UNIVERSITY CARLOS III Master in Economics Master in Industrial Economics and Markets

Game Theory. Quiz 2–November 28th, 2023. 45 minutes.

NAME:

Consider the situation in which player 2 knows which game is played (a or b below). However, player 1 only knows that table a is played with probability $\frac{1}{3}$ *and table b is played with probability* $\frac{2}{3}$.

Player 2

		C	*D*
Player 1	*A*	*24 ,36*	*12, 6*
	B	18, 6	*30, 12*

a

Player 2

		C	*D*
Player 1	*A*	*12 ,3*	*6, 9*
	B	15, 6	*3, 3*

b

(a) Describe the situation as a Bayesian game.

Solution: *There are two players* $N = \{1, 2\}$. *There are two types of player 2:* $T_2 = \{a, b\}$. *There is one type of player 1:* $T_1 = \{t\}$. *The sets of strategies are* $S_2 = \{CC, CD, DC, DD\}$, $S_1 = \{A, B\}$. *The beliefs of the players are*

$$\begin{aligned} p_2(t_1 = t | t_2 = a) &= p_2(t_1 = t | t_2 = b) = 1 \\ p_1(t_2 = a | t_1 = t) &= 1/3 \\ p_1(t_2 = b | t_1 = t) &= 2/3 \end{aligned}$$

The payoffs are given by the above tables.

(b) Find the Bayesian–Nash equilibria in pure strategies and the payoffs of the players. Compute the payoffs of the players in each BNE.

Solution: *Note that*

$$\mathrm{BR}_2(A) = CD, \quad \mathrm{BR}_2(B) = DC$$

We compute now

$$u_1(A, CD) = \frac{1}{3} \times 24 + \frac{2}{3} \times 6 = 12, \quad u_1(B, CD) = \frac{1}{3} \times 18 + \frac{2}{3} \times 3 = 8$$

Hence, $\mathrm{BR}_1(CD) = A$ *and* $\mathrm{BR}_2(A) = CD$. *We see that.* (A, CD) *is a BNE with payoffs* $u_1 = 12$, $u_a = 24$, $u_b = 6$.

We compute now

$$u_1(A, DC) = \frac{1}{3} \times 12 + \frac{2}{3} \times 12 = 12, \quad u_1(B, DC) = \frac{1}{3} \times 30 + \frac{2}{3} \times 15 = 20$$

Hence, $\mathrm{BR}_1(DC) = B$ *and* $\mathrm{BR}_2(B) = DC$. *We see that.* (B, DC) *is a BNE with payoffs* $u_1 = 20$, $u_a = 12$, $u_b = 6$.

Another way to find the BNE is to compute the associated normal form game

	CC	*CD*	*DC*	*DD*
A	(16, 14)	(12, 18)	(12, 4)	(8, 8)
B	(16, 6)	(8, 4)	(20, 8)	(12, 6)

(c) Compute the mixed strategy Nash equilibria. Compute the expected payoffs of the players in each BNE.

Solution: *Let us look for a BNE of the form*

$$(xA + (1 - x)B, (yC + (1 - y)D, zC + (1 - z)D))$$

Let

$$\begin{aligned}\sigma_1 &= xA + (1-x)B \\ \sigma_a &= yC + (1-y)D \\ \sigma_b &= zC + (1-z)D\end{aligned}$$

We have that

$$\begin{aligned}u_1(A;\sigma_a,\sigma_b) &= 4(2+y+z) \\ u_1(B;\sigma_a,\sigma_b) &= 12-4y+8z \\ u_a(\sigma_1,C) &= 6+30x \\ u_a(\sigma_1,D) &= 12-6x \\ u_b(\sigma_1,C) &= 6-3x \\ u_b(\sigma_1,D) &= 3+6x\end{aligned}$$

Suppose first that player 2a is using a completely mixed strategy. Then $u_a(\sigma_1,C) = u_a(\sigma_1,D)$. *Hence,* $6+30x = 12-6x$ *and we conclude that* $x = \frac{1}{6}$. *For this value of* x *we have that* $u_b(\sigma_1,C) = (6-3x)|_{x=\frac{1}{6}} = \frac{11}{4}$ *and* $u_b(\sigma_1,D) = (3+6x)|_{x=\frac{1}{6}} = 4$, *so* $z=1$. *We check if there is a BNE of the form*

$$\left(\frac{1}{6}A + \frac{5}{6}B; (yC + (1-y)D, C)\right)$$

Player 1 must be indifferent between A and B. Hence, $4(2+y+z) = 12-4y+8z$. *Since* $z=1$, *we obtain that* $y=1$. **And we have checked that**

$$\left(\frac{1}{6}A + \frac{5}{6}B; C, C\right)$$

is BNE in mixed strategies with payoffs $u_1 = 16$**,** $u_a = 11$**,** $u_b = 4$**.**

Suppose now that player 2b is using a completely mixed strategy. Then $u_b(\sigma_1,C) = u_b(\sigma_1,D)$. *Hence,* $6-3x = 3+6x$ *and we conclude that* $x = \frac{1}{3}$. *For this value of* x *we have that* $u_a(\sigma_1,C) = (6+30x)|_{x=\frac{1}{3}} = 16$ *and* $u_a(\sigma_1,D) = (12-6x)|_{x=\frac{1}{3}} = 10$, *so* $y=1$. *We check if there is a BNE of the form*

$$\left(\frac{2}{3}A + \frac{1}{3}B; (C, zC + (1-z)D)\right)$$

Player 1 must be indifferent between A and B. Hence, $4(2+y+z) = 12-4y+8z$. *Since* $y=1$, *we obtain that* $z=1$. **And we have checked that**

$$\left(\frac{1}{3}A + \frac{2}{3}B; (C, C)\right)$$

is the other BNE in mixed strategies with payoffs $u_1 = 16$**,** $u_a = 10$**,** $u_b = 5$**.**

Note that we have that $u_1(A;CC) = u_1(B;CC) = 16$. *And for* $\frac{1}{6} < x < \frac{1}{3}$ *we have* $\mathrm{BR}_2(\sigma_1) = CC$. *Hence,* ***we have the following BNE in mixed strategies.***

$$(xA + (1-x)B; (C, C)), \quad \frac{1}{6} \le x \le \frac{1}{3}$$

with payoffs $u_1 = 16$***,*** $u_a = 6+30x$***,*** $u_b = 6-3x$***.***