UNIVERSITY CARLOS III

Master in Economics **Master in Industrial Economics and Markets**

Game Theory

TEST 1–October 15th, 2021

NAME:

Consider the following normal form game:

	X	Y	Z
A	*1,0*	*2,1*	*3,-1*
B	*0,3*	*2, -1*	*5,2*
C	*0, 10*	*1,5*	*10,6*

(a) *What are the strategies that survive the iterated elimination of strictly dominated strategies?*

Solution: Strategy Z is dominated by strategy X for player 2. After eliminating this strategy we obtain the following game

	X	Y
A	1,0	2,1
B	0,3	2, -1
C	0, 10	1,5

Now strategy C is dominated by strategy A for player 1. After eliminating this strategy we obtain the following game

	X	Y
A	1,0	2,1
B	0,3	2, -1

The rationalizable strategies are $\{A, B\} \times \{X, Y\}$.

(b) *Find all pure strategy Nash equilibria and the payoffs of these equilibria.*

(c) *Compute the mixed strategy Nash equilibria and the expected payoffs of these equilibria.*

Solution: The best responses of the players are

	X	Y
A	$\underline{1}$,0	$\underline{2}$,$\underline{1}$
B	0,$\underline{3}$	$\underline{2}$, -1

Hence, the NE is (A, Y). Let us look for a NE of the form

$$\begin{aligned} \sigma_1 &= xA + (1-x)B \\ \sigma_2 &= yX + (1-y)Y \end{aligned}$$

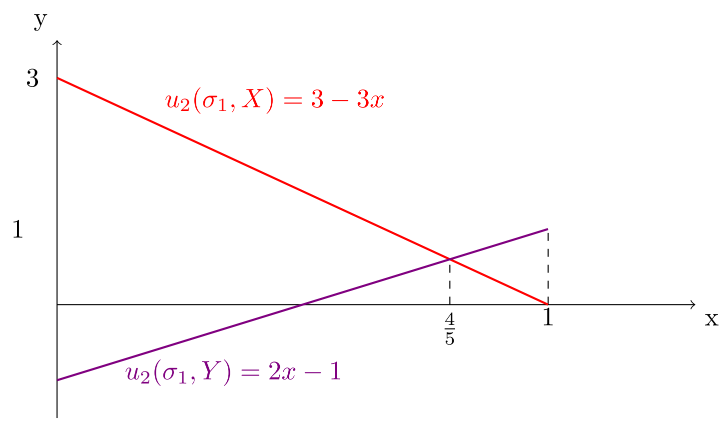
We compute the expected utilities of the players

$$\begin{aligned} u_1(A, \sigma_2) &= y + 2(1-y) = 2 - y \\ u_1(B, \sigma_2) &= 2(1-y) = 2 - 2y \\ u_2(\sigma_1, X) &= 3(1-x) = 3 - 3x \\ u_2(\sigma_1, Y) &= x - (1-x) = 2x - 1 \end{aligned}$$

Since, $2 - y \geq 2 - 2y$ for every $0 \leq y \leq 1$ and the inequality is strict except for $y = 0$, we have that best reply of player 1 is

$$\text{BR}_1(\sigma_2) = \begin{cases} [0,1] & \text{if} \quad y = 0 \\ x = 1 & \text{if} \quad 0 < y \leq 1 \end{cases}$$

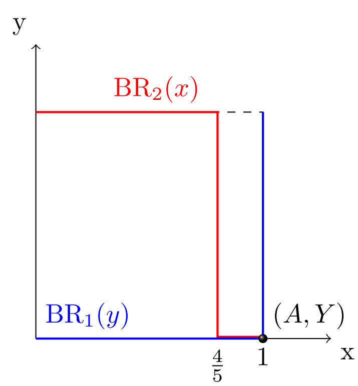
Graphically,



Thus, we have that best reply of player 2 is

$$\text{BR}_2(\sigma_1) = \begin{cases} y = 1 & \text{if} \quad x < \frac{4}{5} \\ y \in [0,1] & \text{if} \quad x = \frac{4}{5} \\ y = 0 & \text{if} \quad x > \frac{4}{5} \end{cases}$$

Graphically,



We obtain The NE

$$(xA + (1-x)B, Y) \qquad \frac{4}{5} \leq x \leq 1 \quad \text{with payoffs } u_1 = 2, u_2 = 2x - 1.$$