ECONOMETRICS RETAKE EXAM

PART B

Universidad Carlos III de Madrid

June 17, 2024

Write your name and group in each answer sheet. Answer the questions in 90'.

1. (40%) The scrap rate for a manufacturing firm is the number of defective items-products (*scrap*) that must be discarded, out of 100 produced. We are interested in using the scrap rate to measure the effect of worker training on productivity. A sample of firms is used to obtain the following OLS model estimate

$$\begin{aligned} \widehat{\ln(scrap_i)} &= \underset{(4.57)}{11.74} - \underset{(0.019)}{0.042} \cdot hrsemp_i - \underset{(0.370)}{0.951} \cdot \ln(sales_i) + \underset{(0.360)}{0.992} \cdot \ln(employ_i), \\ n &= 43,\ RSS = 65.91, \end{aligned}$$

where "ln" means Napierian logarithm, *hrsemp* is the annual hours of training per employee, *sales* is the annual firm sales (in dollars) and *employ* is the number of firm employees. It is reported that $\widehat{Cov}\left(\hat{\beta}_{\ln(sales)}, \hat{\beta}_{\ln(employ)}\right) = -0.11$.

a. (1/5) (i) State the assumption you need on the error term of the model to interpret the coefficient of ln(*sales*) as the expected elasticity of *scrap* with respect to *sales* (50%). (ii) Which additional assumption do we need in order to correctly interpret the estimated coefficient of *hrsemp* as follows: "on average, *scrap* decreases in a 4.2% when *hrsemp* increases in one hour and the rest of explanatory variables remain fixed"? (50%).

b. (2/5) Consider the following slight reformulation of the estimated model

$$\begin{aligned} \widehat{\ln(scrap_i)} &= \underset{(4.57)}{11.74} - \underset{(0.019)}{0.042} \cdot hrsemp_i - \underset{(0.370)}{0.951} \cdot \ln\left(\frac{sales_i}{employ_i}\right) + \underset{(??)}{0.041} \cdot \ln(employ_i), \\ n &= 43,\ RSS = 65.91, \end{aligned}$$

Obtain the omitted standard error of the OLS coefficient of $\ln(employ)$.

c. (2/5) Derive the 95% confidence interval for the expected *scrap* rate elasticity of $sales - to - employee$ ratio (20%). Using this interval, test the hypothesis that a 5% increase in *sales*/*employ* is associated with an average 5% drop in the scrap rate (80%).

2. (30%) Consider the multiple regression model,

$$Y = \beta_0 + \beta_1 X + \beta_2 X^2 + U,$$

where $E(U|X) = 0$, and the rest of conditions for consistency and asymptotic normality of OLS estimators hold. We have *iid* observations $\{Y_i, X_i\}_{i=1}^n$ of (Y, X).

a. (1/3) Provide the value of X where the partial effect of X on Y changes sign (20%). Then, explain carefully how you can test, at the 5% of significance, that this value is larger than 1 (80%).

b. (1/3) Suppose that you incorrectly estimate the partial effect of X by the OLS slope estimator in the simple regression model, i.e.

$$\hat{\gamma}_1 = \frac{\widehat{Cov}(Y, X)}{\widehat{Var}(X)}.$$

Derive a formula for the asymptotic bias of $\hat{\gamma}_1$ as an estimate of β_1 (30%). Can the asymptotic bias be positive when $\beta_2 < 0$ and $X > 0$? (70%).

c. (1/3) Which assumption for the consistency of the OLS estimate of β_1 is not satisfied when X takes only the values 0 and 1? Justify carefully your answer.