ECONOMETRICS RETAKE EXAM: PART A (30%)

June 17, 2024

Universidad Carlos III de Madrid

Write your name and group in each answer sheet. Answer the questions in 50'.

Use the **mroz** dataset from Wooldridge to estimate the labour supply of married women already working and the wage offer they receive. The labour supply is specified as

$$\log(hours) = \beta_0 + \beta_1 \log(wage) + \beta_2 educ + \beta_3 age + \beta_4 kidslt6 + \beta_5 nwifeinc + u, \qquad (1)$$

where $hours$ are annual worked hours, $wage$ is hourly wage in dollars, $educ$ are the years of education, age is the age of the woman in years, $kidslt6$ is the number of children less than 6 years old and $nwifeinc$ is the non wage income of the woman, including husband income.

We consider that the wage offer for the women satisfies the following equation

$$\log(wage) = \alpha_0 + \alpha_1 \log(hours) + \alpha_2 educ + \alpha_3 exper + \alpha_4 exper^2 + v, \qquad (2)$$

where $exper$ are the years of experience. The variables $educ$, age, $kidslt6$, $nwifeinc$, $exper$ and $exper^2$ are taken as exogenous.

1. (40%) Discuss which of the two equations are identified. (1/2) Check, if possible, the relevance and exogeneity of the proposed instruments for each equation explaining in detail the methods used. (1/2)

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   ANSWER:

   (1/2) This is a simultaneous equations model where $\log(hours)$ and $\log(wage)$ are endogenous in both equations.

   Equation (1), labour supply: the endogenous regressor $\log(wage)$, $k = 1$, requires at least one valid instrument which need to be omitted exogenous variables appearing in the other equation, so the candidates are $exper$ and $exper^2$ $(m = 2)$, leading to a (potentially) overidentified equation, $m - k = 1 > 0$, so we can check for instruments exogeneity.

   Equation (2), wage offer: the endogenous regressor $\log(hours)$, $k = 1$, requires some valid instruments which need to be omitted exogenous variables appearing in the other equation, so the candidates are $age, kidslt6$ and $nwifeinc$ $(m = 3)$, leading to a (potentially) overidentified equation, $m - k = 2 > 0$, so we can check for instruments exogeneity.

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   (1/2) To check the relevance and exogeneity of the instruments proposed we can use directly the output of the 2SLS estimation for each equation or estimation of the Reduced Form equations and auxiliary models on TSLS residuals.

Equation (1): In the log($wage$) Reduced Form,

$$\log(wage) = \pi_0 + \pi_1 exper + \pi_2 exper^2 + \pi_3 educ + \pi_4 age + \pi_5 kidslt6 + \pi_6 nwifeinc + e_1,$$

we compute the (robust) First Stage F statistic to test the hypotheses

$$\begin{aligned} H_0 &: \pi_1 = \pi_2 = 0 \\ H_1 &: \pi_1 \neq 0 \text{ and/or } \pi_2 = 0. \end{aligned}$$

Since $F = 6.17263 < 10$, the instruments are weak, despite the null hypothesis H_0 of instruments no relevance is rejected at the usual significance levels.

To test for instruments exogeneity we use the overidentification J-test. For that we can compute directly the (robust) F statistic of significance of the instruments,

$$\begin{aligned} H_0 &: \delta_1 = \delta_2 = 0 \\ H_1 &: \delta_1 \neq 0 \text{ and/or } \delta_2 = 0, \end{aligned}$$

in the auxiliary model

$$\hat{u} = \delta_0 + \delta_1 exper + \delta_2 exper^2 + \delta_3 educ + \delta_4 age + \delta_5 kidslt6 + \delta_6 nwifeinc + error$$

regressing the TSLS residuals $\hat{u}$ on all the exogenous variables, and then obtain the J statistic as $J = mF = 2 \cdot 0.053499 = 0.10700$.

Alternatively, the (non-robust) Sargan version of the overidentification test can be used with the statistic $LM = 0.068$ reported in the TSLS output. In either case, the J or LM statistics do not reject the null of instruments exogeneity when compared to a $\chi^2_{m-k} = \chi^2_{2-1} = \chi^2_1$ distribution (LM p-value $= 0.79348$, $\chi^2_{1,0.05} = 3.841$).

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Model 1: TSLS, using observations 1–428
Dependent variable: lhours
Instrumented: lwage
Instruments: const educ age kidslt6 nwifeinc exper expersq
Heteroskedasticity-robust standard errors, variant HC1

	Coefficient	Std. Error	t-ratio	p-value
const	8.37023	0.723642	11.57	0.0000
lwage	1.99435	0.763713	2.611	0.0093
educ	−0.235461	0.0857156	−2.747	0.0063
age	−0.0135248	0.0126909	−1.066	0.2872
kidslt6	−0.465439	0.243629	−1.910	0.0568
nwifeinc	−0.0139044	0.00708012	−1.964	0.0502
	.	.	.	.

Mean dependent var	6.866960	S.D. dependent var	0.968928
Sum squared resid	1112.900	S.E. of regression	1.623947
R^2	0.006922	Adjusted R^2	-0.004844
$F(5, 422)$	3.032869	P-value(F)	0.010574

Hausman test –

Null hypothesis: OLS estimates are consistent

Asymptotic test statistic: $\chi^2(1) = 41.803$

with p-value = 1.00948e-10

Sargan over-identification test –

Null hypothesis: all instruments are valid

Test statistic: LM = 0.0685365

with p-value = $P(\chi^2(1) > 0.0685365) = 0.79348$

Weak instrument test –

First-stage $F(2, 421) = 6.17263$

Model 1b: OLS, using observations 1–428

Dependent variable: lwage

Heteroskedasticity-robust standard errors, variant HC1

	Coefficient	Std. Error	t-ratio	p-value
const	−0.447161	0.288901	−1.548	0.1224
kidslt6	−0.0532185	0.104790	−0.5079	0.6118
age	−0.00255613	0.00591494	−0.4321	0.6659
educ	0.101111	0.0141358	7.153	0.0000
nwifeinc	0.00555999	0.00274350	2.027	0.0433
exper	0.0418643	0.0151135	2.770	0.0059
expersq	−0.000762482	0.000406466	−1.876	0.0614
	.	.	.	.

Mean dependent var	1.190173	S.D. dependent var	0.723198
Sum squared resid	186.8577	S.E. of regression	0.666215
R^2	0.163302	Adjusted R^2	0.151377
$F(6, 421)$	14.76233	P-value(F)	2.49e–15
Log-likelihood	−429.9477	Akaike criterion	873.8954
Schwarz criterion	902.3093	Hannan–Quinn	885.1173

Test for omission of variables –

Null hypothesis: parameters are zero for the variables

exper

expersq

Test statistic: $F(2, 421) = 6.17263$

with p-value = $P(F(2, 421) > 6.17263) = 0.00227937$

Model 1c: OLS, using observations 1–428

Dependent variable: uhat1

Heteroskedasticity-robust standard errors, variant HC1

	Coefficient	Std. Error	t-ratio	p-value
const	0.0671175	0.670172	0.1001	0.9203
kidslt6	−0.00318799	0.247806	−0.01286	0.9897
age	−0.00125522	0.0147127	−0.08532	0.9321
educ	7.31509e−005	0.0320322	0.002284	0.9982
nwifeinc	0.000320221	0.00741284	0.04320	0.9656
exper	−0.00571369	0.0386083	−0.1480	0.8824
expersq	0.000227995	0.000973947	0.2341	0.8150
	.	.	.	.

Mean dependent var	0.000000	S.D. dependent var	1.614411
Sum squared resid	1112.722	S.E. of regression	1.625744
R^2	0.000160	Adjusted R^2	-0.014089
$F(6, 421)$	0.019696	P-value(F)	0.999967
Log-likelihood	−811.7701	Akaike criterion	1637.540
Schwarz criterion	1665.954	Hannan–Quinn	1648.762

Test for omission of variables –

Null hypothesis: parameters are zero for the variables

exper

expersq

Test statistic: $F(2, 421) = 0.053499$

with p-value $= P(F(2, 421) > 0.053499) = 0.947913$

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Equation (2): In the log($hours$) Reduced Form,

$$\log(hours) = \pi_0 + \pi_1 age + \pi_2 kidslt6 + \pi_3 nwifeinc + \pi_4 educ + \pi_5 exper + \pi_6 exper^2 + e_2$$

the First Stage F statistic for instruments significance,

$$\begin{aligned} H_0 &: \pi_1 = \pi_2 = \pi_3 = 0 \\ H_1 &: \pi_1 \neq 0 \text{ and/or } \pi_2 = 0 \text{ and/or } \pi_3 = 0, \end{aligned}$$

is $F = 6.02742 < 10$, so the instruments are also weak, despite the null hypothesis of no-relevance is rejected at the usual significance levels.

To test for instruments exogeneity we use the overidentification J-test. For that we can compute directly the (robust) F statistic of significance of the instruments,

$$\begin{aligned} H_0 &: \delta_1 = \delta_2 = \delta_3 = 0 \\ H_1 &: \delta_1 \neq 0 \text{ and/or } \delta_2 = 0 \text{ and/or } \delta_3 = 0 \end{aligned}$$

in the auxiliary model

$$\hat{v} = \delta_0 + \delta_1 age + \delta_2 kidslt6 + \delta_3 nwifeinc + \delta_4 educ + \delta_5 exper + \delta_6 exper^2 + error$$

regressing the TSLS residuals $\hat{v}$ on all the exogenous variables, and then obtain the J statistic as $J = mF = 3 \cdot 1.54637 = 4.639\,1$.

Alternatively, the (non-robust) Sargan version of the overidentification test can be used with the statistic $LM = 3.07939$ reported in the TSLS output. In either case, the J or LM statistics do not reject the null of instruments exogeneity when compared to a $\chi^2_{m-k} = \chi^2_{3-1} = \chi^2_2$ distribution (LM p-value $= 0.214446$, $\chi^2_{2,0.05} = 5.991$).

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Model 2: TSLS, using observations 1–428
Dependent variable: lwage
Instrumented: lhours
Instruments: const age kidslt6 nwifeinc educ exper expersq
Heteroskedasticity-robust standard errors, variant HC1

	Coefficient	Std. Error	t-ratio	p-value
const	−0.929121	1.19464	−0.7777	0.4372
lhours	0.0604230	0.167793	0.3601	0.7189
educ	0.110350	0.0156599	7.047	0.0000
exper	0.0363933	0.0177914	2.046	0.0414
expersq	−0.000711528	0.000440867	−1.614	0.1073
	.	.	.	.

Mean dependent var	1.190173	S.D. dependent var	0.723198
Sum squared resid	190.4360	S.E. of regression	0.670972
R^2	0.147643	Adjusted R^2	0.139583
$F(4, 423)$	21.04815	P-value(F)	7.67e–16

Hausman test –
Null hypothesis: OLS estimates are consistent
Asymptotic test statistic: $\chi^2(1) = 0.322786$
with p-value = 0.569938

Sargan over-identification test –
Null hypothesis: all instruments are valid
Test statistic: LM = 3.07939
with p-value = $P(\chi^2(2) > 3.07939) = 0.214446$

Weak instrument test –
First-stage $F(3, 421) = 6.02742$

Model 2b: OLS, using observations 1–428
Dependent variable: lhours
Heteroskedasticity-robust standard errors, variant HC1

	Coefficient	Std. Error	t-ratio	p-value
const	7.54556	0.394932	19.11	0.0000
educ	−0.0337365	0.0200009	−1.687	0.0924
exper	0.0777784	0.0194132	4.006	0.0001
expersq	−0.00129266	0.000529050	−2.443	0.0150
age	−0.0198778	0.00747124	−2.661	0.0081
nwifeinc	−0.00249559	0.00607242	−0.4110	0.6813
kidslt6	−0.574763	0.150146	−3.828	0.0001
	.	.	.	.

Mean dependent var	6.866960	S.D. dependent var	0.968928
Sum squared resid	337.2672	S.E. of regression	0.895047
R^2	0.158677	Adjusted R^2	0.146687
$F(6, 421)$	8.911069	P-value(F)	3.61e–09
Log-likelihood	−556.3207	Akaike criterion	1126.641
Schwarz criterion	1155.055	Hannan–Quinn	1137.863

Test for omission of variables –

Null hypothesis: parameters are zero for the variables

age

nwifeinc

kidslt6

Test statistic: $F(3, 421) = 6.02742$

with p-value $= P(F(3, 421) > 6.02742) = 0.000501035$

Model 2c: OLS, using observations 1–428

Dependent variable: vhat1

Heteroskedasticity-robust standard errors, variant HC1

	Coefficient	Std. Error	t-ratio	p-value
const	0.0260350	0.287849	0.09045	0.9280
educ	−0.00720027	0.0140061	−0.5141	0.6075
exper	0.000771434	0.0153627	0.05021	0.9600
expersq	2.71525e–005	0.000408150	0.06653	0.9470
age	−0.00135505	0.00598676	−0.2263	0.8210
nwifeinc	0.00571078	0.00270223	2.113	0.0352
kidslt6	−0.0184896	0.104487	−0.1770	0.8596
	.	.	.	.

Mean dependent var	0.000000	S.D. dependent var	0.667822
Sum squared resid	189.0659	S.E. of regression	0.670140
R^2	0.007195	Adjusted R^2	-0.006954
$F(6, 421)$	0.780353	P-value(F)	0.585679
Log-likelihood	−432.4618	Akaike criterion	878.9235
Schwarz criterion	907.3374	Hannan–Quinn	890.1455

Test for omission of variables –

Null hypothesis: parameters are zero for the variables

age

nwifeinc

kidslt6

Test statistic: $F(3, 421) = 1.54637$

with p-value $= P(F(3, 421) > 1.54637) = 0.2019$

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2. (20%) How much does the wage increase when the hours of work increase 10% and the rest of variables in the model remain constant? (1/2) Provide a 95% confidence interval for this effect and interpret the result. Is it significant at a 10% significance level? (1/2)

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ANSWER:

(1/2) Since α_1 can be interpreted as an elasticity because both the dependent and independent variables are in logs, the estimated effect of a 10% increment in *hours* is $10 \cdot \hat{\alpha}_1\% = 10 \cdot 0.060423\% = 0.60423\%$, i.e. when hours of work supplied increase a 10%, the average wage offered increases in approximately a 0.6%.

(1/2) The confidence interval for this estimated effect is

$$10 \cdot \hat{\alpha}_1 \pm 1.96 \cdot SE\,(10 \cdot \hat{\alpha}_1)$$

i.e.

$$0.60423 \pm 1.96 \cdot 10 \cdot 0.167793$$

or

$$(-2.6845\%,\ 3.8930\%)\,,$$

so the wage offered moves between a 2.7% reduction and a 3.9% increment with a 95% confidence when *hours* increase a 10%, and therefore is not significative at the 10% level since includes the zero value (because the 10% confidence interval is wider or because the p-value for the significance t-test of $\hat{\alpha}_1$ is larger than 0.10).

3. (25%) We now consider that *educ* in equation (1) is also endogenous because *ability* is omitted in the equation, while the rest of assumptions continue to be valid. We consider two instruments for *educ* : *motheduc* and *fatheduc*, which are the number of years of education of the mother and of the father, respectively. Discuss the identification of this equation with the new assumptions (1/2) and check, if possible, the relevance and exogeneity of the instruments proposed. (1/2)

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ANSWER:

(1/2) The labour supply equation (1) has now two endogenous variables, $\log(wage)$ and *educ*, $k = 2$, and we have available $m = 4$ instruments for them, *exper, exper*^{2}, *motheduc* and *fatheduc*, leading to a (potentially) overidentified equation, $m - k = 2 > 0$, so we can check for instruments exogeneity.

(1/2) In principle we can not test properly for instruments exogeneity as we have two endogenous regressors in this equation. We could test at most the significance of the instruments in each of the reduced forms running OLS regressions:

$\log(wage)$ Reduced Form:

$$\begin{aligned}\log(wage) &= \pi_0 + \pi_1 exper + \pi_2 exper^2 + \pi_3 motheduc + \pi_4 fatheduc \\ &\quad + \pi_5 age + \pi_6 kidslt6 + \pi_7 nwifeinc + e_1,\end{aligned}$$

where we test the instruments significance

$$\begin{aligned}H_0 &: \pi_1 = \pi_2 = \pi_3 = \pi_4 = 0 \\ H_1 &: \text{at least one } \pi_j \neq 0, \ j = 1, 2, 3, 4\end{aligned}$$

using the (robust) First Stage F statistic: $F = 4.35586 < 10$, so the instruments are weak, despite the null hypothesis is rejected at the usual significance levels (p-value=0.0018).

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Model 4: OLS, using observations 1–428
Dependent variable: lwage
Heteroskedasticity-robust standard errors, variant HC1

	Coefficient	Std. Error	t-ratio	p-value
const	0.587953	0.284581	2.066	0.0394
exper	0.0485847	0.0160079	3.035	0.0026
expersq	−0.000877314	0.000442397	−1.983	0.0480
motheduc	0.000989825	0.0123387	0.08022	0.9361
fatheduc	0.0134516	0.0115724	1.162	0.2457
kidslt6	0.0201473	0.107608	0.1872	0.8516
age	−0.00424926	0.00633066	−0.6712	0.5025
nwifeinc	0.0116132	0.00285406	4.069	0.0001
	.	.	.	.

Mean dependent var	1.190173	S.D. dependent var	0.723198
Sum squared resid	206.1377	S.E. of regression	0.700574
R^2	0.076971	Adjusted R^2	0.061587
$F(7, 420)$	5.023181	P-value(F)	0.000017
Log-likelihood	−450.9618	Akaike criterion	917.9237
Schwarz criterion	950.3967	Hannan–Quinn	930.7487

Test for omission of variables –

Null hypothesis: parameters are zero for the variables

exper

expersq

motheduc

fatheduc

Test statistic: $F(4, 420) = 4.35856$

with p-value $= P(F(4, 420) > 4.35856) = 0.00182977$

educ Reduced Form:

$$\begin{aligned} educ &= \gamma_0 + \gamma_1 exper + \gamma_2 exper^2 + \gamma_3 motheduc + \gamma_4 fatheduc \\ &\quad + \gamma_5 age + \gamma_6 kidslt6 + \gamma_7 nwifeinc + e_2, \end{aligned}$$

where we test the instruments significance

$$\begin{aligned} H_0 &: \gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 0 \\ H_1 &: \text{at least one } \gamma_j \neq 0,\ j = 1, 2, 3, 4 \end{aligned}$$

using the (robust) First Stage F statistic: $F = 50.7297 > 10$, so the instruments are not weak and H_0 is rejected at the usual significance levels.

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Model 5: OLS, using observations 1–753

Dependent variable: educ

Heteroskedasticity-robust standard errors, variant HC1

	Coefficient	Std. Error	t-ratio	p-value
const	8.26301	0.512373	16.13	0.0000
exper	0.0964620	0.0251435	3.836	0.0001
expersq	−0.00161100	0.000855960	−1.882	0.0602
motheduc	0.172595	0.0251818	6.854	0.0000
fatheduc	0.158355	0.0236904	6.684	0.0000
kidslt6	0.253897	0.156740	1.620	0.1057
age	−0.0157051	0.0104314	−1.506	0.1326
nwifeinc	0.0447765	0.00711167	6.296	0.0000
	.	.	.	.

Mean dependent var	12.28685	S.D. dependent var	2.280246
Sum squared resid	2676.323	S.E. of regression	1.895357
R^2	0.315525	Adjusted R^2	0.309094
$F(7, 745)$	41.80984	P-value(F)	8.73e–50
Log-likelihood	−1545.913	Akaike criterion	3107.826
Schwarz criterion	3144.819	Hannan–Quinn	3122.078

Test for omission of variables –

Null hypothesis: parameters are zero for the variables

exper

expersq

motheduc

fatheduc

Test statistic: $F(4, 745) = 50.7297$

with p-value = $P(F(4, 745) > 50.7297)$ = 8.67115e-38

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The J statistic $J = m \cdot F = 4 \cdot 0.160878 = 0.643\,51$, where F is the robust F statistic obtained by testing the significance of the instruments,

$$\begin{aligned} H_0 &: \delta_1 = \delta_2 = \delta_3 = \delta_4 = 0 \\ H_1 &: \text{at least one } \delta_j \neq 0, \; j = 1, 2, 3, 4, \end{aligned}$$

in the auxiliary model

$$\begin{aligned} \hat{u} &= \delta_0 + \delta_1 exper + \delta_2 exper^2 + \delta_3 motheduc + \delta_4 fatheduc \\ &\quad + \delta_5 age + \delta_6 kidslt6 + \delta_7 nwifeinc + error \end{aligned}$$

regressing the TSLS residuals $\hat{u}$ on all the exogenous variables (excluding *educ*), or Sargan LM = 0.446375, do not reject the null of instruments exogeneity compared to a $\chi^2_{m-k} = \chi^2_{4-2} = \chi^2_2$ distribution (p-value =0.799965, $\chi^2_{2,0.05} = 5.991$).

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Model 3: TSLS, using observations 1–428

Dependent variable: lhours

Instrumented: lwage educ

Instruments: const age kidslt6 nwifeinc exper expersq motheduc fatheduc

Heteroskedasticity-robust standard errors, variant HC1

	Coefficient	Std. Error	t-ratio	p-value
const	7.26076	0.991588	7.322	0.0000
lwage	1.81092	0.662538	2.733	0.0065
educ	−0.128606	0.0880180	−1.461	0.1447
age	−0.0116012	0.0115872	−1.001	0.3173
kidslt6	−0.543186	0.232447	−2.337	0.0199
nwifeinc	−0.0189058	0.00863895	−2.188	0.0292
	.	.	.	.

Mean dependent var	6.866960	S.D. dependent var	0.968928
Sum squared resid	994.9812	S.E. of regression	1.535505
R^2	0.006555	Adjusted R^2	-0.005216
$F(5, 422)$	3.041034	P-value(F)	0.010403

Hausman test –

Null hypothesis: OLS estimates are consistent

Asymptotic test statistic: $\chi^2(2) = 40.7624$

with p-value = 1.40786e-09

Sargan over-identification test –

Null hypothesis: all instruments are valid

Test statistic: LM = 0.446375

with p-value = $P(\chi^2(2) > 0.446375) = 0.799965$

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Model 3b: OLS, using observations 1–428

Dependent variable: uhat2

Heteroskedasticity-robust standard errors, variant HC1

	Coefficient	Std. Error	t-ratio	p-value
const	0.00417119	0.596862	0.006989	0.9944
age	−0.000573136	0.0141505	−0.04050	0.9677
nwifeinc	0.000567571	0.00699538	0.08114	0.9354
kidslt6	0.00315352	0.232118	0.01359	0.9892
motheduc	0.0144205	0.0244536	0.5897	0.5557
fatheduc	−0.0131371	0.0240577	−0.5461	0.5853
exper	−0.00431842	0.0366166	−0.1179	0.9062
expersq	0.000195319	0.000925187	0.2111	0.8329
	.	.	.	.

Mean dependent var	9.90e–15	S.D. dependent var	1.526488
Sum squared resid	993.9435	S.E. of regression	1.538354
R^2	0.001043	Adjusted R^2	-0.015606
$F(7, 420)$	0.093185	P-value(F)	0.998654
Log-likelihood	−787.6129	Akaike criterion	1591.226
Schwarz criterion	1623.699	Hannan–Quinn	1604.051

Test for omission of variables –

Null hypothesis: parameters are zero for the variables

motheduc

fatheduc

exper

expersq

Test statistic: $F(4, 420) = 0.160878$

with p-value = $P(F(4, 420) > 0.160878) = 0.957987$

4. (15%) Test under the assumptions of part 3 whether labour supply decreases more with an additional year of education than with one more year of age, all other variables remaining constant.

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ANSWER: The hypotheses to test are

$$\begin{aligned} H_0 &: \beta_2 = \beta_3 \\ H_0 &: \beta_2 < \beta_3, \end{aligned}$$

given that we expect that labour supply decreases with both changes so both $\beta's$ are negative.

This is a one side test so we should use a t-test, which can be obtained in different ways:

1. Computing the (robust) SE of $\hat{\beta}_2 - \hat{\beta}_3$ in the TSLS output using the four instruments for the two endogenous regressors, $\log(wage)$ and $educ$, and the corresponding t statistic:

$$\begin{aligned} t &= \frac{\hat{\beta}_2 - \hat{\beta}_3}{SE\left(\hat{\beta}_2 - \hat{\beta}_3\right)} = \frac{\hat{\beta}_2 - \hat{\beta}_3}{\sqrt{\widehat{Var}\left(\hat{\beta}_2\right) + \widehat{Var}\left(\hat{\beta}_3\right) - 2\widehat{Cov}\left(\hat{\beta}_2, \hat{\beta}_3\right)}} \\ &= \frac{\hat{\beta}_2 - \hat{\beta}_3}{\sqrt{SE\left(\hat{\beta}_2\right)^2 + SE\left(\hat{\beta}_3\right)^2 - 2\widehat{Cov}\left(\hat{\beta}_2, \hat{\beta}_3\right)}} \\ &= \frac{-0.128606 - (-0.0116012)}{\sqrt{0.0880180^2 + 0.0115872^2 - 2 \cdot 2.58505 \times 10^{-4}}} = -1.3634 \end{aligned}$$

2. Computing directly the (robust) F test in the TSLS output of Gretl and noting that the t statistic can be obtained as

$$\begin{aligned} t &= sign\left(\hat{\beta}_2 - \hat{\beta}_3\right)\sqrt{F} \\ &= sign\,(-0.117)\sqrt{1.85894} \\ &= -1.3634. \end{aligned}$$

The t statistic is not significative at 5% when compared to the one-sided 5% normal critical value, $t \not< -z_\alpha = -1.645$ (the p-value of the F test is for a 2-sided alternative, so for a one sided test it should be divided by 2, $p - value = 0.172747/2 = 0.086374 > 0.05$), so we do not reject the null hypothesis that both effects are equal in favour that one year of education reduces more the labour supply than one more year or age.

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Restriction: b[educ] - b[age] = 0

Test statistic: Robust F(1) = 1.85894, with p-value = 0.172747

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SOME CRITICAL VALUES: $Z_{0.10} = 1.282$, $Z_{0.05} = 1.645$, $Z_{0.025} = 1.96$, $\chi^2_{1,0.05} = 3.841$, $\chi^2_{1,0.01} = 6.635$, $\chi^2_{2,0.05} = 5.991$, $\chi^2_{2,0.01} = 9.210$, $\chi^2_{3,0.05} = 7.815$, $\chi^2_{3,0.01} 11.345$, $\chi^2_{4,0.05} = 9.488$, $\chi^2_{4,0.01} = 13.277$, $\chi^2_{5,0.05} = 11.071$, $\chi^2_{5,0.01} = 15.086$, $\chi^2_{6,0.05} = 12.592$, $\chi^2_{6,0.01} = 16.812$, where $\mathbb{P}\left(Z > Z_\alpha\right) = \alpha$ and $\mathbb{P}\left(\chi^2_m > \chi^2_{m,\alpha}\right) = \alpha$, Z is distributed as a standard normal random variable of zero mean and unit variance, and χ^2_m as a $chi - squared$ with m degrees of freedom.