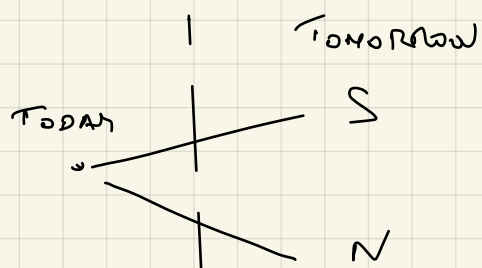


3. Andy's income this year is €20. He is retiring next year, and his income depends on whether Social Security remains financially sound (S), or it goes bankrupt (N). Andy's income in state S will be €20, whereas it will be only €10 in state N (the amount paid by his private pension plan). Beth's income this year is also €20, and her income next year depends on the state of nature: in state S Beth will have to contribute to SS, leaving her with an income of only €10; in state N Beth's income is €20. Andy's preference is represented by the utility function $u^A(x, y_S, y_N) = x + 5 \ln y_S + 6 \ln y_N$, and Beth's by the function $u^B(x, y_S, y_N) = x + 10 \ln y_S + 3 \ln y_N$, where x denotes the individual's spending this year, y_S denotes spending next year in state S , and y_N denotes spending next year in state N . Income is perishable; i.e., one unit of income dated this year is worthless next year.



$$(\bar{x}^A, \bar{y}_S^A, \bar{y}_N^A) = (20; 20, 10)$$

$$(\bar{x}^B, \bar{y}_S^B, \bar{y}_N^B) = (20; 10, 20)$$

(a) Determine the competitive allocation and prices assuming that there markets for all goods.

NOTATION: $P = (1, P_S, P_N)$. BC: $x + P_S y_S + P_N y_N = \bar{x} + P_S \bar{y}_S + P_N \bar{y}_N$

DEMANDS: $MRS_{x y_S}^i = \frac{y_S}{\alpha_i} = \frac{1}{P_S}$, $MRS_{x y_N}^i = \frac{y_N}{\beta_i} = \frac{1}{P_N}$. $(\alpha_A, \beta_A) = (5, 6)$
 $(\alpha_B, \beta_B) = (10, 3)$

ANDY: $y_S^A(P) = \frac{5}{P_S}$, $y_N^A(P) = \frac{6}{P_N}$

BETH: $y_S^B(P) = \frac{10}{P_S}$, $y_N^B(P) = \frac{3}{P_N}$.

MARKET CLEARING

$$\frac{5}{P_S} + \frac{10}{P_S} = 30 \Rightarrow P_S^* = \frac{1}{2}$$

$$\frac{6}{P_N} + \frac{3}{P_N} = 30 \Rightarrow P_N^* = \frac{3}{10}$$

Hence: $y_S^A(P^*) = 10$, $y_N^A(P^*) = 20$, $y_S^B(P^*) = 20$, $y_N^B(P^*) = 10$

$$x^A(P^*) = 20 + \frac{1}{2}(20 - 10) + \frac{3}{10}(10 - 20) = 22$$

$$x^B(P^*) = 20 + \frac{1}{2}(10 - 20) + \frac{3}{10}(20 - 10) = 18$$

CE ALLOCATION: $\left[(22, 10, 20), (18, 20, 10) \right]$

(b) Suppose that only spot markets and a credit market exist. Is the equilibrium allocation Pareto efficient?

Notation. $\hat{p}_1 = \hat{p}_s = \hat{p}_N = 1$, \underline{r} : INTEREST RATE

CONSUMERS' BUDGET CONSTRAINTS:

$$x + b = \bar{x}$$

$$y_s = \bar{y}_s + (1+r)b$$

$$y_N = \bar{y}_N + (1+r)b$$

$$MRS_{y_s, y_N}^A = \frac{S/y_s^A}{G/y_N^A} = \frac{S}{G} \frac{y_N^A}{y_s^A} = \frac{S}{G} \frac{10 + (1+r)b^A}{20 + (1+r)b^A}$$

$$MRS_{y_s, y_N}^B = \frac{10/y_s^B}{3/y_N^B} = \frac{10}{3} \frac{y_N^B}{y_s^B} = \frac{10}{3} \frac{20 + (1+r)b^B}{10 + (1+r)b^B}$$

Clearly, in the CE $y_s^i > 0$, $y_N^i > 0 \quad \forall i \in \{A, B\}$.

Why? If either $y_s^i = 0$ or $y_N^i = 0$ consumer i is worse off in the CE than in the initial allocation.)

$$\Delta C \geq 0: \quad L^A(r^*) = -L^B(r^*)$$

PARETO OPTIMALITY WOULD REQUIRE:

$$MRS_{y_s, y_N}^A = MRS_{y_s, y_N}^B$$

Using $(1+r^*) b^A(r^*) = -(1+r^*) b^B(r^*) = \alpha$, WE MAY WRITE THIS CONDITION

AS

$$\frac{5}{6} \frac{10 + \alpha}{20 + \alpha} = \frac{10}{3} \frac{20 - \alpha}{10 - \alpha}.$$

THE SOLUTIONS TO THIS EQUATION ARE $\alpha = \pm 10\sqrt{5}$.

HOWEVER,

IF $\alpha = 10\sqrt{5} \Rightarrow y_2^B = 10 - 10\sqrt{5} < 0$, WHICH IS A CONTRADICTION

IF $\alpha = -10\sqrt{5} \Rightarrow y_5^A = 10 - 10\sqrt{5} < 0$, WHICH IS A CONTRADICTION

HENCE THE CE ALLOCATION IS NOT PARETO OPTIMAL.

(c) Assume now that there are only spot markets, and the only securities are shares in the firm Gamma Technologies and shares in the firm Delta Insurance. Each share of Gamma will yield €2 in state S and €1 in N . Each share of Delta will yield €1 in state S and €2 in N . Determine the equilibrium security prices and Andy's and Beth's holdings of securities. What portfolio would one have to hold in order to guarantee oneself a return of €1 next year whether the state is S or N ? What would be the cost of that portfolio? What would you say is the interest rate?

$$r_D = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad r_G = \begin{pmatrix} 2 \\ 1 \end{pmatrix}; \quad p_S = p_N = 1, \quad \underline{CE}: -z_G^A = z_D^A = z_G^B = -z_D^B = 10$$

q_D, q_G

IN THIS SETTING A CONSUMER'S BUDGET CONSTRAINTS ARE:

$$\text{(THIS YEAR)} \quad x + q_D z_D + q_G z_G = \bar{x}$$

$$\text{(NEXT YEAR S)} \quad y_S = \bar{y}_S + z_D + 2z_G$$

$$\text{(NEXT YEAR N)} \quad y_N = \bar{y}_N + 2z_D + z_G$$

SOLVING FOR z_D, z_G IN NEXT YEAR CONSTRAINTS WE GET

$$z_D = \frac{1}{3} (\bar{y}_S - y_S) + \frac{2}{3} (\bar{y}_N - y_N)$$

$$z_G = -\frac{2}{3} (\bar{y}_S - y_S) + \frac{1}{3} (\bar{y}_N - y_N)$$

SUBSTITUTING THESE VALUES IN THIS YEAR'S BUDGET CONSTRAINT
WE GET :

$$x + \left(\frac{2}{3} p_D - \frac{1}{3} p_G\right) y_S + \left(\frac{2}{3} p_G - \frac{1}{3} p_D\right) y_N = \bar{x} + \left(\frac{2}{3} p_D - \frac{1}{3} p_G\right) \bar{y}_S + \left(\frac{2}{3} p_G - \frac{1}{3} p_D\right) \bar{y}_N$$

COMPARING THIS BUDGET CONSTRAINT WITH THAT OF PART (a) OF
THE EXERCISE, WE SEE THAT A CONSUMER'S PROBLEM IS THE
SAME, AND THEREFORE IN EQUILIBRIUM THE RELATIVE MARKET VALUES
OF GOODS MUST BE THE SAME. THAT IS :

$$\left. \begin{aligned} \frac{2}{3} p_D^* - \frac{1}{3} p_G^* &= p_S^* = \frac{1}{2} \\ \frac{2}{3} p_G^* - \frac{1}{3} p_D^* &= p_N^* = \frac{3}{10} \end{aligned} \right\} \Leftrightarrow \begin{aligned} p_D^* &= \frac{11}{10} \\ p_G^* &= \frac{13}{10} \end{aligned}$$

As for the IMPLICIT INTEREST RATE, note that giving up 1 unit of consumption today one can buy the portfolio

$$\begin{pmatrix} z_c \\ z_D \end{pmatrix} = \begin{pmatrix} z \\ z \end{pmatrix}, \quad \text{where} \quad 1 = q_D z + q_C z \Leftrightarrow z = \frac{1}{q_D + q_C} = \frac{10}{24}$$

which yields

$$\begin{matrix} S \\ N \end{matrix} \begin{pmatrix} 2z_c + z_D \\ z_c + 2z_D \end{pmatrix} = \begin{pmatrix} 30/24 \\ 30/24 \end{pmatrix} = \begin{pmatrix} 5/4 \\ 5/4 \end{pmatrix}$$

units of consumption in each state. Hence

$$\frac{5}{4} = 1 + r \quad (\Rightarrow) \quad r = 1/4$$

is the "implicit" interest rate.