ECONOMETRICS FINAL EXAM 2022-23: PART A

Universidad Carlos III de Madrid

Answer all questions in 1 hour

Use the **401ksubs** dataset from Wooldridge to estimate a LOGIT model to study the variables that determine the decision of a worker to subscribe a pension plan. The decision to subscribe the pension plan is described by the binary variable $p401k$. The explanatory variables are annual income of the individual in thousands of dollars (inc) and age (age). The income enters in the model in logarithms and age enters as a quadratic polynomial.

1. (6%) For a fixed level of income, when is more likely that the individual subscribes the pension plan?

   Given the logit model

   $$\Pr(p401k = 1|inc, age) = \Lambda\left(\beta_0 + \beta_1 \log(inc) + \beta_2 age + \beta_3 age^2\right)$$

   and since Λ is monotone, the estimated probability is

   $$\widehat{\Pr}(p401k = 1|inc, age) = \Lambda\left(\hat{\beta}_0 + \hat{\beta}_1 \log(inc) + \hat{\beta}_2 age + \hat{\beta}_3 age^2\right)$$

   is maximized when the linear (in parameters) function $\hat{\beta}_2 age + \hat{\beta}_3 age^2$ is maximized in terms of age, i.e. for

   $$age^* = \frac{\hat{\beta}_2}{-2\hat{\beta}_3} = \frac{0.0791561}{-2 \times (-0.000925767)} = 42.752$$

   i.e. approximately at 42.7 years.

   .

   Model 1: Logit, using observations 1–9275
   Dependent variable: p401k
   QML standard errors

	Coefficient	Std. Error	z	p-value
const	−6.72144	0.432940	−15.53	0.0000
loginc	1.16088	0.0455241	25.50	0.0000
age	0.0791561	0.0203789	3.884	0.0001
agesq	−0.000925767	0.000233685	−3.962	0.0001
	.	.	.	.

Mean dependent var	0.276226	S.D. dependent var	0.447154
McFadden R^2	0.072646	Adjusted R^2	0.071914
Log-likelihood	−5069.154	Akaike criterion	10146.31
Schwarz criterion	10174.85	Hannan–Quinn	10156.01

Number of cases 'correctly predicted' = 6722 (72.5 percent)
Likelihood ratio test: $\chi^2(3) = 794.206$ [0.0000]

2. (6%) What is the estimated probability that a 30 years old worker with annual income of \$20,000 subscribes a pension plan?

$$\begin{aligned}\widehat{\Pr}(p401k = 1|inc = 20, age = 30) &= \Lambda\left(\hat{\beta}_0 + \hat{\beta}_1 \log(inc) + \hat{\beta}_2 age + \hat{\beta}_3 age^2\right) \\ &= \Lambda\begin{pmatrix} -6.72144 + 1.16088 \log(20) \\ +0.0791561 * 30 - 0.000925767 * 30^2 \end{pmatrix} \\ &= \Lambda(-1.702\,3) \\ &= \frac{1}{1 + e^{-(-1.702\,3)}} = 0.154\,17.\end{aligned}$$

3. (6%) Calculate the odds of subscribing a pension plan for an individual with the same age and income as the previous question (3%). Interpret the result (3%).

$$\begin{aligned}odd(p401k &= 1|inc = 20, age = 30) = \frac{\widehat{\Pr}(p401k = 1|inc = 20, age = 30)}{1 - \widehat{\Pr}(p401k = 1|inc = 20, age = 30)} \\ &= \frac{0.154\,17}{1 - 0.154\,17} = 0.1823,\end{aligned}$$

i.e. the (estimated) probability of subscribing the plan is only 0.1823 times the probability of not subscribing it.

4. (6%) What is the change in the estimated probability of subscription when the income of an individual with the same characteristics as before increases a 1%?

Now the income becomes \$20,200, so

$$\begin{aligned}\widehat{\Pr}(p401k = 1|inc = 20.2, age = 30) &= \Lambda\left(\hat{\beta}_0 + \hat{\beta}_1 \log(income) + \hat{\beta}_2 age + \hat{\beta}_3 age^2\right) \\ &= \Lambda\begin{pmatrix} -6.72144 + 1.16088 \log(20.2) \\ +0.0791561 * 30 - 0.000925767 * 30^2 \end{pmatrix} \\ &= \Lambda(-1.\,690\,7) \\ &= \frac{1}{1 + e^{-(-1.\,690\,7)}} = 0.155\,68,\end{aligned}$$

so the probability increases in $0.155\,68 - 0.154\,17 = 0.00151$ i.e. 0.15pp.

5. (6%) Compute the odds ratio (OR) for the increment in the previous question (3%) and interpret the result (3%).

We either compute that

$$
\begin{aligned}
odd(p401k &= 1|inc = 20.2, age = 30) = \frac{\widehat{\Pr}\left(p401k = 1|inc = 20.2, age = 30\right)}{1 - \widehat{\Pr}\left(p401k = 1|inc = 20.2, age = 30\right)} \\
&= \frac{0.155\,68}{1 - 0.155\,68} = 0.184\,39,
\end{aligned}
$$

and therefore

$$OR = \frac{0.184\,39}{0.1823} = 1.0115$$

or alternatively with $\Delta x = \Delta \log(inc) \approx 0.01$ (i.e. a 1% increment in income),

$$OR = e^{\hat{\beta}_1 \Delta x} \approx e^{1.16088 \times 0.01} = 1.0117$$

i.e. when income increases a 1%, the odds of subscribing the plan increase approximately a $100 \times \hat{\beta}_1 \Delta x\% = 100 \times 1.16 \times 0.01\% = 1.16\% \approx 1.15\%$.