**UNIVERSITY CARLOS III**

**Master in Economics** **Master in Industrial Economics and Markets**

**Game Theory**

**Quiz 1–October 3rd, 2023**

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NAME:

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*Consider the following normal form game:*

| | $X$ | $Y$ | $Z$ | $U$ |
|---|---|---|---|---|
| $A$ | *1,0* | *2,6* | *0,1* | *20,5* |
| $B$ | *2,12* | *1,3* | *3,8* | *15,2* |
| $C$ | *1,1* | *0,3* | *2,3* | *10,6* |

(a) *What are the strategies that survive the iterated elimination of strictly dominated strategies?*

**Solution:** Strategy $C$ is dominated by strategy $B$ for player 1. After eliminating this strategy we obtain the following game

| | $X$ | $Y$ | $Z$ | $U$ |
|---|---|---|---|---|
| $A$ | 1,0 | 2,6 | 0,1 | 20,5 |
| $B$ | 2,12 | 1,3 | 3,8 | 15,2 |

Now strategy $U$ is dominated by strategy $Y$ for player 2. After eliminating this strategy we obtain the following game

| | $X$ | $Y$ | $Z$ |
|---|---|---|---|
| $A$ | 1,0 | 2,6 | 0,1 |
| $B$ | 2,12 | 1,3 | 3,8 |

Note also that the mixed strategy $\sigma_2 = \frac{2}{3}X + \frac{1}{3}Y$ dominates $Z$, because

$$\begin{aligned} u_2(A, \sigma_2) &= \frac{2}{3} \times 0 + \frac{1}{3} \times 6 = 2 > 1 = u_2(A, Z) \\ u_2(C, \sigma_2) &= \frac{2}{3} \times 12 + \frac{1}{3} \times 3 = 9 > 8 = u_2(Z, Y) \end{aligned}$$

(See also the argument in part (c) below).

The rationalizable strategies are $\{A, B\} \times \{X, Y\}$.

(b) *Find all pure strategy Nash equilibria and the payoffs of these equilibria.*

(c) *Compute the mixed strategy Nash equilibria and the expected payoffs of these equilibria.*

**Solution:**

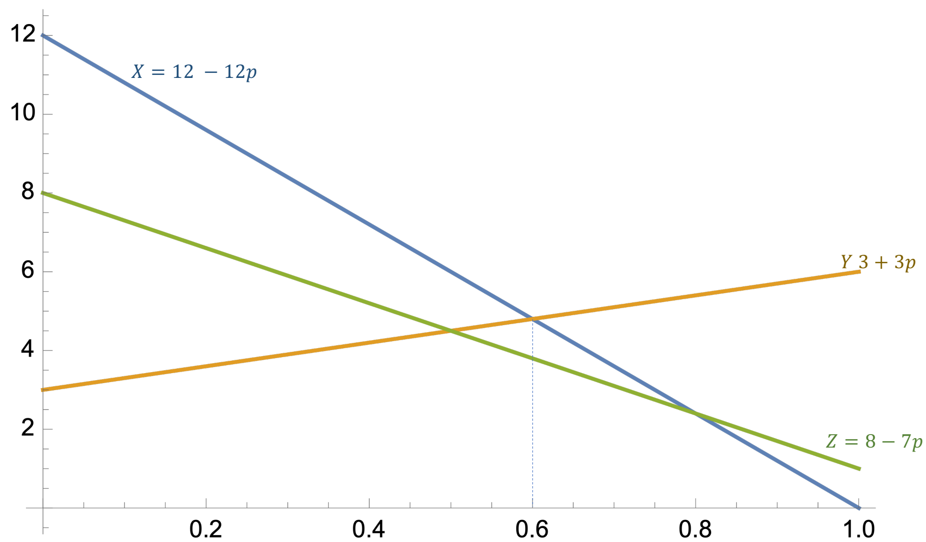
Let us look for a NE of the form

$$\begin{aligned} \sigma_1 &= pA + (1-p)B \\ \sigma_2 &= xX + yY + (1-x-y)Z \end{aligned}$$

We compute the expected utilities of the players

$$\begin{aligned} u_1(A, \sigma_2) &= x + 2y \\ u_1(B, \sigma_2) &= 3 - x - 2y \\ u_2(\sigma_1, X) &= 12 - 12p \\ u_2(\sigma_1, Y) &= 3 + 3p \\ u_2(\sigma_1, Z) &= 8 - 7p \end{aligned}$$

We graph the utilities of player 2.




We see that $Z$ is never best reply for player 2. Hence, we may assume $x+y=1$, $y=1-x$. And

$$\begin{aligned}\sigma_1 &= pA+(1-p)B \\ \sigma_2 &= xX+(1-x)Y\end{aligned}$$

Also, from the picture we see that best reply of player 2 is

$$\mathrm{BR}_2(\sigma_1)=\begin{cases} X \quad (x=1) & \text{if} \quad 0\leq p=\frac{3}{5} \\ \{X,Y\} \quad (0\leq x\leq 1) & \text{if} \quad 0\leq p=\frac{3}{5} \\ \text{Y} \quad (x=0) & \text{if} \quad \frac{3}{5}<p\leq 1 \end{cases}$$

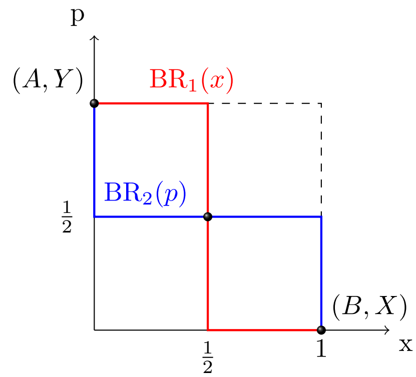
Since,

$$\begin{aligned}u_1\left(A,\sigma_2\right) &= 2-x \\ u_1\left(B,\sigma_2\right) &= 1+x\end{aligned}$$

we have that $u_1\left(A,\sigma_2\right)\geq u_1\left(B,\sigma_2\right)$ iff $x\leq 1/2$. Thus, best reply of player 1 is

$$\mathrm{BR}_1(\sigma_2)=\begin{cases} A \quad (p=1) & \text{if} \quad 0\leq x<\frac{1}{2} \\ \{A,B\} \quad (p\in[0,1]) & \text{if} \quad x=\frac{1}{2} \\ B \quad (y=0) & \text{if} \quad 1\geq x>\frac{1}{2} \end{cases}$$

Graphically,




We obtain the NE

$$\begin{aligned} (x=0, p=1), (A, Y), &\qquad \text{with payoffs } u_1 = 2, u_2 = 6. \\ (x=1, p=0), (B, X), &\qquad \text{with payoffs } u_1 = 2, u_2 = 12. \\ \left(\frac{3}{5}A + \frac{1}{2}C, \frac{2}{5}X + \frac{1}{2}Z\right), &\qquad \text{with payoffs } u_1 = \frac{3}{2}, u_2 = \frac{24}{5}. \end{aligned}$$

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