The Agency Problem: Moral Hazard

Example 4. Consider an agency problem in which $\pi(z) = z$, $u(w) = \sqrt{w}$, $c(e) = e^2/2$, and $\underline{u} = 1$. The set of feasible effort levels e is $[0, 1]$. Effort is not verifiable. Revenue X takes two values $x_1 = 4$, and $x_2 = 8$ with probabilities $p(e) = (2 - e)/3$ and $1 - p(e) = (1 + e)/3$, respectively. Identify the optimal contract.

The Agency Problem: Moral Hazard

Without moral hazard the Principal offers the wage

$$w(e) = \left(1 + \frac{e^2}{2}\right)^2$$

and chooses the effort that solves

$$\max_{e \in [0,1]} \mathbb{E}[X(e)] - w(e) = \left(4\left(\frac{2-e}{3}\right) + 8\left(1 - \frac{2-e}{3}\right)\right) - \left(1 + \frac{e^2}{2}\right)^2$$

whose solution is $e^* = 0.57273$. The expected profit is

$$\mathbb{E}[X(e^*)] - w(e^*) = 4.7421.$$

The Agency Problem: Moral Hazard

With moral hazard, if the agent the contract (w_1, w_2) his effort solves the problem

$$\max_{e\in[0,1]} \mathbb{E}[u(W(e))] - C(e) = \sqrt{w_1}\left(\frac{2-e}{3}\right) + \sqrt{w2}\left(1 - \frac{2-e}{3}\right) - \frac{e^2}{2}$$

Hence, on optimal contract inducing the agent to exert effort e must solve the system:

$$\begin{aligned} \frac{1}{3}(\sqrt{w_1} - \sqrt{w_2}) &= e \\ \sqrt{w_1}\left(\frac{2-e}{3}\right) + \sqrt{w_2}\left(1 - \frac{2-e}{3}\right) &= 1 + \frac{e^2}{2} \end{aligned}$$

Solving for (w_1, w_2) as a function of e we get

$$w_1(e) = \left(\frac{3}{2}e^2 + e + 1\right)^2, w_2(e) = \left(\frac{3}{2}e^2 - 2e + 1\right)^2.$$

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Hence, the Principal chooses the effort to solve

$$\max_{e\in[0,1]} \mathbb{E}[X(e) - W(e)] = (4 - w_1(e))\, p(e) + (8 - w_2(e))\,(1 - p(e))$$

whose solution is

$$\tilde{e}^* = 0.20545$$

leading to the expected profit

$$\mathbb{E}[X(\tilde{e}^*)] - w(\tilde{e}^*) = 4.4733.$$

The cost of moral hazard to the Principal is

$$\mathbb{E}[X(e^*)] - w(e^*) - (\mathbb{E}[X(\tilde{e}^*)] - w(\tilde{e}^*)) = 4.7421 - 4.4733 = 0.2688.$$

The Agency Problem: Moral Hazard

Example 4'. Consider an agency problem in which $\pi(z) = z$, $u(w) = \sqrt{w}$, $c(e) = e/2$, and $\underline{u} = 1$. The set of feasible effort levels is $[0, 1]$. Effort is not verifiable. Revenue X takes two values $x_1 = 4$, and $x_2 = 8$ with probabilities $p(e) = (2 - e)/3$ and $1 - p(e) = (1 + e)/3$, respectively. Identify the optimal contract.

The Agency Problem: Moral Hazard

Without moral hazard the Principal offers the wage

$$w(e) = (1 + e)^2$$

and chooses the effort that solves

$$\max_{e \in [0,1]} \mathbb{E}[X(e)] - w(e) = \left(4\left(\frac{2-e}{3}\right) + 8\left(1 - \frac{2-e}{3}\right)\right) - \left(1 + \frac{e}{2}\right)^2$$

whose solution,

$$\frac{d\mathbb{E}[X(e)]}{de} - w'(e) = 0$$

is $e^* = 2/3$. The expected profit is

$$\mathbb{E}[X(e^*)] - w(e^*) = \frac{40}{9}.$$

The Agency Problem: Moral Hazard

With moral hazard, if the agent accepts the contract (w_1, w_2), its payoff is

$$\sqrt{w_1}\left(\frac{2-e}{3}\right)+\sqrt{w_2}\left(1-\frac{2-e}{3}\right)-\frac{e}{2}$$
$$=\quad \frac{2\sqrt{w_1}+\sqrt{w_2}}{3}+\frac{1}{3}\left(\sqrt{w_2}-\sqrt{w_1}-\frac{3}{2}\right)e.$$

Hence the agent effort is $e^* = 0$ if $\sqrt{w_2} < \sqrt{w_1} + \frac{3}{2}$ and it is $e^* = 1$ otherwise. Also, the participation constraint is

$$\sqrt{w_1}\left(\frac{2-e}{3}\right)+\sqrt{w_2}\left(1-\frac{2-e}{3}\right)=1+\frac{e}{2}$$

Therefore the Principal may offer a contract involving the fixed wage $w(0) = 1$, inducing the agent to exert no effort and leading the profits

$$\mathbb{E}[X(0)] - w(0) = 4\left(\frac{2}{3}\right)+8\left(1-\frac{2}{3}\right)-1=\frac{13}{3}.$$

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Or she may offer the contract that solves the system

$$\begin{aligned} \sqrt{w_2} &= \sqrt{w_1} + \frac{3}{2} \\ \left(\frac{1}{3}\right)\sqrt{w_1} + \left(\frac{2}{3}\right)\sqrt{w_2} &= 1 + \frac{1}{2}, \end{aligned}$$

whose solution is

$$w_1 = \frac{1}{4}, w_2 = 4,$$

which induces the agent to exert maximum effort $e = 1$, and leads to profits

$$\mathbb{E}[X(1)] - \mathbb{E}[W(1)] = (4 - \frac{1}{4})\left(\frac{1}{3}\right) + (8-4)\left(1 - \frac{1}{3}\right) = \frac{47}{12} < \frac{13}{3}.$$

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Hence the optimal contract is $(w_1, w_2, e) = (1, 1, 0)$, and the profit is $13/3$. The reduction of profit (and surplus) due to moral hazard is

$$\frac{40}{9} - \frac{13}{3} = \frac{1}{9}.$$