UNIVERSITY CARLOS III

Master in Economics **Master in Industrial Economics and Markets**

Game Theory

Quiz 1–December 3rd, 2024

NAME:

Consider a market with two firms and a homogeneous product. The inverse demand function is $p = 40 - 4q$*, where* $q = q_1 + q_2$*,* q_1 *is the output of firm 1 and* q_2 *is the output of firm 2. The firms have constant marginal cost* $c = 8$*. Firm 1 (the leader) chooses its output level* q_1 *first. Then, firm 2 (the follower) chooses its output level* q_2*, after observing* q_1*.*

(a) *(80 points) Compute the subgame perfect Nash equilibrium of this game. Write the strategies of the players at each node. Write the equilibrium path and payoffs of the players.*

Solution: *Firm 2 maximizes its profit function. The first order condition is*

$$32 - 4q_1 - 8q_2 = 0$$

Firm 2's best response to a price set by firm 1 is

$$\mathrm{BR}_2(q_1) = \max\left\{0, \frac{8 - q_1}{2}\right\}$$

Now, in taking its decision, Firm 1 anticipates the reaction of Firm 2. Firm 1 maximizes

$$\pi_1\left(q_1, \frac{8 - q_1}{2}\right) = -4q_1\left(\frac{8 - q_1}{2} + q_1 - 8\right) = 16q_1 - 2q_1^2$$

The first order condition is

$$16 - 4q_1 = 0$$

and we obtain that the SPN $q_1^S = 4$*,* $q_2^S = \max\left\{0, \frac{8-q_1}{2}\right\}$*.*
The equilibrium path is $q_1 = 4$*,* $q_2 = 2$*. The profits are* $\pi_1^S(4,2) = 32$ *and* $\pi_2^S(4,2) = 16$*.*

(b) *(10 points) Suppose now that firm 2 can purchase firm 1 and behaves like a monopolist. That is firm 2 maximizes* $pq - cq$*. What is the maximum amount firm 2 would pay for the purchase of firm 1? What is the minimum amount firm 1 would accept for the purchase? Would the deal take place?*

Solution: *Firm 2 maximizes*

$$\Pi(q)(p - c)q = 32q - 4q^2$$

The FOC is

$$32 = 8q$$

so, $q^m = 4$ *and the profit is* $\Pi(4) = 64$*. The maximum amount firm 2 is willing to pay for the purchase of firm 1 is* $64 - 16 = 48$*. The minimum amount firm 1 is willing accept for the purchase is* 32*. Hence, the deal is possible.*

(c) *(10 points) Is* $q_1 = q_2 = \frac{8}{3}$ *a NE equilibria of the game? Is it a SPNE of the game? Why?*

Solution: $q_1 = q_2 = \frac{8}{3}$ *is a NE, because*

$$\mathrm{BR}_2(q_1) = \max\left\{0, \frac{8 - q_1}{2}\right\}, \quad \mathrm{BR}_1(q_2) = \max\left\{0, \frac{8 - q_2}{2}\right\}$$

So, $\mathrm{BR}_i(\frac{8}{3}) = \frac{8}{3}$*,* $i = 1, 2$*. It is not A SPNE*
