6. A monopoly produces a good with cost $c(q) = q$ that sells to two consumers whose demands are $D_L(p) = \max\{8 - p, 0\}$ and $D_H(p) = \max\{10 - p, 0\}$.

(a) Calculate the monopoly equilibrium in the absence of price discrimination.

(b) Assume that the monopoly sells the good using a two-part tariff (T, p), where T is a fee a consumer must pay to access the market, and p is the unit price at which the good is sold. Calculate the two-part tariff the monopoly will offer to each consumer.

(c) Now assume that the monopoly cannot discriminate consumers because it does not observe their types. Calculate the menu of two-part tariffs the monopoly will offer in this case.

(a)

$$D(p) := D_L(p) + D_H(p) = \begin{cases} 18 - 2p & \text{if } p \in [0, 8] \\ 10 - 2p & \text{if } p \in (8, 10] \\ 0 & \text{if } p > 10 \end{cases}$$

For $q \in [0, 2]$, $\pi(q) = P(q)q - C(q) = (5 - \frac{q}{2})q - q = 4q - \frac{q^2}{2}$, $\pi'(q) = 4 - q > 0$.

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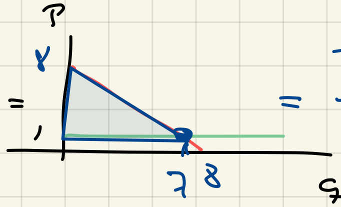
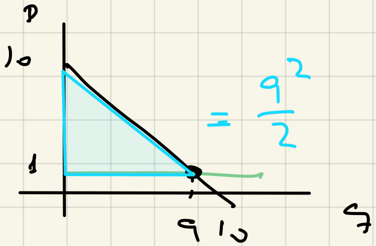
For $q \in [2, 18]$: $\pi(q) = P(q)q - C(q) = (9 - q/2)q - q = 8q - \frac{q^2}{2}$

$\rightarrow$ $\pi'(q) = 8 - q \gtreqless 0 \iff q \lesseqgtr 8$

Hence, in the absence of price discrimination $q^* = 8$, $p^* = 5$.

(b) If the monopoly can offer a different two-part tariff to each consumer, it will offer the contracts

$p_i = MC(q) = 1$, and $T_L =$ 

$= \frac{7^2}{2}$, $T_H =$ 

$= \frac{9^2}{2}$

(c) If the monopoly cannot distinguish L from H, then it may offer the single contract $(\bar{T}, \bar{p})$ designed to solve the problem:

$$\max_{p \in [0,8]} \quad D(p)(p-1) + 2T = \Pi(p,T)$$

$$T = \min\left\{\frac{(8-p)^2}{2}, \frac{(10-p)^2}{2}\right\} = \frac{(8-p)^2}{2}$$

Thus, $\bar{p}$ solves:

$$\max_{p \in [0,8]} \quad \bar{\Pi}(p) := \Pi\left(p, \frac{(8-p)^2}{2}\right) = (18-2p)(p-1) + (8-p)^2$$

i.e., $\bar{\Pi}'(p) = (18-2p) - 2(p-1) - 2(8-p) = 4 - 2p = 0$

Hence $\bar{p} = 2$, $\bar{T} = \frac{(8-2)^2}{2} = 18$; $\bar{\Pi} = 50$

Alternatively, the monopoly may offer a menu

$$\{(\hat{p}_L, \hat{T}_L), (\hat{p}_H, \hat{T}_H)\}$$

by solving the problem

$$\max_{(p_L, T_L, p_H, T_H) \in \mathbb{R}_+^4} (8 - p_L)(p_L - 1) + (10 - p_H)(p_H - 1) + T_L + T_H$$

s.t.
$$\frac{(8 - p_L)^2}{2} \geq T_L \qquad (PC_L)$$

$$\frac{(10 - p_H)^2}{2} \geq T_H \qquad (PC_H)$$

$$\frac{(8 - p_L)^2}{2} - T_L \geq \frac{(8 - p_H)^2}{2} - T_H \qquad (IC_L)$$

$$\frac{(10 - p_H)^2}{2} - T_H \geq \frac{(10 - p_L)^2}{2} - T_L \qquad (IC_H)$$

<u>Note</u>: $PC_L + IC_H \Rightarrow PC_H$ is <u>not</u> binding

IC_L is <u>not</u> binding (This can be shown by contradiction)

Hence $T_L = \frac{(8 - p_L)^2}{2}$ and $T_H = \frac{(10 - p_H)^2}{2} - \left[\frac{(10 - p_L)^2}{2} - \frac{(8 - p_L)^2}{2}\right]$

Substituting T_L and T_C in the objective function and solving w.r.t. $(p_L, p_H) \in [0, 8] \times [0, 10]$ we get

$$\hat{p}_L = 3 \quad , \quad \hat{p}_H = 1 \, ,$$

and therefore

$$\hat{T}_L = \frac{(8 - \hat{p}_L)^2}{2} = \frac{25}{2} \, ,$$

$$\hat{T}_H = \frac{(10 - \hat{p}_H)^2}{2} - \left[\frac{(10 - \hat{p}_L)^2}{2} - \frac{(8 - \hat{p}_L)^2}{2} \right] =$$

$$= \frac{81}{2} - \left(\frac{49}{2} - \frac{25}{2} \right) = \frac{57}{2}$$

The profit with the menu is

$$\hat{\pi} = (8-3)(3-1) + \frac{25}{2} + (10-1)(1-1) + \frac{57}{2} = \frac{102}{2} = 51 > \overline{\pi} \, .$$

Hence the monopoly will offer the <u>menu</u>.