University Carlos III of Madrid **Departament of Economics**
Game Theory: Problem set 5: Bayesian Games.

Problem 1: Consider a Cornout duopoly which operates in a market with the following inverse demand function

$$P(Q) = \begin{cases} 90 - Q & \text{if } Q \leq 90, \\ 0 & \text{if } Q > 90. \end{cases}$$

where $Q = q_1 + q_2$ is the total output in the market. The cost of firm 2 is $c_2(q_2) = 9q_2$ with probability 1/3 and $c_2(q_2) = 27q_2$ with probability 2/3. The cost of firm 1 is $c_1(q_1) = 18q_1$. Firm 2 knows its own cost, but firm 1 only knows the types of costs of firm 2 and its probabilities. The above description is common knowledge.

1. Represent the above situation as a Bayesian Game. That is, describe the set set of players, their types, the set of strategies, their beliefs and their utilities.
2. Compute the Bayesian Equilibrium and the benefits of the firms in this equilibrium.
3. Suppose now that firm 1 knows that the costs of firm 2 is $c_2(q_2) = 9q_2$. Compute the Nash equilibrium and the benefits of the firms in this equilibrium.
4. Suppose now that firm 1 knows that the costs of firm 2 is $c_2(q_2) = 27q_2$. Compute the Nash equilibrium and the benefits of the firms in this equilibrium.
5. In view of the above computation there is one type of firm 2 which prefers complete information and one type that prefers the situation with incomplete information. Identify those types.

Problem 2: Consider a Cornout duopoly which operates in a market with the following inverse demand function

$$P(Q) = \begin{cases} 60 - Q & \text{if } Q \leq 60, \\ 0 & \text{if } Q > 60. \end{cases}$$

where $Q = q_1 + q_2$ is the total output in the market. The cost of firm 2 is $c_2(q_2) = 12q_2$ with probability 1/4 and $c_2(q_2) = 24q_2$ with probability 3/4. The cost of firm 1 is $c_1(q_1) = 18q_1$. Firm 2 knows its own cost, but firm 1 only knows the types of costs of firm 2 and its probabilities. The above description is common knowledge.

1. Represent the above situation as a Bayesian Game. That is, describe the set set of players, their types, the set of strategies, their beliefs and their utilities.
2. Compute the Bayesian Equilibrium and the benefits of the firms in this equilibrium.
3. Suppose now that firm 1 knows that the costs of firm 2 is $c_2(q_2) = 12q_2$. Compute the Nash equilibrium and the benefits of the firms in this equilibrium.
4. Suppose now that firm 1 knows that the costs of firm 2 is $c_2(q_2) = 24q_2$. Compute the Nash equilibrium and the benefits of the firms in this equilibrium.
5. In view of the above computation there is one type of firm 2 which prefers complete information and one type that prefers the situation with incomplete information. Identify those types.

Problem 3: Consider a Cournot duopoly which operates in a market with inverse demand function $P(q) = a - q$, where $q = q_1 + q_2$ is total output in the market. Firm 2 knows if the value of total demand a is high ($a = a_h = 27$) or low ($a = a_l = 9$). The value of the parameter a is uncertain for firm 1. Firm 1 believes that with probability 2/3 it could be high ($a = a_h = 27$) and with probability 1/3 it could be low ($a = a_l = 9$). All the above is common knowledge and both firms choose simultaneously their production plans. Both firms have zero cost.

1. Represent this situation as a bayesian game.
2. Compute the bayesian equilibrium and the profits in equilibrium.
3. Compare the above result with the one in which firm 1 knows the value of a.

Problem 4: (The battle of the sexes) A couple is deciding wether to go to the soccer match or to the ballet. Each of the partners has to make the decision simultaneously and independently (they cannot communicate). She (player 1) likes soccer very much, but would prefer to go with her partner. He (player 2) enjoys ballet way more than soccer. Sometimes he prefers to go with his partner, but some other times he prefers to go alone (you can imagine the reasons). He knows his mood tonight. But, she does not. She thinks that the probability that he would enjoy her company tonight is 1/2. The situation is summarized in the following tables.

He prefers her company

She \ He	S	B
S	4,2	0,0
B	0, 0	2,4

He prefers to be alone

She \ He	S	B
S	4,0	0,4
B	0, 2	2,0

(a) Describe the situation as a Bayesian game.

(b) Find the Bayesian equililibria.

Problem 5: Consider the situation in which player 2 knows what game is played (A or B below). But player 1 only knows that A is played with probability p and B is played with probability $1 - p$.

A

Player 1 \ Player 2	S	B
S	2,2	1,0
B	1, 5	0,2

B

Player 1 \ Player 2	S	B
S	2,2	0,5
B	0, 0	4,2

(a) Describe the situation as a Bayesian game.

(b) Find the Bayesian equililibria.

Problem 6: Consider the situation in which player 1 knows what game is played (A or B below). But player 2 only knows that A is played with probability 1/3 and B is played with probability 2/3.

A

Player 1 \ Player 2	S	B
S	1,1	0,0
B	0, 0	0,0

B

Player 1 \ Player 2	S	B
S	0,0	0,0
B	0, 0	2,2

(a) Describe the situation as a Bayesian game.

(b) Find the Bayesian equililibria.

Problem 7: Two individuals consider donating towards a public good. If any of the agents contributes to the public good, then it is implemented. If agent agent $i = 1, 2$ contributes to the public good his utility is $u_i = 2 - c_i$. If agent $i = 1, 2$ does not contribute to the public good, but agent $j \neq i$ contributes to the public, the utility of agent i is $u_i = 2$. It is known that $c_1 = 1$. Only agent 2 knows c_2. Agent 1 knows that $c_2 = 1$ with probability p and $c_2 = 3$ with probability $1 - p$. That is player 2 knows what game is played (A or B below). But player 1 only knows that A is played with probability p and B is played with probability $1 - p$.

Game A:

		Player 2: C	Player 2: N
Player 1	C	1,1	1,2
	N	2, 1	0,0

Game B:

		Player 2: C	Player 2: N
Player 1	C	1,-1	1,2
	N	$2, -1$	0,0

(a) Describe the situation as a Bayesian game.

(b) Find the Bayesian equililibria.

Problem 8: Two individuals consider donating towards a public good. If any of the agents contributes to the public good, then it is implemented. If agent agent $i = 1, 2$ contributes to the public good his utility is $u_i = 2 - c_i$. If agent $i = 1, 2$ does not contribute to the public good, but agent $j \neq i$ contributes to the public, the utility of agent i is $u_i = 2$. That is

		Player 2: C	Player 2: N
Player 1	C	$2 - c_1,\ 2 - c_2$	$2 - c_1$,2
	N	$2,\ 2 - c_2$	0,0

where c_1 and c_2 are distributed uniformly on the interval $[1, 3]$.

(a) Describe the situation as a Bayesian game.

(b) Show that there is a Bayesian equilibrium of the form $(s_1(c_1), s_2(c_2))$ with

$$s_i(c_i) = \begin{cases} C & \text{if } c_i \leq a \\ N & \text{if } c_i > a \end{cases}, \qquad i = 1, 2$$

for some $a \in [1, 3]$:

Problem 9:

Two risk averse individuals with utility functions $u(x) = \sqrt{x}$, where x represents money, face a first price auction. Agent $i = 1, 2$ ($i = 1, 2$) values the good in v_i monetary units. This valuation is private information, but it is known that the v_i's are random variables independently and uniformly distributed in the interval $[0, 1]$.

(a) Describe the situation as a Bayesian game.

(b) Find a bayesian Nash equilibrium of the form $b_i(v_i) = \alpha_i v_i$. What is the utility of each individual in equilibrium?