Public Goods and Externalities: An Example

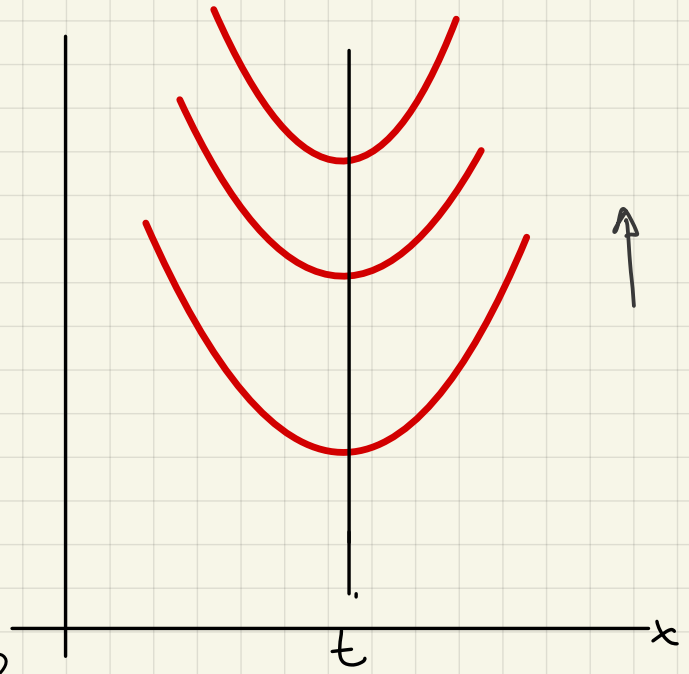
Ann and Bob share an apartment. Central heating is provided free of charge.

Their preferences for room temperature (x) and income (y) are represented by utility functions

$$u_i(x,y) = y - \alpha_i(t_i - x)^2,$$

where $t_A = 25$, $\alpha_A = 3/2$

$t_B = 20$, $\alpha_B = 1$.



Thus, the apartment's temperature is a public good to Ann and Bob.

At which temperature should be set the apartment's thermostat?

Let us try and identify "good outcomes". To begin, Let us set the temperature to some intermediate value $t \in [t_B, t_A]$,

$$\text{e.g.,} \quad x = 21^\circ.$$

Is this temperature Pareto optimal?

Since

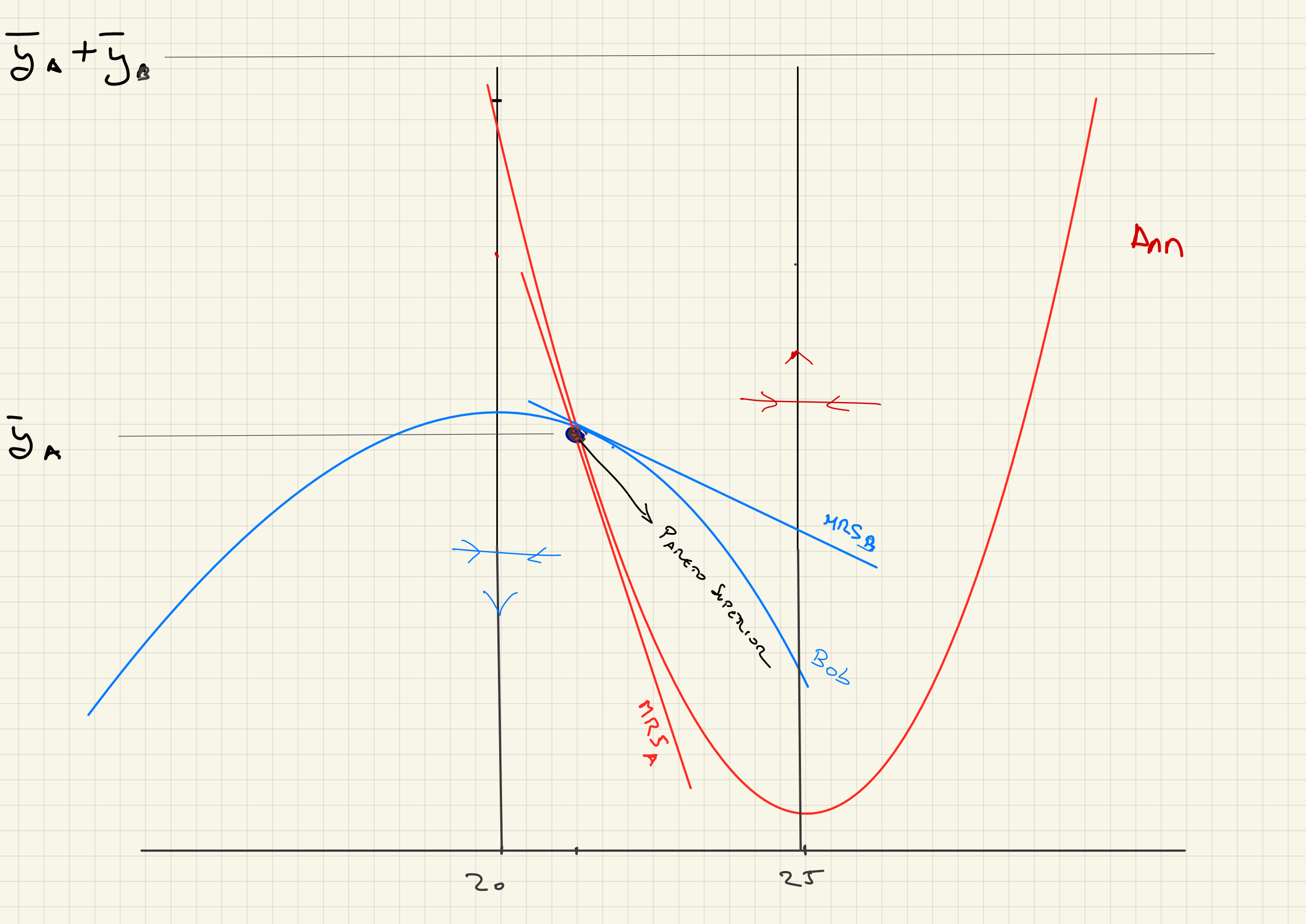
$$MRS_i(x, y) = 2\alpha_i(t_i - x),$$

we have

$$MRS_A(21, y) = 75 - 3(21) = 12$$

$$MRS_B(21, y) = 40 - 2(21) = -2.$$

How to interpret these numbers?


$\bar{y}_A + \bar{y}_B$
$\bar{y}_A$
Ann
MRS$_B$
Parent Superior
Bob
MRS$_A$
20
25

Ann proposes to increase the temperature by $1°$, and offers Bob 5 euros as compensation

Would Bob accept ?

$$u_B(\bar{y}_B, 21) = \bar{y}_B - (20-21)^2 = \bar{y}_B - 1.$$

$$u_B(\bar{y}_B + 5, 22) = \bar{y}_B + 5 - (20-22)^2 = \bar{y}_B + 1.$$

Would Ann make such offer?

$$u_A(\bar{y}_A, 21) = \bar{y}_A - \frac{3}{2}(25-21)^2 = \bar{y}_A - 24.$$

$$u_A(\bar{y}_A - 5, 22) = \bar{y}_A - 5 - \frac{3}{2}(25-22)^2 = \bar{y}_A - 18.5$$

Both are better off !

?

Note $MRS_A(21) + MRS_B(21) = 12 - 2 = 10 > 0$

Ray should raise the temperature as long as

$$MRS_A(x) + MRS_B(x) > 0$$

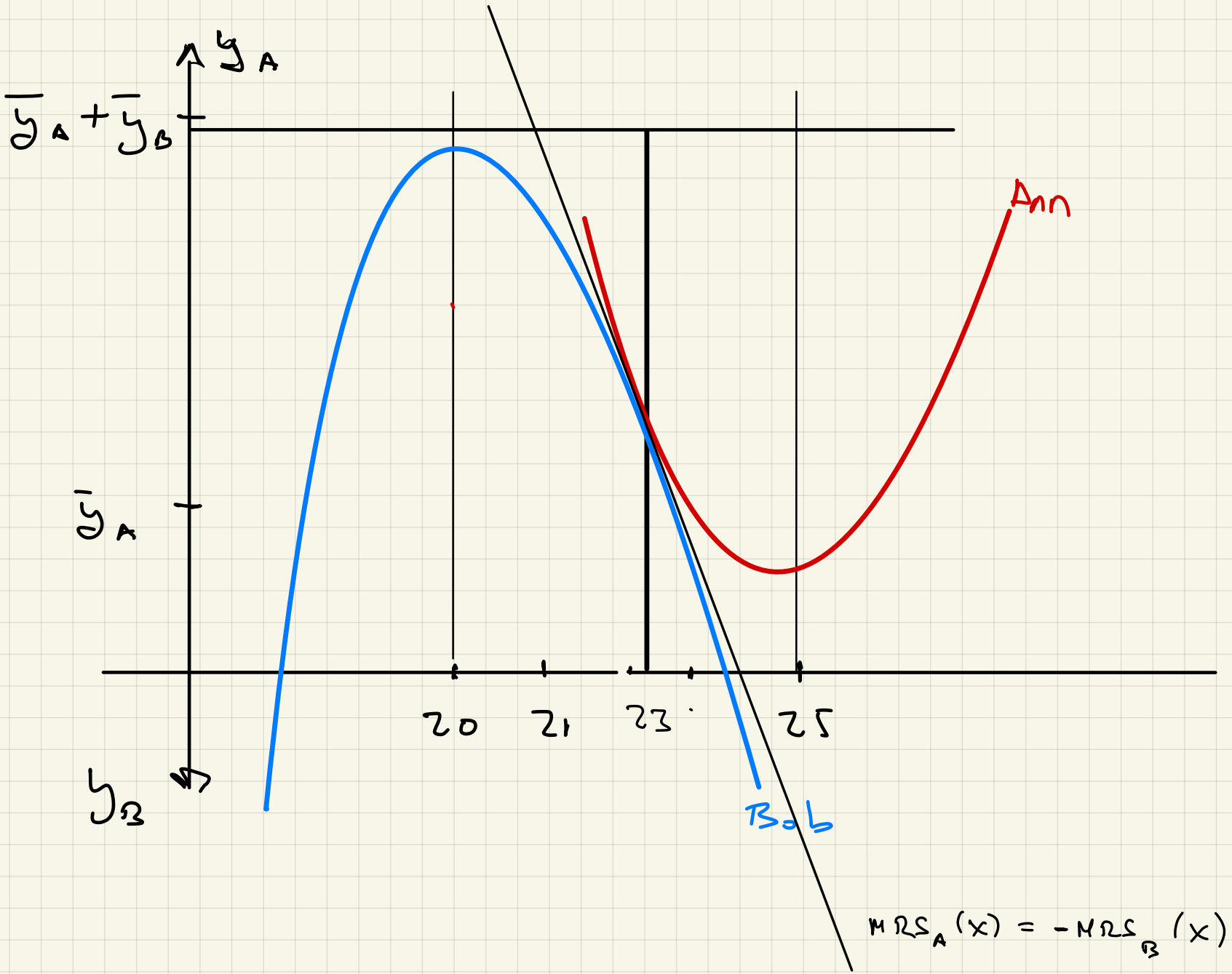
Likewise, if $MRS_A(x) + MRS(B) < 0$, Ray can both improve by reducing the temperature (and accord some compensations).

PO requires:

$$MRS_A(x) + MRS_B(x) = 0$$

$$75 - 3x + 40 - 2x = 0$$

i.e., $$x^* = \frac{115}{5} = 23^\circ$$


y_A
$\bar{y}_A + \bar{y}_B$
$\bar{y}_A$
y_B
20
21
23
25
Ann
B.L
$MRS_A(x) = -MRS_B(x)$

... but how is the apartment's temperature decided?

And how to solve this problem?

Perhaps a compromise may be reached by bargaining!

BARGAINING

Set $\bar{y}_A = \bar{y}_B = 0$. For $t = 23°$,

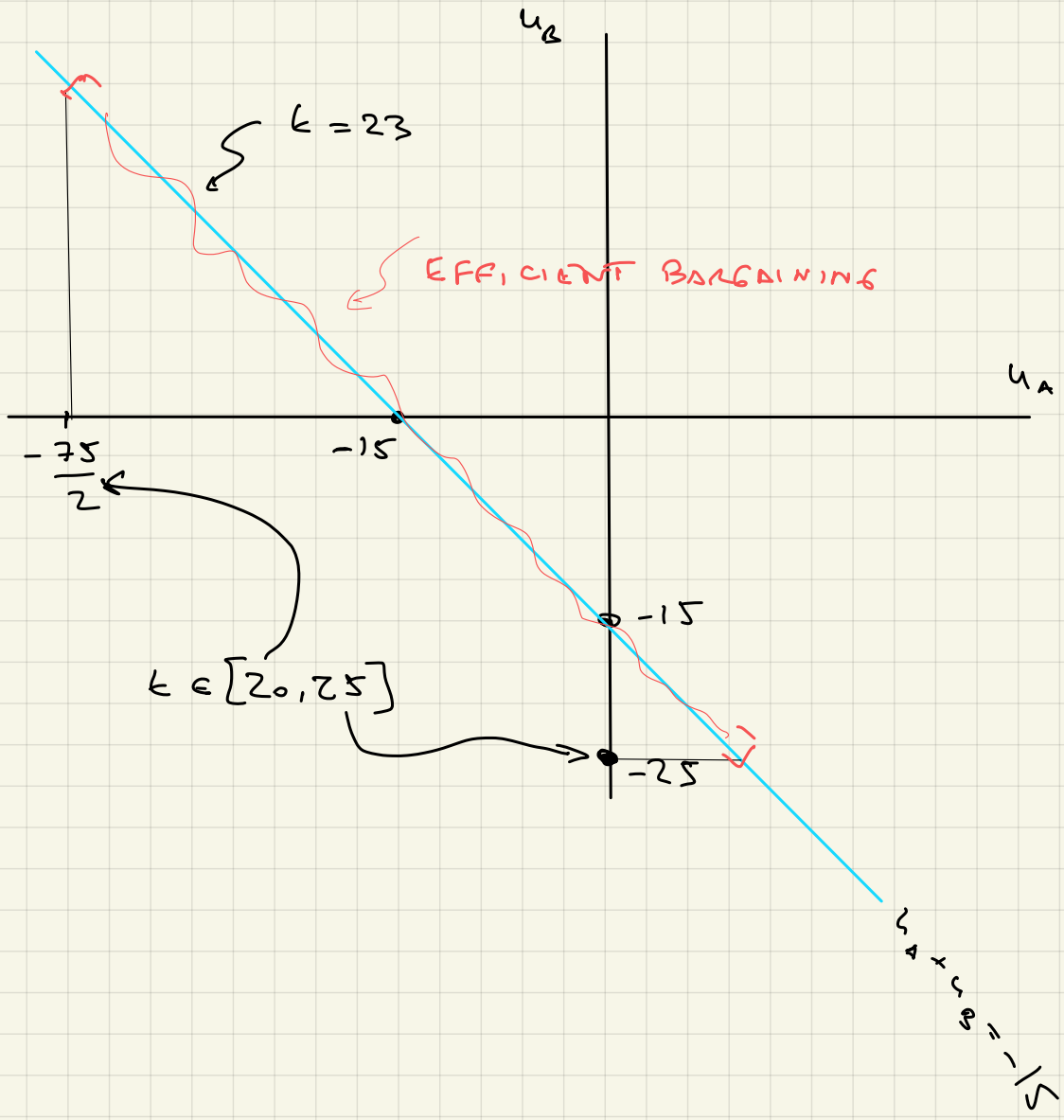
$$u_A + u_B = -\frac{3}{2}(2)^2 - (3)^2 = -15.$$

ABSENT AN AGREEMENT:

Ann's Owner $\longrightarrow$ $t = 25°$

Bob's Owner $\longrightarrow$ $t = 20°$

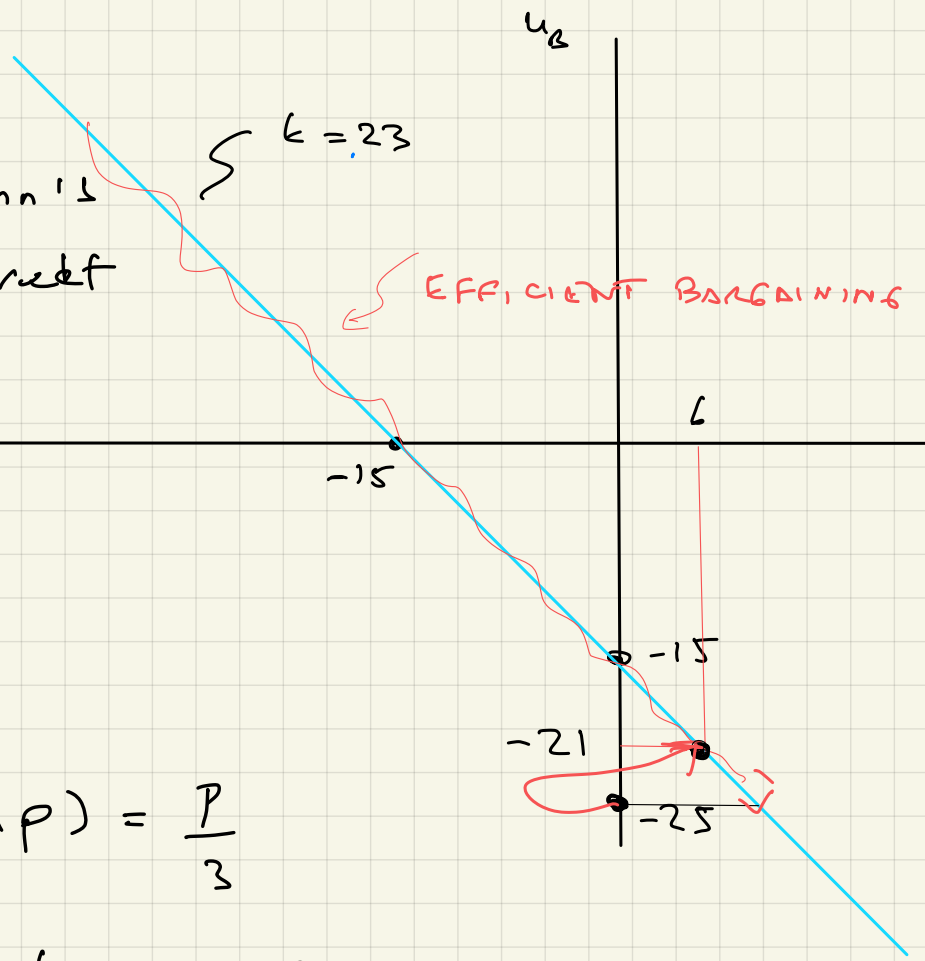
Plenty of room to reach a compromise.



A MARKET SOLUTION

Assume instead that starting from Ann's ideal temperature (25°), a market is created whereby Ann (Bob) supplies (demands) rights to lower (raise) temperature.

p: price of 1 degree.



Ann $\max_r \; pr - \frac{3}{2}[25-(25-r)]^2 \Rightarrow r_A(p) = \frac{p}{3}$

Bob $\max_r \; -pr - [20-(25-r)]^2 \Rightarrow r_B(p) = 5 - \frac{p}{2}$

Market Clearing. $\frac{p}{3} = 5 - \frac{p}{2} \iff p^* = 6 \; ; \; r^* = 2.$

Market Outcome: $x^* = 23 \Rightarrow$

$u_A^* = -\frac{3}{2}(25-23)^2 + p^* r^* = -6 + 12 = 6 > 0$

$u_B^* = -(20-23)^2 - p^* r^* = -9 - 12 = -21 > -25$

Exercises:

(1) If there is another apartment's resident, Conrad, whose preferences parameters are $\alpha_C = 1$, $t_C = 22$, what is the apartment's optimal temperature?

(2) If the cost of maintaining the temperature at t degrees is

$$C(t) = \frac{(t-20)^2}{2}$$

what is the apartment's optimal temperature?

(3) What would be the temperature if Ann, Bob and Conrad vote, and the thermostat is set at the median temperature?