**UNIVERSITY CARLOS III**

**Master in Economics** **Master in Industrial Economics and Markets**

**Game Theory**

**TEST 1–October 25th, 2018**

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NAME:

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*Consider the following normal form game:*

| | $X$ | $Y$ | $Z$ |
|---|---|---|---|
| $A$ | 1,0 | 0,2 | 4, 5 |
| $B$ | 2, 5 | 2, -1 | 4,0 |
| $C$ | 0, 3 | -1,3 | 3,-2 |

(a) *What are the strategies that survive the iterated elimination of strictly dominated strategies?*

**Solution:** Strategy $C$ is dominated by strategy $A$ for player 1. After eliminating this strategy we obtain the following game

| | $X$ | $Y$ | $Z$ |
|---|---|---|---|
| $A$ | 1,0 | 0,2 | 4, 5 |
| $B$ | 2, 5 | 2, -1 | 4,0 |

Strategy $Y$ is dominated by strategy $Z$ for player 2. After eliminating this strategy we obtain the following game

| | $X$ | $Z$ |
|---|---|---|
| $A$ | 1,0 | 4, 5 |
| $B$ | 2, 5 | 4,0 |

The rationalizable strategies are $\{A, B\} \times \{X, Z\}$.

(b) *Find all pure strategy Nash equilibria and the payoffs of these equilibria.*

**Solution:** Let us look for a NE of the form

$$\begin{aligned} \sigma_1 &= xA + (1-x)B \\ \sigma_2 &= yX + (1-y)Z \end{aligned}$$

We compute the expected utilities of the players

$$\begin{aligned} u_1(A, \sigma_2) &= y + 4(1-y) = 4 - 3y \\ u_1(B, \sigma_2) &= 2y + 4(1-y) = 4 - 2y \\ u_2(\sigma_1, X) &= 5(1-x) \\ u_2(\sigma_1, Z) &= 5x \end{aligned}$$

Note that

(a) $4 - 3y < 4 - 2y$ for $0 < y \leq 1$.

(b) $4 - 3y = 4 - 2y$ for $y = 0$.

Thus, we have that best reply of player 1 is

$$\mathrm{BR}_1(\sigma_2) = \begin{cases} [0,1] & \text{if} \quad y = 0 \\ x = 0 & \text{if} \quad 0 < y \leq 1 \end{cases}$$

Note that

(a) $5(1-x) < 5x$ for $x > \frac{1}{2}$.

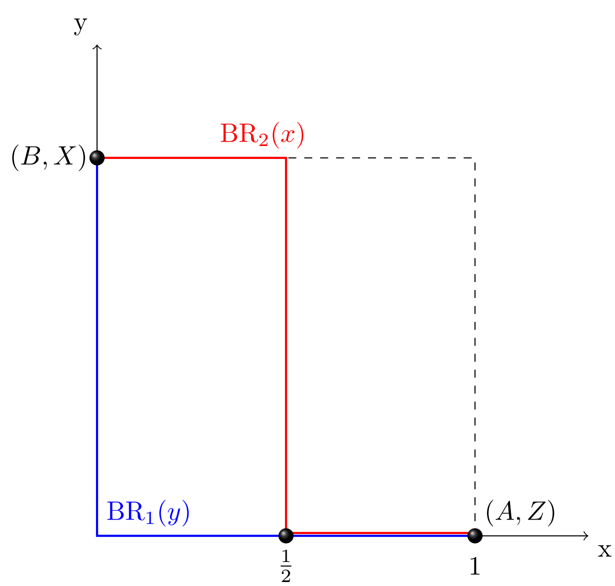
(b) $5(1-x) > 5x$ for $x < \frac{1}{2}$.

(c) $5(1-x) = 5x$ for $x = \frac{1}{2}$.

Thus, we have that best reply of player 2 is

$$\mathrm{BR}_2(\sigma_1) = \begin{cases} y = 1 & \text{if} \quad x < \frac{1}{2} \\ [0,1] & \text{if} \quad x = \frac{1}{2} \\ y = 0 & \text{if} \quad x > \frac{1}{2} \end{cases}$$

Graphically,




We obtain two NE in pure strategies: $x = 0$, $y = 1$ and $x = 1$, $y = 0$. That is we obtain the NE

$$(B, X) \quad \text{with payoffs} \quad u_1 = 2, u_2 = 5$$

and

$$(B, X) \quad \text{with payoffs} \quad u_1 = 4, u_2 = 5$$

*(c) Compute the mixed strategy Nash equilibria and the expected payoffs of these equilibria.*

**Solution:** From the previous part we see that the NE in mixed strategies correspond to $\frac{1}{2} \leq x \leq 1$, $y = 0$. That is,

$$\begin{aligned} \sigma_1 &= xA + (1-x)B \quad \frac{1}{2} \leq x \leq 1 \\ \sigma_2 &= Z \end{aligned}$$

with payoffs $u_1 = 4$, $u_2 = 5x$.