Exercise 1. A good of two qualities, high (H) and low (L), is traded in competitive markets in which each seller has a single unit and each buyer wants to buy a single unit. There are n_H sellers with a unit of high quality whose opportunity cost is c_H euros, n_L sellers with a unit of low quality whose opportunity cost is c_L euros, and n buyers who value a unit of high quality in u_H euros and a unit of low quality in u_L euros. Assume that $u_H > c_H > u_L > c_L$.

(a) Suppose that quality is observable. Calculate the competitive prices for the cases $n > n^H + n^L$ and $n < n_H + n_L$. Discuss if these competitive equilibria generate the maximum surplus. (If you find it helpful, assume $u_H = 10$, $c_H = 7$, $u_L = 5$, $c_L = 0$, $n_H = 1$, $n_L = 1$ and $n \in \{1, 3\}$.)

(b) Now suppose that quality is not observable and both qualities trade in the same market. Also assume that $n = n_H + n_L$. Represent the supply and demand schedules in the plane (q, p) and calculate the competitive equilibria of this market when the expected value of a random unit,

$$u(n^H, n^L) = \frac{n_H}{n_H + n_L} u_H + \frac{n_L}{n_H + n_L} u_L,$$

is greater than c^H, and when it is less than c^H. (If you find it helpful, use the parameter values suggested in part (a), and consider the cases $n_L = 1$ and $n_L = 2$.)

PARAMETERS: $u_H > c_H > u_L > c_L$

$$n_B > \max\{n_H, n_L\}$$

CE WHEN QUALITY IS OBSERVABLE:

DIFFERENCIATED MARKETS FOR <u>HIGH</u> AND <u>LOW</u> QUALITY.

LET τ: $u_\tau - c_\tau = \max\{u_H - c_H, u_L - c_L\}$.

<u>CE</u>:

① $n_B < n_H + n_L$

- n_B UNITS TRADE
- ALL τ-QUALITY UNITS TRADE
- $n_B - n_H - n_L$ UNITS OF THE QUALITY $\tau' \neq \tau$ <u>DO NOT</u> TRADE.
- $P_{\tau'} = c_{\tau'}$
- $u_\tau - P_\tau = u_{\tau'} - c_{\tau'}$

THE MAXIMUN SURPLUS IS REALIZED.

(2) $n_B > n_H + n_L$

- BUYERS' SURPLUS IS ZERO $\Rightarrow$ $P_H = u_H$, $P_L = C_L$
- ALL UNITS TRADE $\Rightarrow$ MAXIMUM SURPLUS IS REALIZED.

(3) $n_B = n_H + n_L$

- MULTIPLE CE EXIST. SPECIFICALLY, THE SET OF CE PRICES IS

$$\{ (P_H, P_L) \in [C_H, u_H] \times [C_L, u_L] \mid u_H - P_H = u_L - P_L \}$$

- ALL UNITS TRADE $\Rightarrow$ MAXIMUM SURPLUS IS REALIZED.

(a) $n_H = n_L = 1$, $n \in \{1, 2, 3\}$.

$n_B = 1 \implies (p_H, p_L) \in [0, 7] \times [0, 2]$; e.g., $(p_H^*, p_L^*) = (5, 1)$

$n_B = 2 \implies \{(p_H, p_L) \in [7, 10] \times [0, 5] \mid 10 - p_H = 5 - p_L\}$;

e.g., $(p_H^*, p_L^*) = (8, 3)$

$n_B = 3 \implies p_H^* = u_H = 10$, $p_L^* = u_L = 5$.

UNOBSERVABLE QUALITY: ALL UNITS TRADE IN THE SAME MARKET. THE QUALITY IS OBSERVED ONL UPON PURCHASE.

LET US WRITE

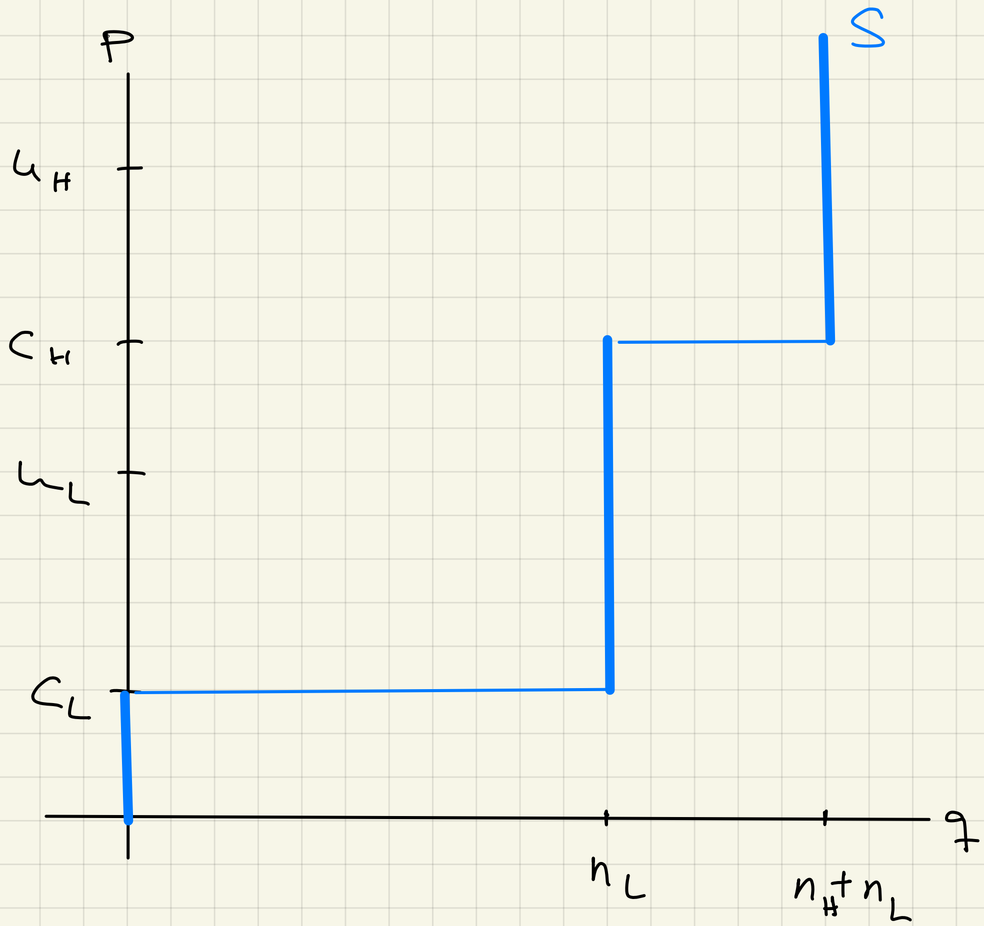
$$\bar{u} := \frac{n_H}{n_H + n_L} u_H + \frac{n_L}{n_H + n_L} u_L$$

FOR THE BUYERS' EXPECTED VALUE OF A RANDOM UNIT WHEN ALL SELLERS SUPPLY. (WE ASSUME THAT BUYERS ARE RISK-NEUTRAL.)

WHAT ARE THE SUPPLY AND DEMAND

OF THIS GOOD OF RANDOM QUALITY?

Supply: Easy!



Demand?

Buyers, who know the costs of sellers, can make inferences about the quality that are supplied at each price.

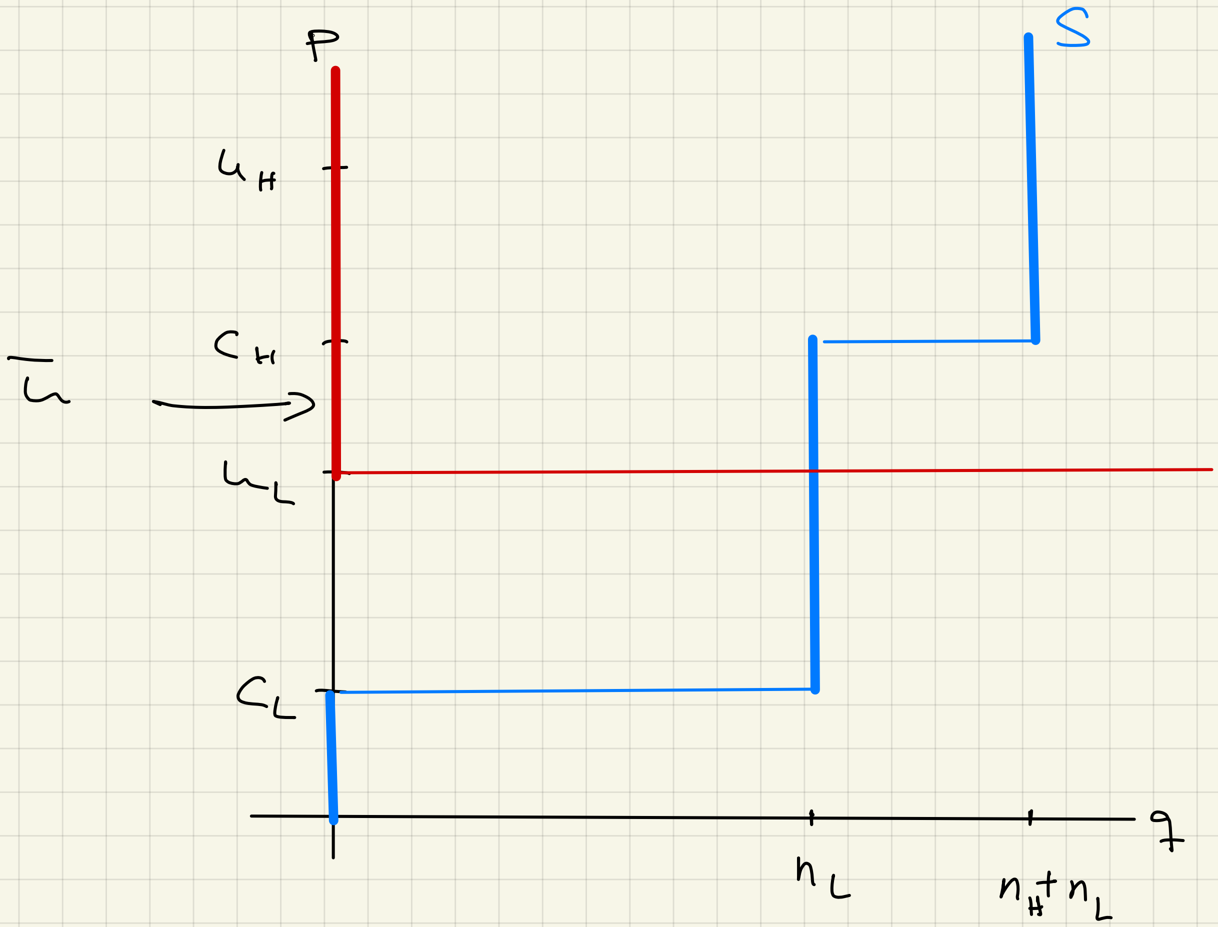
$$E[u \mid p] = \begin{cases} \bar{u} & \text{if } p > c_H \\ u_L & \text{if } p \in (c_L, c_H) \\ ? & p < c_L \end{cases}$$

$$D(p) = \begin{cases} n_B & \text{if } E[u \mid p] > p \\ \{0, 1, \ldots, n_B\} & \text{if } E[u \mid p] = p \\ 0 & \text{if } E[u \mid p] < p \end{cases}$$

(i) $\bar{u} < c_H$

Since $n_B > n_L$,

$p^* = u_L$



THE CE IS INEFFICIENT!

(b) (c)

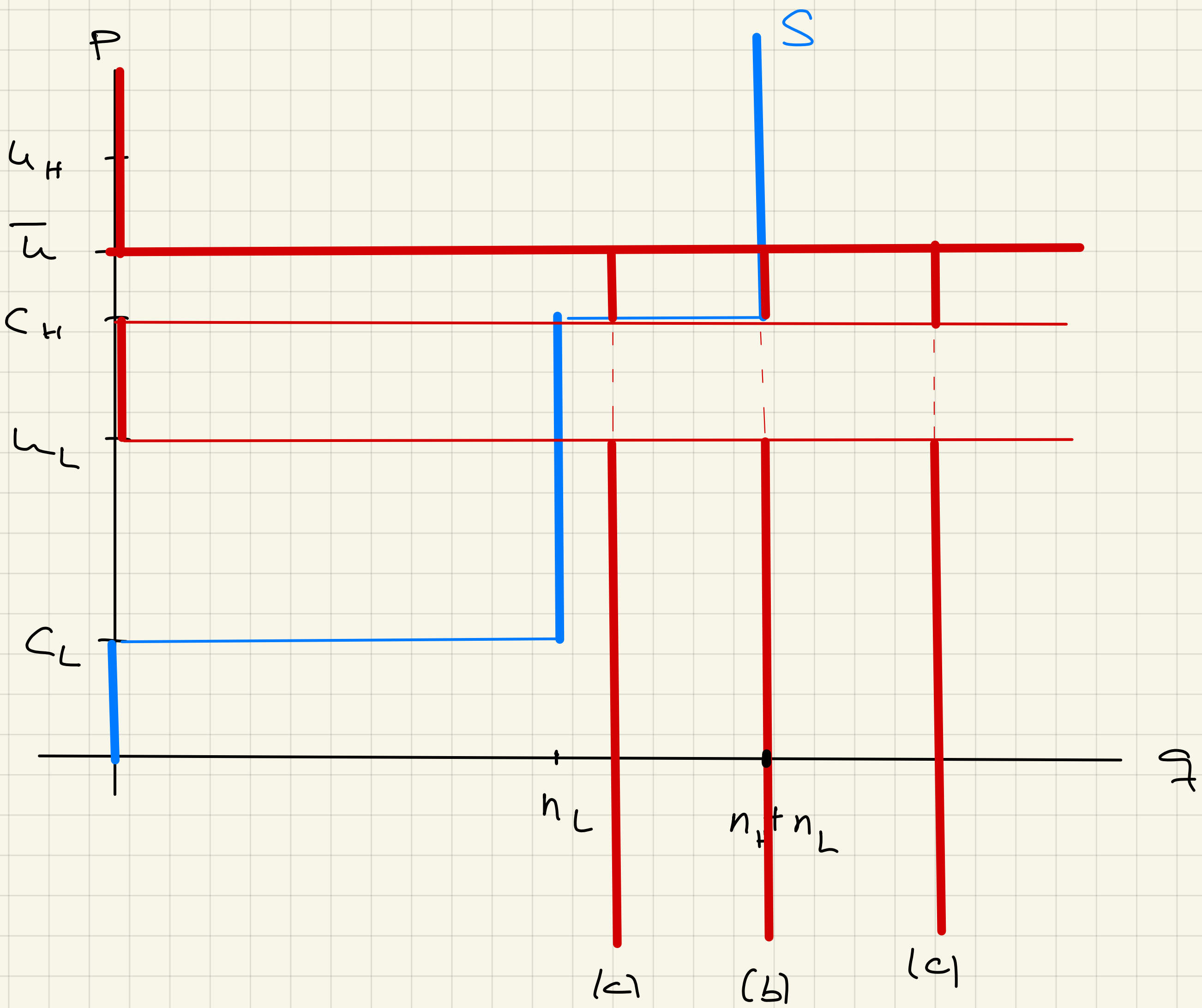
(ALL HIGH-QUALITY SELLERS AND SOME BUYERS DO NOT TRADE EVEN THOUGH THERE ARE GAINS TO TRADE — RECALL THAT $n_B > n_L$.)

(ii) $\bar{u} > c_H$

(a) $n_B < n_H + n_L$

(b) $n_B = n_H + n_L$

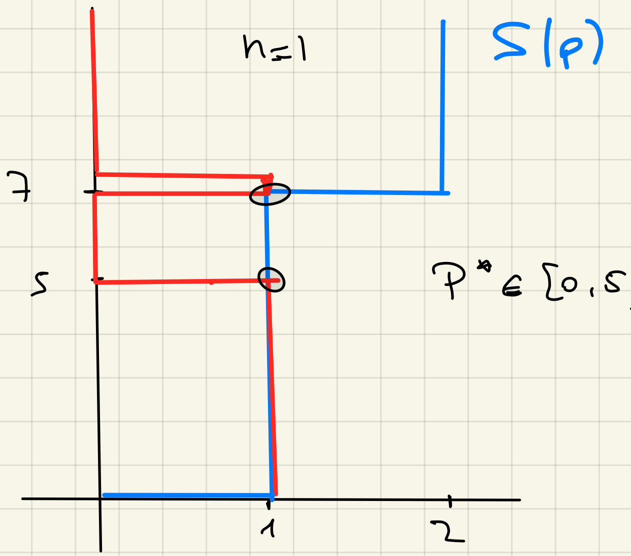
(c) $n_B > n_H + n_L$



THERE EFFICIENT AS WELL AS INEFFICIENT CE!

(b) $n^H = n^L = 1$:

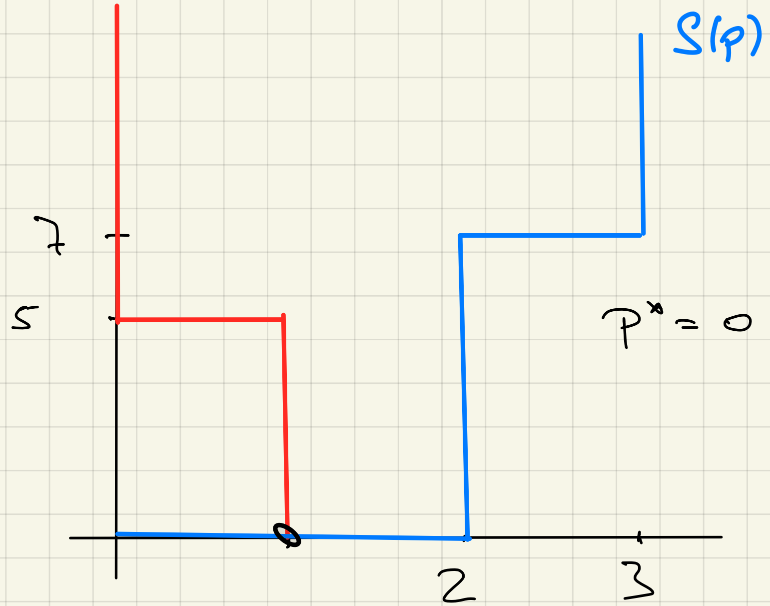
$$E[u] = \frac{u^H + u^L}{2} = 7.5$$



$P^* \in [0, 5]$

$n^H = 1, \; n_L = 2$:

$$E[u] = \frac{u^H + 2u^L}{3} = \frac{20}{3} < 7$$



$P^* = 0$