Micro II Midterm (uc3m-Masters in Economics, March 20, 2026)

Exercise 1. In a pure exchange economy that extends over two dates, today and tomorrow, there is a single perishable good, consumption, and two consumers, 1 and 2. The state of nature tomorrow can be either *hot* or *cold*. Consumers' preferences for consumption today (x), tomorrow if hot (y), and tomorrow if cold (z) are represented by the utility functions $u_1(x, y, z) = x + y^2 z$, and $u_2(x, y, z) = x + yz^2$, and their endowments are $(\bar{x}_1, \bar{y}_1, \bar{z}_1) = (2, 1, 2)$ and $(\bar{x}_2, \bar{y}_2, \bar{z}_2) = (1, 2, 1)$, respectively.

(a) (30 points) Calculate the competitive equilibrium (CE) prices an allocation assuming that there are markets for contingent contracts for all goods. (Normalize the price of good x to 1, and denote by p_y and p_z the prices of y and z, respectively. It is easy to calculate the consumers' demands for y and z using the marginal rates of substitution of xy and xz, respectively.)

(b) (20 points) Assume that no contingent markets exits, but there are markets for credit and for a security that pays one unit of consumption tomorrow if hot, which operate today. Determine the competitive equilibrium interest rate and security price, (r^*, q^*), how much each consumer borrows/lends and how many units of the security she buys/sells.

Exercise 2. Two fishermen, Art (A) and Bob (B), have free access to a local lake. Each must choose how many days a week to fish, z_i. The total catch of fish obtained by each fisherman is $(10 - \bar{z})\, z_i$, where z_i is the number of days $i \in \{A, B\}$ fishes, and $\bar{z} = (z_A + z_B)/2$ is the average number of days the two men fish a weak. Their preferences for weakly fish consumption x and days of leisure time y are represented by utility functions of the form $u_i(x, y) = x + \alpha_i y$, where $(\alpha_A, \alpha_B) = (3, 2)$.

(a) (20 points) Calculate how many days a weak will each fisherman allocates to leisure and fishing, and determine whether the resulting allocation is Pareto optimal.

(b) (20 points) Art and Bob create a firm, sharing its property equally, to which they transfer the exclusive right to fish in the lake. The firm hires fishermen from the local community (formed by Art and Bob) and sell the fish catch in the local market (in which Art and Bob are the only buyers). Normalize the price of fish to 1, and denote by w the daily wage. Determine the competitive equilibrium wage w^* and allocation, $[(x_A^*, y_A^*), (x_B^*, y_B^*)]$.

(c) (10 points) Calculate the equilibrium wage and allocation if the firm acts as a monopoly in the market for fish and a monopsony in the market for labor, i.e., the firm chooses the its labor (hence its output) and the wage given the labor supply. (You may still normalize the price of fish to one.) Discuss the effects of market power on efficiency (i.e., surplus) and distribution.

(d) (For extra credit: 10 points) Determine the set of Pareto optimal allocations. (Keep in mind that time cannot be redistributed, that no man can supply more the seven days of labor, and Pareto optimality requires maximizing the surplus generated by the economics activity, which depends on which man supplies labor.)

Solutions

1(a). Obviously, the CE prices are positive. Let us calculate the consumers' demands for y and z, identified for $i \in \{1,2\}$ by the solution to the system

$$MRS^i_{xy}(x,y,z) = \frac{\partial u_i}{\partial x} / \frac{\partial u_i}{\partial y} = \frac{1}{p_y} \text{ and } MRS^i_{xz}(x,y,z) = \frac{\partial u_i}{\partial x} / \frac{\partial u_i}{\partial z} = \frac{1}{p_z}.$$

Hence, for $i = 1$

$$\frac{1}{2y_1 z_1} = \frac{1}{p_y} \text{ and } \frac{1}{y_1^2} = \frac{1}{p_z},$$

i.e.,

$$y_1(p_y, p_z) = \sqrt{p_z} \text{ and } z_1(p_y, p_z) = \frac{p_y}{2\sqrt{p_z}}.$$

Symmetrically,

$$y_2(p_y, p_z) = \frac{p_z}{2\sqrt{p_y}} \text{ and } z_2(p_y, p_z) = \sqrt{p_y}.$$

Market clearing requires

$$\begin{aligned} y_1(p_y, p_z) + y_2(p_y, p_z) &= \sqrt{p_z} + \frac{p_z}{2\sqrt{p_y}} = 1 + 2 \\ z_1(p_y, p_z) + z_2(p_y, p_z) &= \frac{p_y}{2\sqrt{p_z}} + \sqrt{p_y} = 2 + 1. \end{aligned}$$

Hence, the CE equilibrium prices are

$$p_y^* = p_z^* = 4,$$

and the CE allocation is

$$[(x_1^*, y_1^*, z_1^*), (x_2^*, y_2^*, z_2^*)] = [(2,2,1),(1,1,2)].$$

1(b). Consumer i's budget constraints are

$$\begin{aligned} x_i + q s_i &= \bar{x}_i + b_i \\ y_i &= \bar{y}_i - (1+r)b + s_i \\ z_i &= \bar{z}_i - (1+r)b_i. \end{aligned}$$

Hence

$$x_i + q y_i + \left(\frac{1}{1+r} - q\right) z_i = \bar{x}_i + q\bar{y}_i + \left(\frac{1}{1+r} - q\right) \bar{z}_i.$$

Since in the CE $1+r>0$, *the rank the returns matrix,*

$$R = \begin{pmatrix} 1+r & 1 \\ 1+r & 0 \end{pmatrix},$$

is 2. Therefore the CE allocation of this economy is that of the economy in part (a). Hence

$$p_y^* = 4 = q^*, \text{ and } p_z^* = 4 = \frac{1}{1+r} - 4 \Leftrightarrow r^* = -\frac{7}{8}.$$

Moreover, using consumer 1's budget constraints and market clearing conditions,

$$\begin{aligned} y_1^* &= 2 = 1 - \frac{b_1^*}{8} + s_1^*, \; z_1^* = 1 = 2 - \frac{b_1^*}{8} \\ s_1 + s_2 &= 0 \text{ and } b_1 + b_2 = 0. \end{aligned}$$

we get

$$b_1^* = 8 = -b_2^* \text{ and } s_1^* = 2 = -s_2^*.$$

2(a) Each fisherman chooses the number of fishing days z_i by solving the problem

$$\max_{z_i \in [0,7]} \left(10 - \frac{z_i + z_{-i}}{2}\right) z_i + \alpha_i (7 - z_i)$$

An interior solution is

$$z_i = 10 - \alpha_i - \frac{z_{-i}}{2}.$$

Hence in an interior equilibrium the number of days Art and Bob fish satisfy the system

$$\begin{aligned} z_A &= 7 - \frac{z_B}{2} \\ z_B &= 8 - \frac{z_A}{2}. \end{aligned}$$

The unique solution to this system is

$$(\hat{z}_A, \hat{z}_B) = (4, 6)$$

The weakly fish consumption and days of leisure of Art and Bob are

$$\begin{aligned} (\hat{x}_A, \hat{y}_A) &= ((10 - 5) 4, 7 - 4) = (20, 3) \\ (\hat{x}_B, \hat{y}_B) &= ((10 - 5) 6, 7 - 6) = (30, 1) \end{aligned}$$

and their utilities are

$$(\hat{u}_A, \hat{u}_B) = (20 + 3 (3), 30 + 2 (1)) = (29, 32).$$

This allocation of time is not Pareto optimal: for example, if both men reduce their fishing one day to $(\tilde{z}_A, \tilde{z}_B) = (3, 5)$, then the resulting allocation is

$$\begin{aligned} (\tilde{x}_A, \tilde{y}_A) &= ((10 - 4) 3, 7 - 3) = (18, 4) \\ (\tilde{x}_B, \tilde{y}_B) &= ((10 - 4) 5, 7 - 5) = (30, 2) \end{aligned}$$

and their utilities are

$$(\tilde{u}_A, \tilde{u}_B) = (18 + 3 (4), 30 + 2 (2)) = (30, 34),$$

i.e., the resulting allocation is Pareto superior.

2(b) Now Art and Bob choose their weakly fish consumption and supply of labor ℓ by solving the problem

$$\max_{(\ell,c)\in[0,7]\times\mathbb{R}_+} \quad c+\alpha_i\left(7-\ell\right)$$
$$s.t. \quad : \quad c = w\ell + \pi(w)/2,$$

where $\pi(w)$ is the firm's profit. Equivalently,

$$\max_{\ell\in[0,7]} w\ell + \pi(w)/2 + \alpha_i\left(7-\ell\right) = \left(\pi(w)/2 + 7\alpha_i\right) + \left(w-\alpha_i\right)\ell,$$

Since the objective function in this problem in linear in ℓ with coefficient $w-\alpha_i$, the labor supply of each fisherman is

$$\ell_i\left(w\right) = \begin{cases} 0 & \text{if } w<\alpha_i \\ [0,7] & \text{if } w=\alpha_i \\ 7 & \text{if } w>\alpha_i. \end{cases}$$

And the market labor supply is

$$L^S\left(w\right) = \begin{cases} 0 & \text{if } w<2 \\ [0,7] & \text{if } w=2 \\ 7 & \text{if } w\in(2,3). \\ [7,14] & \text{if } w=3 \\ 14 & \text{if } w>3. \end{cases}$$

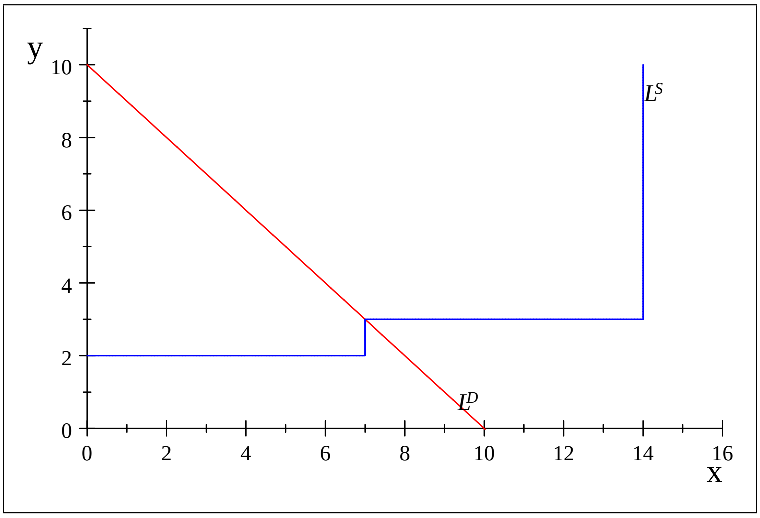
The firm's labor demand is the solution to the problem

$$\max_{\ell\in\mathbb{R}_+} \left(10-\frac{\ell}{2}\right)\ell - w\ell,$$

i.e.,

$$L^D(w) = 10 - w.$$

The CE equilibrium wage is solves the labor market clearing condition $L^S\left(w\right) = L^D(w)$, i.e., $w^* = 3$.



Firms profits are

$$\left(10 - \frac{7}{2}\right) 7 - 3\,(7) = 24.5.$$

and the CE allocation is

$$\begin{aligned} (x_A^*, y_A^*) &= (12.25, 7) \\ (\tilde{x}_B, \tilde{y}_B) &= (12.25 + 3\,(7)\,, 0) = (33.25, 0), \end{aligned}$$

and the fishermen utilities are

$$(u_A^*, u_B^*) = (12.25 + 3\,(7)\,, 33.25 + 2\,(0)) = (33.25, 33.25).$$

2(c) If the firm exercises market power, it will choose between setting the wage $w = 2$ and the labor $\ell \in L^S(2) = [0, 7]$ that maximizes profits,

$$\hat{\pi}(2, \ell) = \left(10 - \frac{\ell}{2}\right)\ell - 2\ell,$$

which is $\ell = 7$, leading to profit

$$\hat{\pi}(2, 7) = \left(10 - \frac{7}{2}\right)7 - 2\,(7) = 31.5$$

or setting the wage $w = 3$ and the labor $\ell \in L^S(2) = [7, 14]$ that maximizes profits

$$\hat{\pi}(3, \ell) = \left(10 - \frac{\ell}{2}\right)\ell - 3\ell,$$

which $\ell = 7$, leading to profit

$$\hat{\pi}(3, 7) = \left(10 - \frac{7}{2}\right)7 - 3\,(7) = 24.5.$$

Hence the monopoly equilibrium is $(w_M, \ell_M) = (2, 7)$ and profits are $\pi_M = 31.5$, leading to the allocation

$$\begin{aligned} (x_A^M, y_A^M) &= (15.75, 7) \\ (x_B^M, y_B^M) &= (15.75 + 2\,(7)\,, 0) = (29.75, 0). \end{aligned}$$

The fishermen utilities are

$$(u_A^M, u_B^M) = (15.75 + 3\,(7)\,, 29.75 + 2\,(0)) = (36.75, 29.75).$$

Note that market power does not lead to inefficiencies, but implies a redistribution of the surplus against the individual who supplies labor.

2(d) Let us identify the conditions for Pareto optimality by solving the surplus maximization problem

$$\max_{(l_1,l_2)\in[0,7]^2} W(l_A, l_B) = \left(10 - \frac{l_A + l_B}{2}\right)(l_A + l_B) + 3\,(7 - l_A) + 2\,(7 - l_B)\,.$$

Since

$$\frac{\partial W(l_A, l_B)}{\partial l_B} = 8 - l_A - l_B > 7 - l_A - l_B = \frac{\partial W(l_A, l_B)}{\partial l_A},$$

(because Bob's cost of labor is smaller Art's) and

$$\frac{\partial W(0, l_B)}{\partial l_B} = 8 - l_B > 0 \text{ on } [0, 7],$$

maximizing the surplus entails setting $l_B^* = 7$.

Moreover, since $\partial W(l_A, 7)/\partial l_A = -l_A < 0$ for $l_A > 0$ (i.e., when fishing more than seven days the marginal productivity of additional fishing falls below Art's cost of labor), the maximum surplus is reached for $l_A^* = 0$.

Hence, the fish catch that maximizes the surplus is $(7 - 7/2)\,7 = 49/2$, and therefore the set of Pareto optimal allocations is

$$P := \{[(x_A, y_A), (x_B, y_A)] \in \mathbb{R}_+ \mid x_A + x_B = \frac{49}{2},\ y_A = 7,\ y_B = 0\}.$$

Obviously, the allocation of part (b) is in P.