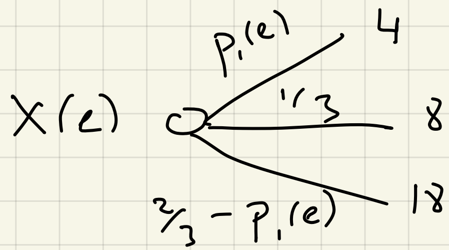
Example 3'. Consider an agency problem in which $\pi(z) = z$, $u(w) = \sqrt{w}$, $c(e) = e$, and $\underline{u} = 1$. Effort may be either $e = 0$ or $e = 1$, and it is not verifiable. Revenue X takes three values $x_1 = 4$, $x_2 = 8$ and $x_3 = 18$ with probabilities $p_1(e) = (2-e)/3$ and $p_2(e) = 1/3$, respectively. Identify the optimal contract. (Hint: $w_1 = 1/4$, $w_2 = 4$, $w_3 = 49/4$, $\lambda = 4$, $\mu = 3$.)



$E[X(0)] = \frac{16}{3}$

$E[X(1)] = 10.$

$(e_\ell, \bar{w}(e_\ell))$: $\sqrt{w} = 0 + 1 \Rightarrow \bar{w}(e_\ell) = 1$, $E[\pi(e_\ell, \bar{w}_\ell)] = \frac{13}{3}$

$(e_h, \bar{w}(e_h))^{*}$: $\sqrt{w} = 1 + 1 \Rightarrow \bar{w}(e_h) = 4 \Rightarrow E[\pi(e_h, \bar{w}(e_h))] = 10 - 4 = 6.$

$\underline{\underline{e_h}}$:

(PC) $\frac{1}{3}\left(\sqrt{w_1} + \sqrt{w_2} + \sqrt{w_3}\right) = 2$

(IC) $\frac{1}{3}\left(\sqrt{w_1} + \sqrt{w_2} + \sqrt{w_3}\right) - 1 = \frac{2}{3}\sqrt{w_1} + \frac{1}{3}\sqrt{w_2} - 0$

$$2\sqrt{w_1} = \lambda + \mu\left(1 - \frac{2/3}{1/3}\right)$$

$$2\sqrt{w_2} = \lambda + \mu\left(1 - \frac{1/3}{1/3}\right)$$

$$2\sqrt{w_3} = \lambda + \mu\left(1 - \frac{0}{1/3}\right)$$

$$E[X(0)] - \bar{w}(0) = \frac{2}{3}(4) + \frac{1}{3}(8) + 0(18) - 1 = \frac{13}{3} \doteq 4.3$$

$$E[X(1) - W(1)] = \frac{1}{3}\left(4 - \frac{1}{4}\right) + \frac{1}{3}(8 - 4) + \frac{1}{3}\left(18 - \frac{49}{4}\right)$$

$$= \frac{1}{3}\left(\frac{15}{4} + \frac{16}{4} + \frac{23}{4}\right)$$

$$= \frac{1}{3}\,\frac{54}{4} = \frac{7}{2} = 4.5$$

$$\geq$$

Optimal Contract: $W = (w_1, w_2, w_3) = \left(\frac{1}{4}, 4, \frac{49}{4}\right)$

$$E[W] = \frac{1}{3}\left(\frac{1}{4} + 4 + \frac{49}{4}\right) = 4 + \frac{5}{6}$$

Surplus Loss $= \frac{5}{6}$