# FINAL EXAM

Econometrics

Universidad Carlos III de Madrid

21/05/24

Write your name and group in each answer sheet. Answer the questions in 2:00 hours.

1. (30%) Consider the multiple regression model,

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 \ln(X_2) + U,$$

where $E\left(U \middle| X_1, X_2\right) = 0$, and the rest of conditions for consistency and asymptotic normality of OLS estimators hold. We have *iid* observations $\{Y_i, X_{1i}, X_{2i}\}_{i=1}^n$ of $(Y, X_1, X_2)$.

   a. (1/3) Show that the OLS estimator of $\beta_1$ is

$$\hat{\beta}_1 = \frac{\widehat{Cov}\left(Y, \hat{\varepsilon}_1\right)}{\widehat{Var}\left(\hat{\varepsilon}_1\right)},$$

   where $\hat{\varepsilon}_{1i} = X_{1i} - \hat{X}_{1i}$, and $\hat{X}_{1i}$ are the OLS fitted values of $X_1$ given $\ln(X_2)$.

   b. (1/3) Suppose that $\beta_1$ is estimated by $\hat{\alpha}_1 = \widehat{Cov}\left(Y, X_1\right) \big/ \widehat{Var}\left(X_1\right)$. Derive the asymptotic bias of $\hat{\alpha}_1$ as an estimator of $\beta_1$.

   c. (1/3) Provide an estimator of the expected elasticity of $Y$ with respect to $X_2$ evaluated at $X_1 = x_1$ and $X_2 = x_2$.

2. (30%) Consider the multiple regression model

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + U,$$

where $E\left(U \middle| X_1, X_2, X_3\right) = 0$, $E\left(U^2 \middle| X_1, X_2, X_3\right) = \sigma^2\left(X_1, X_2, X_3\right)$ and $\sigma^2(\cdot)$ is an unknown function, and all the remainder assumptions for the consistency and asymptotic normality of the *OLS* estimator hold. We have a random sample (*iid* observations) $\{Y_i, X_{1i}, X_{2i}, X_{3i}\}_{i=1}^n$ of $(Y, X_1, X_2, X_3)$.

a. (3/5) Explain how to test the following three hypotheses at 5% of significance after suitably reparameterizing the models in such a way that the hypotheses can be expressed as significance tests of the resulting transformed parameters (in each case, you must provide the test statistic, its approximated distribution when the sample size is large, and the rejection rule).

i. (1/3) $H_0 : \beta_1/\beta_2 = 1$ & $\beta_2 = -2$ vs $H_1 : \beta_1/\beta_2 \neq 1$ and/or $\beta_2 \neq -2$.

ii. (1/3) $H_0 : \beta_1 + \beta_2 + \beta_3 = 0$ vs $H_1 : \beta_1 + \beta_2 + \beta_3 < 0$.

iii. (1/3) $H_0 : \beta_1 + \beta_2 + \beta_3 = 1$ vs $H_1 : \beta_1 + \beta_2 + \beta_3 \neq 1$.

b. (2/5) Explain, assuming that $\sigma^2(X_{1i}, X_{2i}, X_{3i}) = E(U_i^2)$, whether or not you could test the above hypotheses comparing either the coefficients of determination ($R^2$) or the sum of squared residuals ($SSR$) in the restricted and unrestricted models. Provide the corresponding test statistics in each case.

3. (40%) In order to estimate the relationship between aggregate wages, $W$, aggregate profits, $P$, and aggregate income, $Y$, consider the following model,

$$\begin{aligned} W &= \beta_0 + \beta_1 Y + u, \qquad (1) \\ P &= \alpha_0 + \alpha_1 Y + \alpha_2 K + v, \\ Y &= W + P, \end{aligned}$$

where $K$ is aggregate stock of capital and $u$, $v$ are disturbance terms, assumed independent of each other. The third equation is an identity, all forms of income are classified as wages and profits. The coefficients are intended to be estimated using data from a sample of industrialized countries, with the variables measured on a per capita basis in a common currency. $K$ may be assumed to be exogenous.

a. (2/5) Show that $Y$ is endogenous in the structural equation (1).

b. (1/5) Explain how to obtain the two stages least squares (2SLS) estimator of $\beta_1$.

c. (2/5) Show that $\beta_1$ is the ratio between the slopes in the reduced form equations of the endogenous variables in the structural equation (1), and provide the resulting estimator based on the OLS estimators of the slope parameters in the reduced forms (indirect least squares estimator). Show that this estimator is identical to the estimator in (b).