T

MARKETS FOR LEMONS

G. Akerlof : The market for lemons: quality .... QJE (1970)

AN EXAMPLE

A good of two qualities: High and Low.

There are b buyers, each of whom wants to buy one unit.

There are s^H sellers, each with one unit of H quality,

and s^L " , " " " " L " .

Assumption: $$u^H > c^H > u^L > c^L$$

I. Assume that quality is observable.

CE : Two competitive markets:

- a market for good H
- a market for good L.

Supply:

For $\tau \in \{H, L\}$:

$$S^{\tau}(p) = \begin{cases} 0 & \text{if } p < c^{\tau} \\ [0, s^{\tau}] & \text{if } p = c^{\tau} \\ s^{\tau} & \text{if } p > c^{\tau}. \end{cases}$$

Demand:

The demands of goods H and L depend on (p^L, p^H):

Consumers demand the quality that gives them the largest utility:

$$u^I - p^I = \max\{u^L - p^L,\ u^H - p^H\} \geq 0.$$

For $(p^L, p^H) \in [0, u^L] \times [0, u^H]$: (AVOID!)

$$\left(D^L(p^L, p^H),\ D^H(p^L, p^H)\right) = \begin{cases} (0, b) & \text{if } u^H - p^H = \max\{u^L - p^L,\ u^H - p^H\} > 0 \\ \{(x,y) \mid x+y = b\} & \text{if } u^L - p^L = u^H - p^H > 0 \\ (b, 0) & \text{if } u^L - p^L = \max\{u^L - p^L,\ u^H - p^H\} > 0 \\ \{(x,y) \mid x+y \leq b\} & \text{if } u^L - p^L = u^H - p^H = 0 \\ (0, 0) & \text{if } \max\{u^L - p^L,\ u^H - p^H\} < 0. \end{cases}$$

CE Assume $b > \max\{s^L, s^H\}$. Then:

- If $b > s^L + s^H$, then $p^L = u^L$, $p^H = u^H$, and all units trade.
- If $b < s^L + s^H$, then $\begin{cases} u^L - c^L < u^H - c^H \implies p^L = c^L, \; p^H = u^H - (u^L - c^L) \\ \qquad\quad > \qquad\qquad \implies p^H = c^H, \; p^L = u^L - (u^H - c^H) \end{cases}$

Moreover, b units trade.

CE is Pareto efficient (i.e., maximum surplus is realized).

Example: $b = 3$, $s^H = s^L = 2$, $u^H = 10$, $u^L = 4$, $c^H = 6$, $c^L = 2$.

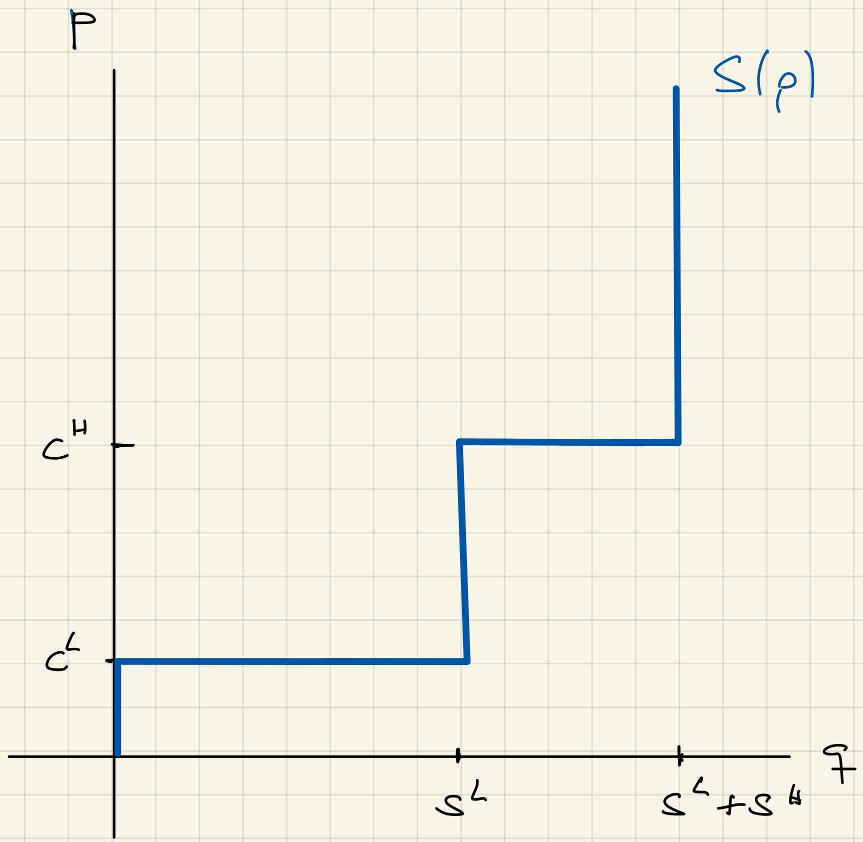
Then: $p^L = 2$, $p^H = 10 - (4-2) = \underline{8}$.

Surplus realized: $2(10-6) + (4-2) = 10$

Each buyer : 2.
H seller : 2
L sellers : 0

II. Assume quality is known to sellers, but is observed by buyers only upon purchase.

In this case there is a single market where both qualities trade. Hence buyers are uncertain about the quality of the good they are offered.



CE.

Supply:

$$S(p) = \begin{cases} 0 & \text{if } p < c^L \\ s^L & \text{if } c^L < p < c^H \\ s^L + s^H & \text{if } p > c^H. \end{cases}$$

Demand (buyers are risk-neutral):

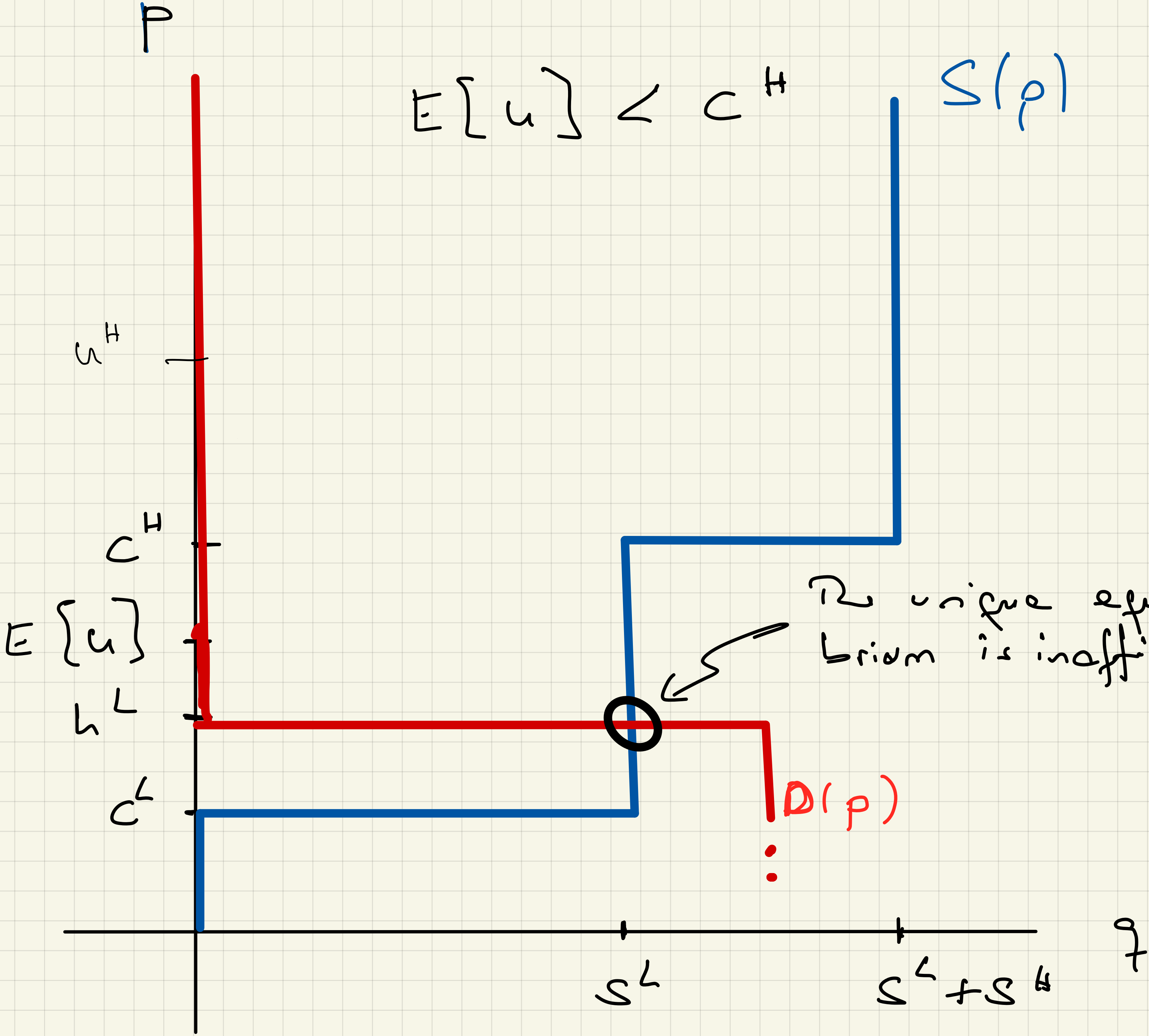
$$E[u] = \frac{s^L}{s^L + s^H} u^L + \frac{s^H}{s^L + s^H} u^H$$

Assume $E[u] < c^H$. Hence

(*) All sellers supply at π's price

$$D(p) = \begin{cases} 0 & \text{if} \quad p > u^L \\ b & \text{if} \quad c^L \leq p \leq u^L \\ ? & \text{if} \quad p < c^L \end{cases}$$

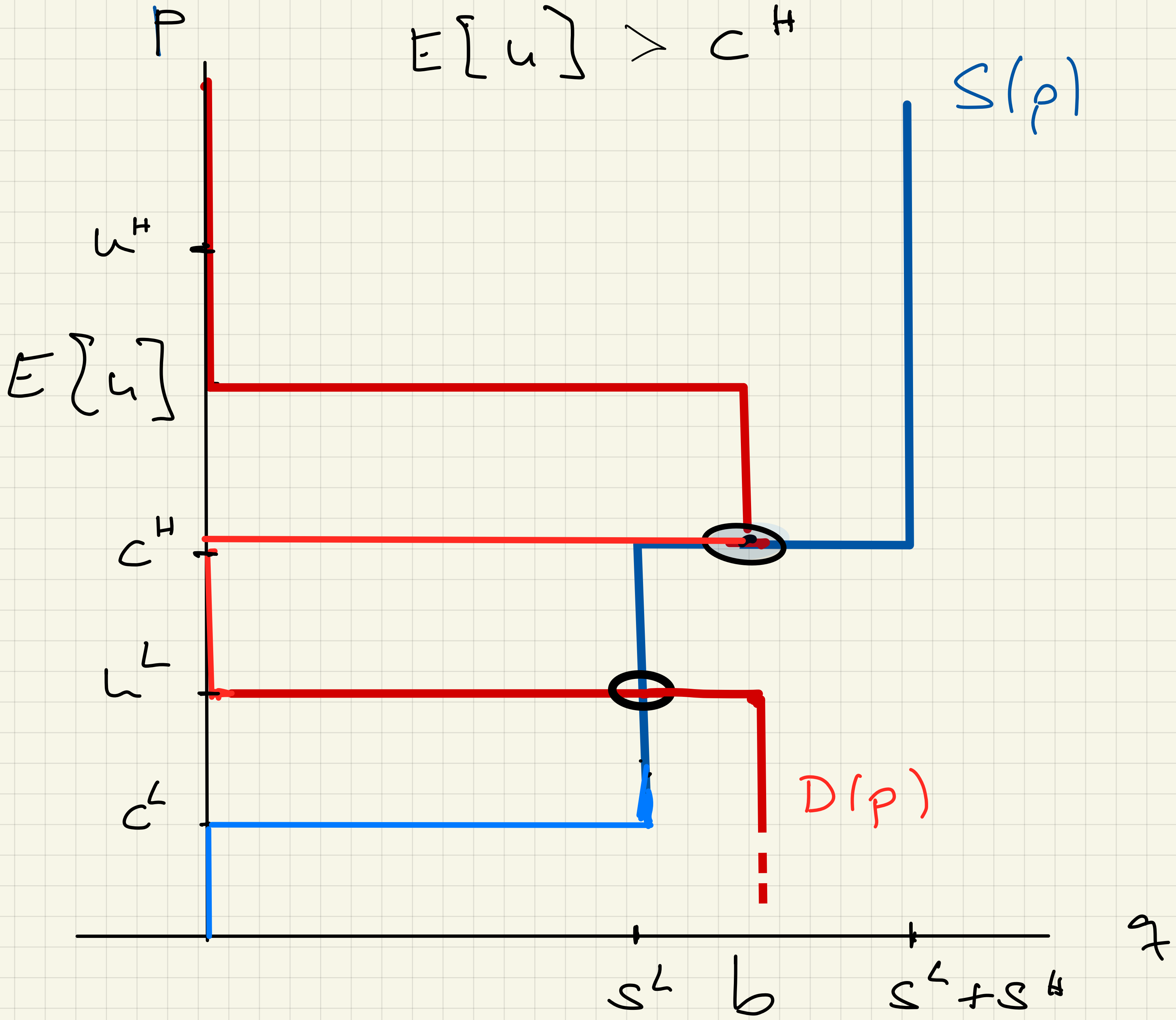
$E[u] < c^H$



p
S(p)
u^H
c^H
$E[u]$
h^L
c^L
The unique equilibrium is inefficient.
D(p)
s^L
$s^L + s^H$
q

Assume $E[u] < c^H$. Hence

$$D(p) = \begin{cases} 0 & \text{if} \quad p > u^H \\ b & \text{if} \quad c^H \leq p \leq u \\ 0 & \text{if} \quad c^L < p < c^H \\ b & \text{if} \quad c^L \leq p \leq u^L \\ ? & \text{if} \quad p < c^L \end{cases}$$



$E[u] > c^H$
p
$S(p)$
u^H
$E[u]$
c^H
u^L
c^L
$D(p)$
q
s^L
b
$s^L + s^H$

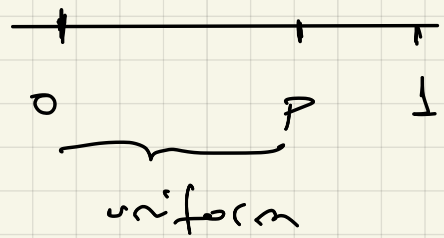
A more extreme example.

A measure 1 of sellers. The quality of the good supplied by the seller $s \in [0,1]$ is $q(s) = s$, and its opportunity cost is $c(s) = s$. Buyers' value of quality $s \in [0,1]$ is $u(s) = \alpha s$, $\alpha \in (1,2)$.

CE

Supply: $p \in \mathbb{R}_+$, $S(p) = \min\{p, 1\}$.

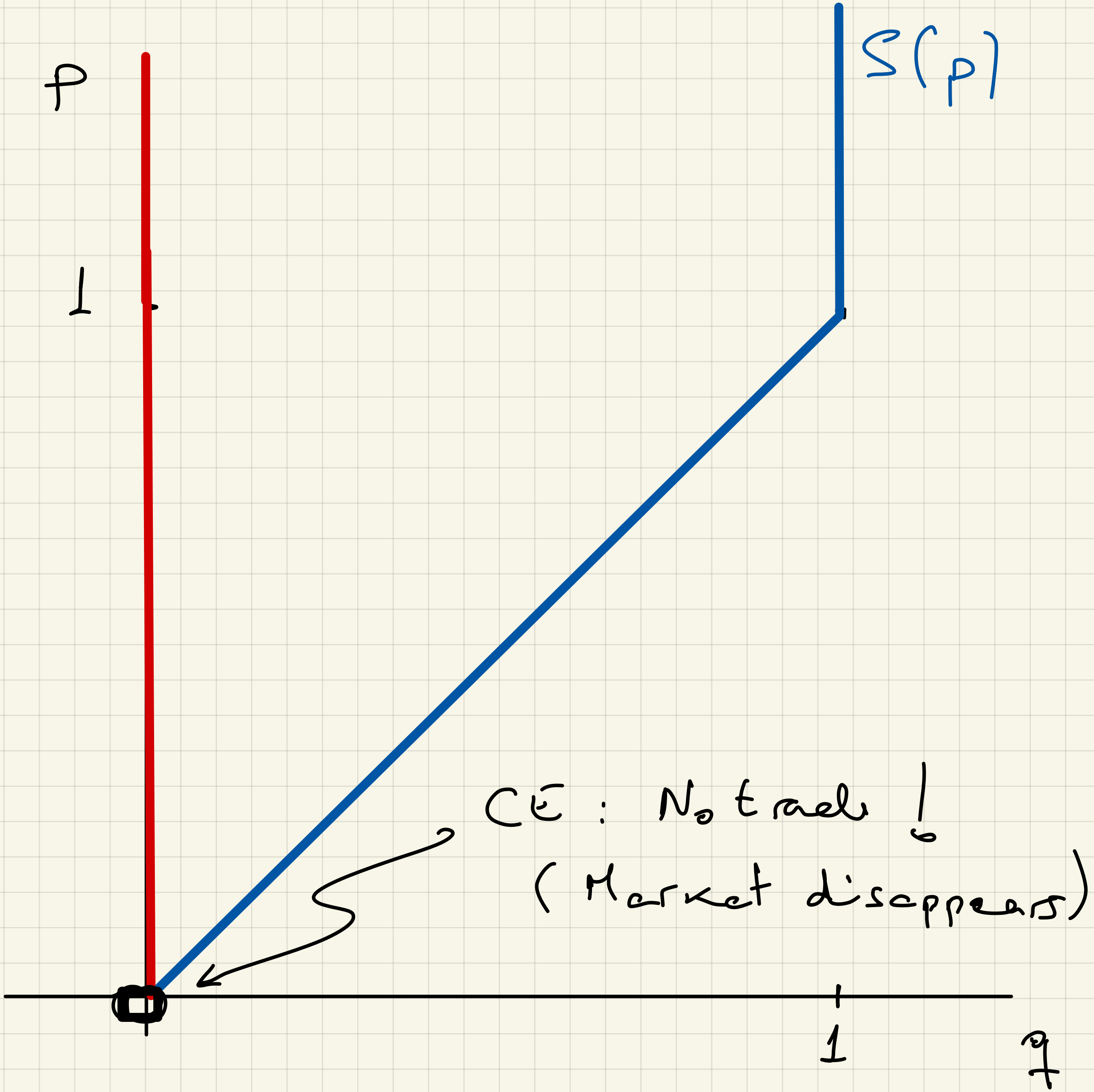
Demand: $D(p) = \begin{cases} 0 & \text{if } p > E[u(S) \mid s \leq p] \\ 1 & \text{otherwise.} \end{cases}$



$\mathbb{E}[u(s) \mid s \leq p]$?

If $p > 1$, $E[u(S) \mid s \leq p] = \alpha E[S \mid s \leq p] = \alpha/2 < 1 < p \Rightarrow D(p) = 0$

If $p \leq 1$: $E[u(S) \mid s \leq p] = \alpha E[S \mid s \leq p] = \alpha\left(\frac{p}{2}\right) < p \Rightarrow D(p) = 0$


p
1
S(p)
CE : No trade !
(Market disappears)
1
q

Let $p \leq 1$:

$$\mathbb{E}[u(S) \mid S \leq p] = \mathbb{E}[\alpha S \mid S \leq p]$$
$$= \alpha \, \mathbb{E}[S \mid S \leq p]$$

$$S \sim U[0,1], \quad f(s) = 1 \qquad \left(\int_0^1 f(s)\,ds = [x]_0^1 = 1 \right)$$

$$S \mid S \leq p: \quad F(s \mid S \leq p) = \Pr[S \leq s \mid S \leq p]$$
$$= \frac{\Pr[S \leq \min\{p, s\}]}{\Pr[S \leq p]}$$
$$= \begin{cases} 1 & \text{if } s > p \\ \frac{F(s)}{F(p)} = \frac{s}{p} & \text{if } s \leq p. \end{cases}$$

Hence

$$f(s \mid S \leq p) = \begin{cases} 0 & \text{if } s > p \\ \frac{1}{p} & \text{if } s \leq p \end{cases}$$

$$\mathbb{E}[s \mid S \leq p] = \int_0^1 s\, f(s \mid S \leq p)\, ds$$

$$= \int_0^p \frac{s}{p}\, ds$$

$$= \frac{1}{p}\left[\frac{x^2}{2}\right]_0^p = \frac{1}{p}\left(\frac{p^2}{2}\right) = \frac{p}{2}.$$