ 1 \leq x^2 + y^2 \leq 9\right\} \cup \left\{(x,y) \in \mathbb{R}^2 : y \geq 0,\ 1 \leq |x| \leq 3,\ y \leq 3 - |x|\right\}$$

ordered with the Pareto order $\leq_P$ (Recall that $(x,y) \leq_P (a,b) \Leftrightarrow x \leq a$ and $y \leq b$). The set is represented in the following figure:



(a) (10 points) Find the maximal, minimal points, the maximum and the minimum of A, if they exist. Justify your answers.

(b) (10 points) On set A, consider the function defined by $f(x,y) = y - x$. Justify that f has a maximum and a minimum. Then, using a graphical reasoning on the level curves, find the points at which the maximum and minimum of f are reached and calculate their values.

2

Consider the function $f(x, y) = -y^2 - 3x^4 + 4x^3$.

(a) (10 points) Find the largest open convex set in $\mathbb{R}^2$ where f is strictly concave.

(b) (10 points) Find and classify the critical points of f.

3

Consider the following Lagrange problem:

Optimize $f(x, y, z) = 2x^2 + 2y^2 + 2z^2 + 2xy - 7x - 9y - 27z$
subject to: $x + y + z = 10$, with $x, y, z \in \mathbb{R}$.

(a) (15 points) Write the Lagrangian of the problem and find all its critical points.

(b) (10 points) Apply the sufficient conditions to the critical points obtained in the previous section and classify them using the Hessian matrix of the Lagrangian. Can we affirm that any solution is global? Justify your answer.

(c) (5 points) Approximately how much does the optimal value of the objective function change when the constraint changes from $x + y + z = 10$ to $x + y + z = 9.5$?

4

Consider the program

$$\begin{array}{ll} \text{Maximize} & f(x,y) = -x^2 - y^2 + 6y \\ \text{subject to:} & (x,y) \in A, \end{array}$$

where the feasible set A is given by two inequalities:

$$A = \{(x,y) \in \mathbb{R}^2 \,:\, x^2 \leq y,\ y \leq 2x + 3\}.$$

(a) (15 points) Obtain the solutions of the Kuhn–Tucker equations corresponding to the program.

(b) (5 points) Are the points found in part (a) local and/or global maxima? Justify your answer.

(c) (10 points) What happens if we substitute the feasible set A with the following feasible set B:

$$B = \{(x,y) \,:\, x \geqslant -1,\ y \geqslant 2\}?$$

Is it necessary to solve the problem again to obtain the solution? Justify your answer.

Observation: it is recommended to draw sets A and B.
