UC3M
TOPICS Advanced Mathematics for Economics
Final January 13th, 2025

Niu: _____ Group: _____

Name: _____

1

(a) (5 points) Find the general solution of the difference equation

$$x_{t+2} - 2x_{t+1} + 4x_t = 0.$$

Note: $\sqrt{12} = 2\sqrt{3}$.

(b) (5 points) Find the general solution of the difference equation

$$x_{t+2} - 2x_{t+1} + 4x_t = t + 1.$$

(c) (5 points) Find the solution of the initial value problem

$$x_{t+2} - 2x_{t+1} + 4x_t = t + 1, \quad x_0 = \frac{2}{3},\ x_1 = \frac{2}{3}.$$

2

Let the matrix

$$A = \begin{pmatrix} a & 0 & 0 \\ b & -2 & 2 \\ 2 & -2 & 3 \end{pmatrix},$$

where a and b are parameters.

(a) (5 points) Determine the eigenvalues of A.

(b) (5 points) For which values of the parameters a, b is the matrix A diagonalizable?

(c) (5 points) For the values $a = \frac{1}{2}$ and $b = 1$, determine a diagonal matrix D and an invertible matrix P such that $P^{-1}AP = D$.

3

Consider the following system of linear difference equations.

$$\begin{pmatrix} x_{t+1} \\ y_{t+1} \\ z_{t+1} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 1 & -2 & 2 \\ 2 & -2 & 3 \end{pmatrix} \begin{pmatrix} x_t \\ y_t \\ z_t \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}.$$

Note that the matrix of the system is the matrix A of Problem 2 above when $a = \frac{1}{2}$ and $b = 1$.

(a) (5 points) Find the stationary or equilibrium point of the system. Study the stability of the equilibrium point (stable, locally stable, unstable, saddle point), finding the stable manifold in the case that the point is a saddle.

(b) (5 points) Determine the general solution of the system.

4

Consider the ODE

$$2tx' + x = t^2, \quad t \neq 0.$$

(a) (5 points) Find the general solution.

(b) (5 points) Find the solution of the initial value problem

$$2tx' + x = t^2, \quad x(25) = 126.$$

What is the interval of definition?

5

(a) (5 points) Find the general solution of the ODE

$$x'' - x' - 6x = 0$$

(b) (5 points) Find the general solution of the ODE

$$x'' - x' - 6x = 30t + 5$$

(c) (5 points) Find the solution of the following initial value problem

$$x'' - x' - 6x = 30t + 5, \quad x(0) = 4 \quad x'(0) = 2$$

(d) (5 points) Find the solution $x(t)$ of the following initial value problem

$$x'' - x' - 6x = 0, \quad x(0) = 1 \quad \lim_{t \to \infty} x(t) = 0$$

6

Consider the following system of ODE's

$$\begin{cases} x' &= -15 - 3x + 5y + xy \\ y' &= -2x + xy \end{cases}$$

(a) (5 points) Determine the stationary points.

(b) (5 points) Determine the stability of the stationary points.
