UNIVERSITY CARLOS III

Master in Economics **Master in Industrial Economics and Markets**

Game Theory

Quiz 1–October 2nd, 2024

NAME:

Consider the following normal form game:

	X	Y	Z	T
A	*2,0*	*12,4*	*4,12*	*15,3*
B	*1,6*	*10,7*	*3,8*	*6,5*
C	*4,10*	*10,5*	*2,0*	*10 , 1*

(a) *(10 points) What are the strategies that survive the iterated elimination of strictly dominated strategies?*

Solution: Strategy T is dominated by strategy Y for player 2. After eliminating this strategy we obtain the following game

	X	Y	Z
A	2,0	12,4	4,12
B	1,6	10,7	3,8
C	4,10	10,5	2,0

Now strategy B is dominated by strategy A for player 1. After eliminating this strategy we obtain the following game

	X	Y	Z
A	2,0	12,4	4,12
C	4,10	10,5	2,0

Note also that the mixed strategy $\sigma_2 = \frac{3}{5}X + \frac{2}{5}Z$ dominates Y, because

$$\begin{aligned} u_2(A, \sigma_2) &= \frac{3}{5} \times 0 + \frac{2}{5} \times 12 = \frac{24}{5} > 4 = u_2(A, Y) \\ u_2(C, \sigma_2) &= \frac{3}{5} \times 10 + \frac{2}{5} \times 0 = 6 > 5 = u_2(C, Y) \end{aligned}$$

(See also the argument in part (c) below).

The rationalizable strategies are $\{A, C\} \times \{X, Z\}$.

(b) *(10 points) Find all pure strategy Nash equilibria and the payoffs of these equilibria.*

(c) *(80 points) Compute the mixed strategy Nash equilibria and the expected payoffs of these equilibria.*

Solution:

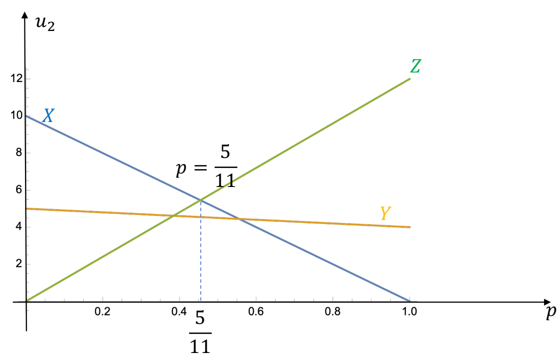
Let us look for a NE of the form

$$\begin{aligned} \sigma_1 &= pA + (1-p)C \\ \sigma_2 &= xX + yY + (1-x-y)Z \end{aligned}$$

We compute the expected utilities of the players

$$\begin{aligned}
u_1(A, \sigma_2) &= 4 - 2x + 8y \\
u_1(C, \sigma_2) &= 2 + 2x + 8y \\
u_2(\sigma_1, X) &= 10 - 10p \\
u_2(\sigma_1, Y) &= 5 - p \\
u_2(\sigma_1, Z) &= 12p
\end{aligned}$$

We graph the utilities of player 2.



We see that Y is not part of any best reply for player 2. Hence, we may assume $y = 0$. And

$$\begin{aligned}
\sigma_1 &= pA + (1-p)C \\
\sigma_2 &= xX + (1-x)Z
\end{aligned}$$

Also, from the picture we see that best reply of player 2 is

$$\mathrm{BR}_2(\sigma_1) = \begin{cases} X \quad (x=1) & \text{if} \quad 0 \le p = 5/11 \\ \{X, Z\} \quad (0 \le x \le 1) & \text{if} \quad p = 5/11 \\ \mathrm{Z} \quad (x=0) & \text{if} \quad 5/11 < p \le 1 \end{cases}$$

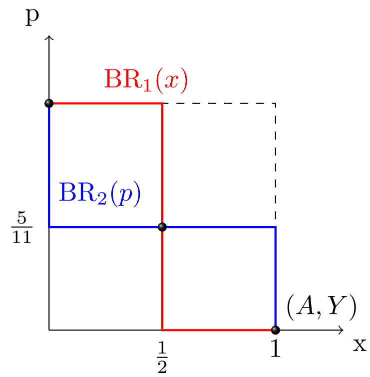
Since,

$$\begin{aligned}
u_1(A, \sigma_2) &= 4 - 2x \\
u_1(C, \sigma_2) &= 2 + 2x
\end{aligned}$$

we have that $u_1(A, \sigma_2) \ge u_1(Z, \sigma_2)$ iff $x \le 1/2$. Thus, best reply of player 1 is

$$\mathrm{BR}_1(\sigma_2) = \begin{cases} A \quad (p=1) & \text{if} \quad 0 \le x < \frac{1}{2} \\ \{A, C\} \quad (p \in [0,1]) & \text{if} \quad x = \frac{1}{2} \\ C \quad (y=0) & \text{if} \quad 1 \ge x > \frac{1}{2} \end{cases}$$

Graphically,



We obtain the NE

$$\begin{aligned} (x = 0, p = 1), (A, Z), & \quad \text{with payoffs } u_1 = 4, u_2 = 12. \\ (x = 1, p = 0), (C, X), & \quad \text{with payoffs } u_1 = 4, u_2 = 10. \\ \left(\frac{5}{11}A + \frac{6}{11}C, \frac{1}{2}X + \frac{1}{2}Z\right), & \quad \text{with payoffs } u_1 = 3, u_2 = \frac{60}{11}. \end{aligned}$$

(d) (10 points) Consider the game

	X	Y	Z	T
A	*2,0*	*12,4*	*4,12*	*15,3*
C	*4,10*	*10,5*	*2,0*	*10 , 1*

Find a mixed strategy for player 2, $\sigma_2 = \alpha X + (1-\alpha)Z$*, that strictly dominates* Y

Solution: *We must have that*

$$\begin{aligned} u_2(A, \sigma_2) &= 12 - 12\alpha > 4 = u_2(A, Y) \\ u_2(C, \sigma_2) &= 10\alpha > 5 = u_2(C, Y) \end{aligned}$$

That is,

$$\frac{1}{2} < \alpha < \frac{2}{3}$$

The mixed strategy $\sigma_2 = \frac{3}{5}X + \frac{2}{5}Z$ *dominates* Y*, because*

$$\begin{aligned} u_2(A, \sigma_2) &= \frac{3}{5} \times 0 + \frac{2}{5} \times 12 = \frac{24}{5} > 4 = u_2(A, Y) \\ u_2(C, \sigma_2) &= \frac{3}{5} \times 10 + \frac{2}{5} \times 0 = 6 > 5 = u_2(C, Y) \end{aligned}$$