Consider an economy in which there are two goods x and y, one firm whose production set is

$$Y = \{(-x, y) \in \mathbb{R}_- \times \mathbb{R}_+ \mid y \leq 2\sqrt{x}),$$

and two consumers with identical preferences, represented by $u(x, y) = xy$, and endowments, $(\bar{x}_i, \bar{y}_i) = (24, 0)$, and with fractions $\theta \in [0, 1]$ and $1 - \theta$ of the firm's property.

Calculate the set of Pareto optimal and competitive equilibrium allocations for $\theta = 1$ and $\theta = 1/2$.

THE CE OF THE ECONOMY

Normalize prices: $P_x = 1$, $P_y = p$.

Firm

$$\max_{x \geq 0} \; p(2\sqrt{x}) - x \iff \max_{x \geq 0} \; p2\sqrt{x} - x$$

Solution: $\frac{p}{\sqrt{x}} - 1 = 0 \Rightarrow \boxed{x_f(p) = p^2}$ (INPUT DEMAND)

$y_f(p) = 2p$ (OUTPUT SUPPLY)

$\pi(p) = p(2p) - p^2 = p^2$ (PROFITS)

Consumers:

$$\max_{(x,y)\in\mathbb{R}^2_+} xy \quad \text{s.t.} \quad x + py \le 24 + \theta_i \underbrace{\pi(p)}_{p^2} \iff \max_{x\ge 0} \; x\left(\frac{24+\theta_i p^2 - x}{p}\right)$$

F.O.C: $24 + \theta_i p^2 - 2x = 0$

Hence,

$$\boxed{x_i(p) = 12 + \frac{\theta_i p^2}{2}}$$

$$\boxed{y_i(p) = \frac{12}{p} + \frac{\theta_i p}{2}}$$

Market Clearing: $(24 - x_1(p)) + (24 - x_2(p)) = x_f(p)$

i.e.,

$$24 - \frac{\theta_1+\theta_2}{2}p^2 = p^2 \iff 24 = \frac{3}{2}p^2 \iff \boxed{p^* = 4}$$

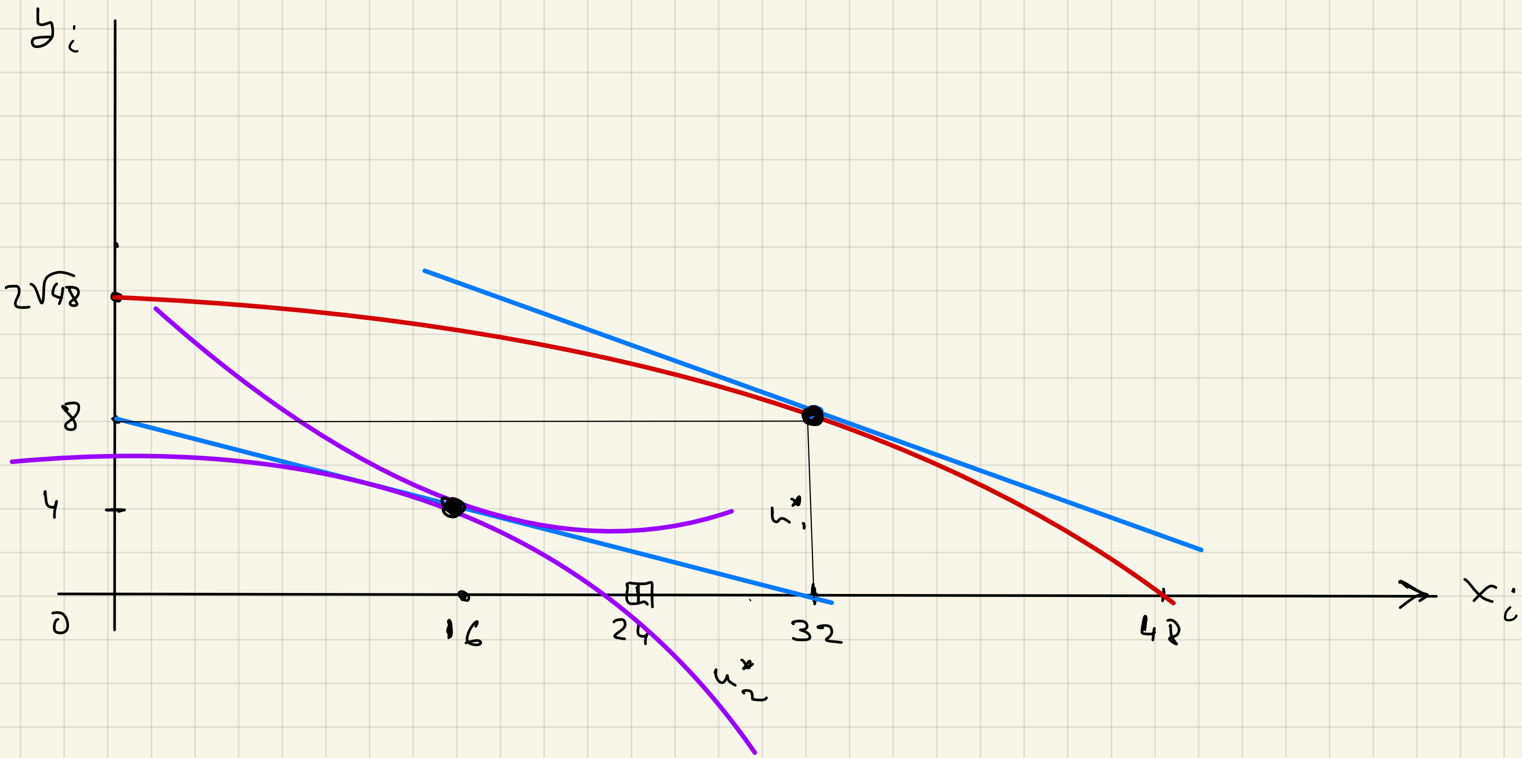
Thus, in the CE firm's

Input is $x_f(p^*) = (p^*)^2 = 16$

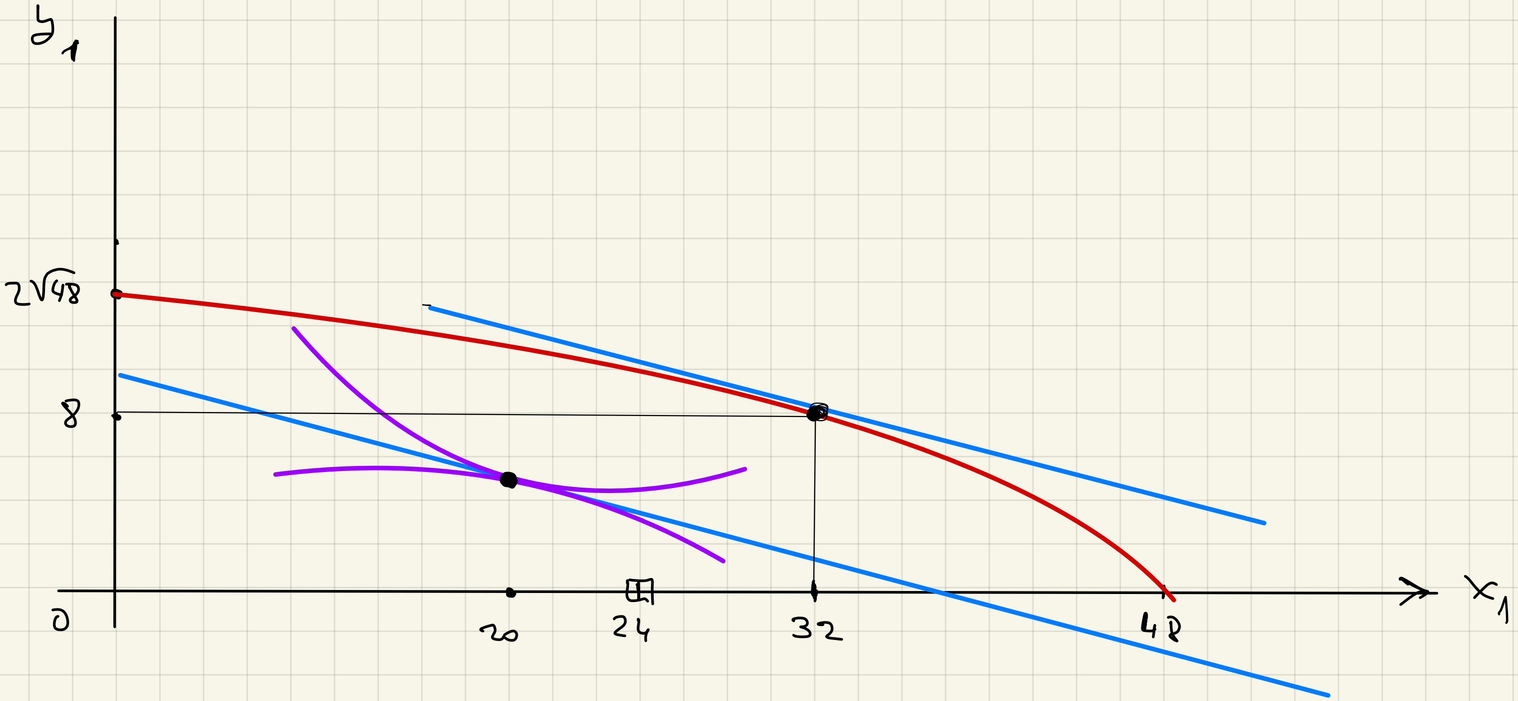
Output is $y_f(p^*) = 2p^* = \underline{\underline{8}}$.

Profit is $p^2 = 16$

$\theta_1 = \theta_2 = \frac{1}{2} \Rightarrow (x_i^*, y_i^*) = (16, 4) \quad , \quad i \in \{1, 2\}$



$\theta_1 = 1, \ \theta_2 = 0 \quad \Rightarrow \quad (x_1, y_1) = (20, 5), \quad (x_2, y_2) = (12, 3)$



TWO FIRMS (RATHER THAN 1), SAME TECHNOLOGY, $\theta_1=(1,0)$, $\theta_2=(0,1)$

Market Clearing:

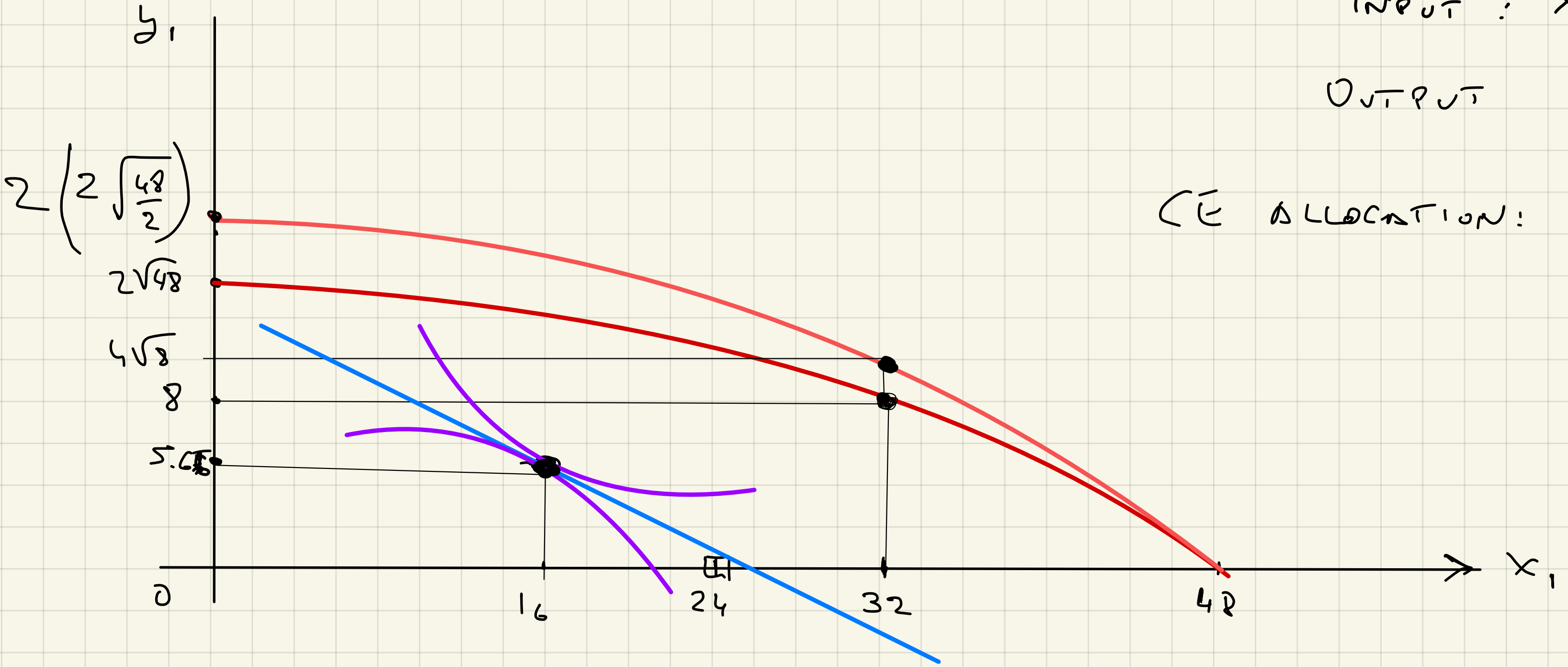
$$[24-x_1(p)]+[24-x_2(p)] = x_{f_1}(p) + x_{f_2}(p)$$

$$2\left(12+\frac{p^2}{2}\right)=2p^2 \iff 24=3p^2 \iff p^*=\sqrt{8}.$$

INPUT : $x_{f_j}(p^*)=8$

OUTPUT $y_{f_j}(p^*) = 2\sqrt{8} \approx 5.66$

CE ALLOCATION: $[(16, 5.66), (16, 5.66)]$



1. There is only one perishable commodity available for consumption today and tomorrow, and two consumers, Ann and Bob, whose preferences for alternative consumption streams today and tomorrow, $(x, y) \in \mathbb{R}^2_+$, are described by the utility functions $u^A(x, y) = \min\{x, y\}$, and $u^B(x, y) = xy$, respectively.

(a) Assume that the endowment streams are (0,30) for Ann and (20,0) for Bob. Ann and Bob participate in a competitive credit market in which they are the only participants. Calculate the competitive equilibrium rate of interest and consumption streams, and determine whether this allocation is Pareto optimal.

Let us normalize (spot) prices: $\hat{P}_x = \hat{P}_y = 1$.

Ann's demand of consumption today :

$$\left.\begin{array}{l} \left.\begin{array}{l} x = b \\ y = 30 - (1+r) b \end{array}\right\} \text{B.C.} \\ \left.\begin{array}{l} x = y \end{array}\right\} \text{OPTIMALITY} \end{array}\right\} \Rightarrow \boxed{b^A(r) = \frac{30}{2+r}}$$

CONSOLIDATING THE B.C: $(1+r)\, x + y = 30 \iff x + \frac{y}{1+r} = \frac{30}{1+r}$

Bob's demand of consumption today :

$$\left.\begin{aligned} x &= 20 + b \\ y &= -(1+r)\, b \end{aligned}\right\} \text{ B.C. } \iff (1+r)\,x + y = 20\,(1+r)$$

$$\frac{y}{x} = (1+r) \quad \} \text{ OPTIMALITY}$$

$$\boxed{b^B(r) = -10}$$

ALTERNATIVELY :

$$\boxed{\max_{b \in \mathbb{R}} \ (20+b)\left(-(1+r)\,b\right) \qquad \underline{\text{FOC}} : \ 20 + 2b = 0}$$

<u>CREDIT MARKET CLEARING</u>

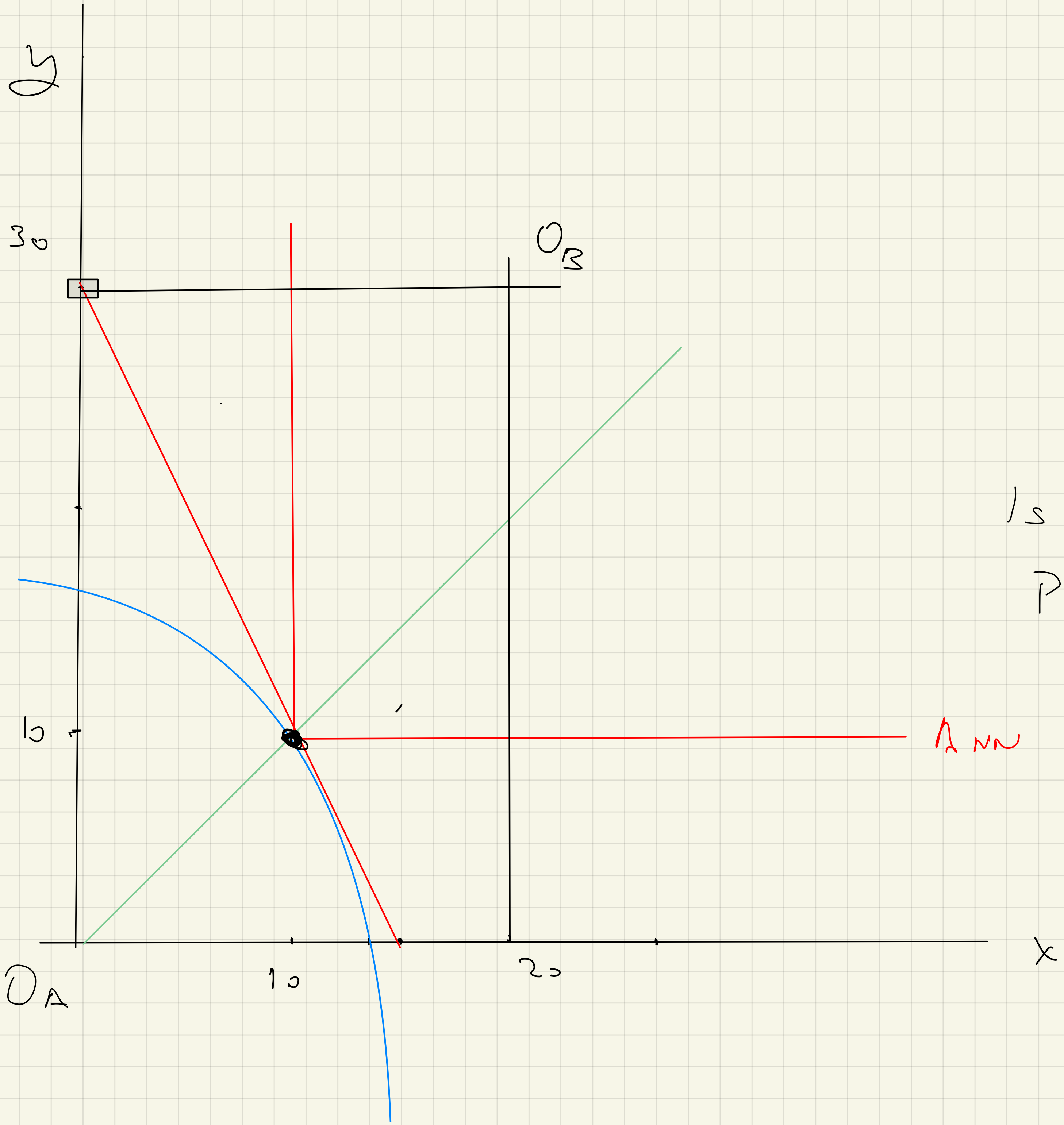
$$b^A(r) + b^B(r) = 0$$

$$\frac{30}{2+r} - 10 = 0 \iff \underline{\underline{r^* = 1}}$$

<u>RADNER</u> CE :

$$(x_A^*, y_A^*) = (10, 10)$$

$$(x_B^*, y_B^*) = (10, 20)$$



Is the Radner CE Pareto optimal?