SELF-ENFORCING CONTRACTS WITH PERSISTENT SHOCKS

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ABSTRACT. The use of self-enforcing contracts in an environment with persistent shocks rationalizes: (i) that compensation may appear to "reward luck"; (ii) that adverse shocks are compounded via a 'low morale effect' leading to even worse performance; (iii) that countries with poorer contract enforcement would have a larger dispersion of firm productivities and a greater aggregate TFP and GDP volatility.

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1. Introduction

Many, if not most, economic interactions are carried out with very incomplete or no formal contracts at all. In their absence, to overcome the possible hold-up problems, economic agents often rely on the repeated nature of their interactions to establish relational or self-enforcing contracts. It is also natural that the surplus from the relationship fluctuates over time in a persistent manner. In this paper we show how limited external enforceability coupled with persistent productivity shocks can generate several interesting empirical predictions that are consistent with both micro and macro data.

Concretely, we extend the model of Levin (2003) to allow for a persistent productivity state and show the following three related results:

(i) agents can appear to be rewarded for luck,

(ii) in periods of distress the firm's situation is worsened by "low morale", the difficulty of incentivizing workers to work hard and / or not to leave the firm,

(iii) the enforceability constraint generates a multiplier effect amplifying the exogenous productivity shocks. This has two implications for countries / industries where enforceability constraints are more binding:

(a) idiosyncratic shocks induce a larger dispersion of measured firm productivity levels within a country or industry
(b) correlated shocks result in larger TFP and GDP volatility.

We start by showing that even when the exogenous productivity shocks are perfectly observable and independent of the agent's effort, bonuses are dependent on the realization of the shocks beyond the agent's influence. This is contrary to the "Informativeness Principle" (see Hölmstrom (1979)), according to which a performance measure should only affect compensation if it provides information about the agent's hidden effort. Importantly, the empirical literature has documented several cases in which contracts seem to be rewarding luck. One of the best known of such studies, by Bertrand and Mullainathan (2001), documents that the compensation of executives in major oil companies in the US positively correlates with the price of oil, an element outside the executive's control. Garvey and Milbourn (2006) show also that there is reward for luck and further argue it is not symmetric: good luck is rewarded while bad luck is not punished.[1] Finally, DeVaro et al. (which we discuss further below) show empirically that persistence of the state is important for

[1]See also Frydman and Saks (2010) for a historical perspective on executive compensation.

"reward for luck." They also provide evidence against capture of boards by CEOs being the main driver of the observed compensation patterns.

The economic rationale for "rewarding luck" in our model hinges on the fact that the persistent productivity shock implies that the continuation value of the firm is higher in the good state vis a vis the bad state. As a result, the principal has a stronger desire to maintain the relationship in the good state and thus can credibly promise to pay higher bonuses in such a state. The bonus payments are thus higher in the high state than in the low state. It is important to note that although luck determines the size of the bonus, the bonus is still paid only when the output measure directly related to the agent's effort is indicative of effort. More specifically, the agent is being rewarded, conditional upon success, with a lottery, and for incentives what matters is the expected payoff of that lottery.

Our second result highlights the effect of persistent productivity during bad times. When the current state is bad, the future value of the relationship is low and the principal can no longer credibly promise to make large payments. Thus, it can no longer motivate the agent to exert very high effort. This "low morale effect" is consistent with the evidence that labor productivity levels are procyclical (Baily et al., 2001). We can alternatively recast the model as in Fuchs (2015) where every period the agent receives a stochastic outside option and must decide whether to stay with the firm or leave. In such a formulation, when the future of the firm looks bleak, the agent would be willing to accept lower outside options. This is consistent with the findings presented by Baghai et al. (2016) which show how Swedish firms lose their key talents when their financial health deteriorates. Similarly, using an online search platform, Brown and Matsa (2016) show that job applicants avoid companies with poor financial conditions.

In his detailed review on the determinants of productivity Syverson (2011) states that one:

> "robust finding in the literature ... is that higher productivity producers are more likely to survive than their less efficient industry competitors. Productivity is quite literally a matter of survival for businesses"

Our predictions are consistent with this finding and further suggest that the causality can go both ways. In particular, if a firm is more likely to survive, then it will be able to become more productive. This happens because with higher productivity it becomes easier to enforce contracts: i.e., the dynamic enforceability constraint is less likely to be binding. Empirically, the survival prospects of the firm at time t might not be fully observable to the econometrician but influence time $t+1$ productivity. Leading to a positive correlation between productivity at t and survival to $t+1$, yet, it is survivability that determines productivity and not the converse.

Syverson (2011) also highlights how dramatic the productivity differences can be within a given sector. Particularly related to the third implication of our model he highlights that these differences vary dramatically across countries. While in the US the most productive firms within an industry can be twice as productive as the least productive ones, for India and China there can be a fivefold difference.[2]

There are naturally many possible reasons for these cross country differences; our explanation for this finding is that it is partly driven by the fact that the US has an effective legal system that facilitates the enforceability of contracts while those institutions are much weaker in developing countries.[3] Thus, a larger fraction of business relationships in India and China are governed by relational contracts. As a result, when we shock two firms in the US and two firms in India with a pair of low and high exogenous productivity shocks, the spread in measured productivity in India would be larger than in the US. To see why this is the case, suppose the productivity shock does not affect the optimal level of effort. With complete contracts (i.e. in the US) there will be no change in worker effort at both firms. As a result, the measured productivity differences in the US firms would be just given by the spread of the exogenous shock. Instead, with imperfect enforceability (i.e. the case of India), the equilibrium effort that can be elicited from the workers depends on the shock realization. When the shock is high, the firm can credibly commit to higher bonuses and, as a result, demand higher effort and become more productive. The opposite effect takes place when the shock is negative. This implies there is an extra wedge in the measured productivity of Indian firms given by the fact that the equilibrium effort also varies with the exogenous shock.

We build on this idea and show that this finding holds more generally. For a large sample of countries we document there is a negative relationship between the spread in productivity measures within industries of a given country and the quality of the legal system of that country.

Our model also makes predictions about the aggregate fluctuations. In particular, if we consider aggregate shocks (abstracting from general equilibrium effects)

[2]See references therein in particular Syverson (2004) and Hsieh and Klenow (2009).
[3]See Syverson (2011) and references therein for a rich discussion of other possible factors influencing firm productivity.

the implication of our model is that the volatility of aggregate TFP and GDP of economies with poorer enforcement would be greater. We again proxy the share of firms relying on self-enforcing contracts by a measure of how ineffective the legal system is or how large the informal sector of the economy is.[4] Consistent with the predictions of our model, we find there is a positive correlation between poor enforcement and the volatility of GDP or aggregate TFP measures.

There is certainly a huge amount of additional heterogeneity and richness that is not captured by our simple model. Yet, we believe our model provides a very useful lens through which we can start to uncover how seemingly unrelated findings at the firm, industry and aggregate economy level can actually be manifestations of a natural friction economic agents must constantly deal with: namely, how to realize the gains from trade without falling prey to the hold-up problem.

The paper is organized as follows: first we discuss the related theoretical literature, in Section II we present the model, Section III establishes the sufficiency of Markovian contracts which allows us to simplify the analysis, the analysis is carried out in Section IV where we present our main results and in Section V we provide some final remarks.

Related Theoretical Literature:

Our model belongs to the rich literature on relational incentive contracts that builds on Levin (2003) and the earlier work by Bull (1987) and MacLeod and Malcomson (1989). From a technical perspective the most closely related work to ours within this literature are Kwon (2016) and DeVaro et al.. The focus of Kwon (2016) and her main contribution is on proving that the optimality property of stationary contracts identified by Levin (2003) can be adapted and extended to a richer setting with Markovian shocks. We go beyond her analysis and complement it with a characterization of the optimal contracts. Using a similar model DeVaro et al. like us focus on using their model to offer a theoretical rationale for rewarding luck. In addition, as mentioned earlier, they complement their theoretical analysis with some corroborating evidence on CEO compensation. Although our main argument for rewarding luck is similar in spirit, our model uses a different timing of shocks vis a vis theirs. In our view, our timing, with the shock revealed after the effort choice, offers a broader modeling flexibility: in particular, for non-separable production

[4]Several such indices are now available. For our findings in Section 4 we use the rule of law measure from the Worldwide Governance Indicators (WGI).

functions, our timing implies that positive (luck) innovations can lead to higher bonuses without affecting the current effort choice. Instead, with the DeVaro et al. timing where the shock is revealed before the effort choice, it would become less clear whether the increase in compensation is instead driven by the need to elicit higher effort in the good states. From an econometrician's perspective this is particularly problematic since, like the principal, the econometrician is unlikely to observe the agent's effort. In addition to this, our main contribution relative to their work is on highlighting the "morale effect" and the related amplification of shocks that is induced by the lack of enforceability.

To our knowledge three other quite different theoretical explanations have been put forward for the well-documented "reward for luck" phenomenon. The first is by Oyer (2004) who argues that the shocks identified in some of the empirical studies (such as the price of oil) not only affect one firm, but rather more broadly the entire sector or industry. As a result, when oil prices are high the aggregate demand for the agent's services may naturally increase and the agent's outside option is likely to be higher as a result. Thus, in order to retain the agent (CEO), the principal must increase the agent's wage. His model does not have implications for the agent's effort thus, unlike our model does, and hence it does not speak to the " morale effect" or the correlation between enforceability and volatility.

The second explanation for the "reward for luck" was put forward by Hoffmann and Pfeil (2010) and is also present in DeMarzo et al. (2012). Both papers study persistent productivity shocks in the context of the DeMarzo and Sannikov (2006) cash flow diversion model. There is no enforceability constraint in their models. Instead, in addition to hidden effort (cash flow diversion) the main agency friction in their model is the agent's limited liability and the need for financing up-front. In their model, when productivity is higher, the potential cash flows are larger. Thus, the principal has stronger incentives to avoid reducing the size of the firm or triggering termination (which is the main way of providing incentives to the agent within their model). More importantly, the latter observation implies that the principal must leave more rents to the agent. Thus, the "reward for luck" effect arises. It is important to note however that due to the linearity in how effort translates into expected profits, the equilibrium effort in their model is independent of the productivity shock. Thus, again, their models do not allow for a "morale effect" nor can they be used to study the correlation between enforceability and output volatility.

The third explanation, put forward by DeMarzo and Kaniel (2017), is of a very different nature. They consider agents that not only care about their own compensation but also about their compensation relative to others. They show that when this second component of the preferences is strong, it might not be optimal to use relative performance pay (to filter out the common noise in the output). As a result, they argue, under certain parameter configurations it might appear as if agents are rewarded for luck.

Finally, Barron et al. (2018) is also quite related. They study the problem of an entrepreneur which needs both: (i) to take an initial loan to finance its venture and (ii) to hire an agent to work on it. The contract between the entrepreneur and the worker is not externally enforceable. Thus the continuation value of the entrepreneur plays an important role in determining the extent to which the enforceability constraint binds. In their model an entrepreneur that has a lot of debt is similar to the principal in our model that experiences a low productivity shock. In both cases, their equity value is low and thus there is a limit on how large a bonus they can credibly promise to the agent. Thus, when the firm is very leveraged, as when it is experiencing adverse persistent shocks (which are not present in their model), it cannot induce the worker to work hard. This is similar to the "morale effect" except it is induced by an excess of leverage rather than a persistent adverse shock. Their paper also provides some nice empirical evidence supporting the mechanism in their model.

2. Setup

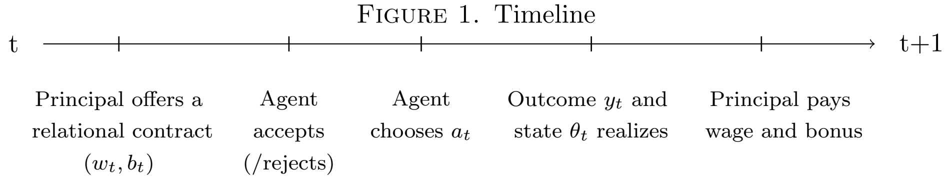
A principal ("she") and an agent ("he") interact repeatedly over time. Time is discrete and indexed via $t \in \{1, 2, ...\}$. At the beginning of every period, the principal makes the agent an offer consisting of an enforceable wage payment w_t, a schedule of non-enforceable bonus payments $\{b_t\}$ and recommends the agent take an action a_t. If the agent rejects, the game ends and both receive their outside options. If the agent accepts, the agent privately chooses his action $\tilde{a}_t \in [0, 1]$ incurring a cost $c(\cdot)$. We assume $c(\cdot)$ is continuously differentiable, strictly increasing and strictly convex, with $c(0) = c'(0) = 0$ and $1 < c'(1) \leq \infty$. [5]

After the agent takes the action a, the publicly observable output $\pi_t = f(y(\tilde{a}_t, \varepsilon), \theta_t)$ is realized. In addition to the agent's action, two random shocks ε_t and θ_t determine

[5]We make these assumptions on the effort cost function since they imply solutions are interior thus allowing us to simplify the exposition.

π_t. While $\varepsilon_t \overset{iid}{\sim} U([0,1])$ is not observable, θ_t becomes publicly observable upon the realization of π_t. Furthermore, we assume $\theta_t \in \{L, H\} =: \Theta$ follows a first-order Markov chain with a symmetric persistence parameter $\lambda := \mathsf{Prob}\,(\theta_t = \theta_{t-1}|\theta_{t-1}) > 1/2$ for both $\theta_{t-1} = H, L$ and $H > L > 0$.[6] To make the analysis cleaner we will further assume that $y_t = 1(\varepsilon_t \leq \tilde{a}_t) \in \{0,1\} =: \mathcal{Y}$ so that $\mathsf{Prob}\,(y_t = 1) = \tilde{a}_t$, that is, the action directly determines the probability that the first component of the output is high. Furthermore, we assume in our formulation that total output is additively separable i.e. $\pi_t = y\left(\tilde{a}_t, \varepsilon\right) + \theta_t$ The model on the other hand is flexible enough to allow for the alternative specifications of π and indeed it is natural to consider complementarities between the state and the action. [7] The advantage of the additive structure is that the first best action is independent of the state θ_{t-1} and thus it provides a very clear benchmark.

Figure 1. Timeline



At the end of each period, the principal pays the wage w_t and decides what, if any, discretionary transfer b_t to make to the agent. Alternatively, once the agent decides to terminate the relationship ($d_t^A = 0$), both the principal and agent receive outside options with 0 values forever.

Both the principal and agent are risk-neutral, discount the future with a common discount factor δ, and have deep pockets. The principal maximizes the present value of the expected discounted output streams minus her payments to the agent:

$$\Pi_t = (1-\delta)\sum_{\tau=t}^{\infty} \delta^{\tau-t}\mathbb{E}_t\left[d_\tau^A(\pi_\tau - w_\tau) - b_\tau\right]$$

The agent maximizes the expected discounted payments received minus his cost of effort which we assume enters additively:

[6]The model could be extended to richer stochastic processes. We avoid doing so to maximize the clarity of the exposition.

[7]$\pi_t = \alpha y_t\theta_t + (1-\alpha)(y_t + \theta_t)$, $\forall\ \alpha \in [0,1]$. For example, we could have $\pi_t = y_t\theta_t$ when $\alpha = 1$, θ_t might represent the price of corn and y_t the amount of corn the agent produces. We avoid doing so to maximize the clarity of the exposition.

$$v_t = (1-\delta)\sum_{\tau=t}^{\infty}\delta^{\tau-t}\mathbb{E}_t\left[d_\tau^A\left(w_\tau - c\left(a_\tau\right)\right) + b_\tau\right]$$

Given any period t and public history $h^t = \{\theta_0, \theta_1, ...\theta_{t-1}, y_1, ...y_{t-1}, b_1, ...b_{t-1}, w_1, ...w_{t-1}\} \in \mathcal{H}^t$, a relational contract specifies the compensation mix the principal offers; whether or not the agent accepts it; and, if the agent accepts, the effort and dicretionary bonus payment decisions. The compensation mix (w_t, b_t) are allowed to be functions of the public history, and are functions of the following form: $w_t : \mathcal{H}^t \to \mathbb{R}$, $b_t : \mathcal{H}^t \times (\Theta \times \mathcal{Y}) \to \mathbb{R}^+$.

A relational contract is self-enforcing if it is a perfect public equilibrium of the repeated game under relational contracting. As is usual in this literature we are interested in characterizing the efficient self-enforcing arrangements that would govern this relationship. As a first step, using an extension of the methods by Spear and Srivastava (1987) and Abreu et al. (1990) we can conveniently surmise a given public history of the game h^t, into a continuation value for the agent v and the past state θ_{t-1}. This allows us to capture the principal's problem with the following recursive representation:

$$\begin{aligned}
\Pi\left(v,\theta_{-1}\right) &= \max_{a,w,b,v'} \mathbb{E}\left[(1-\delta)\left(\pi - w - b\left(y,\theta_{-1},\theta,v\right)\right) + \delta\Pi\left(v',\theta\right)|a,\theta_{-1}\right] \\
& \quad s.t. \\
v &= \mathbb{E}\left[(1-\delta)\left(w + b\left(y,\theta_{-1},\theta,v\right) - c\left(a\right)\right) + \delta v'|a,\theta_{-1}\right] && \text{[PK]} \\
a &\in \arg\max_{\tilde{a}\in[0,1]} \mathbb{E}\left[(1-\delta)\left(w + b\left(y,\theta_{-1},\theta,v\right) - c\left(\tilde{a}\right)\right) + \delta v'|\tilde{a},\theta_{-1}\right] && \text{[IC]} \\
\delta\Pi(v',\theta) &\geq (1-\delta)\, b\left(y,\theta_{-1},\theta,v\right) \qquad \forall y\times\theta_{-1}\times\theta\times v && \text{[DEP]} \\
\Pi(v',\theta) &\geq 0 && \text{[PCP]} \\
v' &\geq 0 && \text{[PCA]}
\end{aligned}$$

where [PK] is the promise keeping constraint, [IC] the incentive compatibility constraint for the agent to follow the recommended action a, [DEP] is the dynamic enforceability constraints guaranteeing that the principal prefers to pay the promised bonus rather than reneging and effectively terminating the relationship, [PC]'s imply that each party needs to be at least as well off as the outside option every period.

3. Basic Properties of Optimal Markovian Contracts

In order to obtain a characterization of the optimal arrangements we first show that the general problem can be significantly simplified by restricting our analysis to Markovian contracts. This is analogous to the main contribution in Levin (2003) of showing that it is without loss to focus on stationary contracts when shocks are i.i.d. over time. Indeed, to show this result we extend and apply analogous ideas from Theorem 2 in Levin (2003) to accommodate state dependence.

Proposition 1 (Optimality of Markovian Relational Contracts)**.** There exists a Markovian relational contract such that

(R-1) it attains maximum surplus state-by-state and it is without loss to set $v' = v$; and

(R-2) it is without loss to set $b\left(y = 0, \theta_{-1}, \theta, v\right) = 0$

Remark 1. This result says that for any contract that maximizes the expected surplus we can find a Markovian relational contract that does the job. The offered base wages and bonuses are only functions of the last and current realization θ. Importantly, since beyond participation, continuation values don't play a role, we can focus on simply maximizing the current total surplus generated by the pair to characterize the optimal arrangements.

Since we have established that bonuses are only paid after success we will simplify are notation and denote the bonus payment upon success simply as $b(\theta_{-1}, \theta)$. The only role for the base wage is to ensure that the contract delivers the right promised value to the agent. We can thus solve the problem of figuring out the effort and bonus choices that maximize surplus, which we denote by $\Pi(\theta_{-1})$, first and then simply divide it between the principal and the agent by adjusting the base wages. Also note that we can drop the participation constraints since a contract that implements zero effort is always available and generates strictly positive surplus. Thus, the problem can be simplified to:

$$\Pi(\theta_{-1}) = \max_{a(\theta_{-1}), b(\theta_{-1}, \theta)} (1-\delta)\left(a(\theta_{-1}) + \mathbb{E}(\theta|\theta_{-1}) - c(a(\theta_{-1}))\right) + \delta\mathbb{E}(\Pi(\theta)|\theta_{-1}) \tag{1}$$

subject to:

$$[\text{IC}] \quad a(\theta_{-1}) \in \underset{\tilde{a}}{\operatorname{argmax}}\, \mathbb{E}(b(\theta_{-1}, \theta)|\theta_{-1})\tilde{a} - c(\tilde{a})$$

$$[\text{DEP}] \quad \delta\Pi(\theta) \geq (1-\delta)b(\theta_{-1}, \theta)$$

Denote the optimal effort and bonus payment by $a^*(\theta_{-1})$ and $b(\theta_{-1}, \theta)$, respectively. The optimal bonus scheme $b(\theta_{-1}, \theta)$ and the optimal effort $a^*(\theta_{-1})$ that solves the reduced problem (1) are related as follows

Lemma 1. In an optimal Markovian relation contract, we have that $c'(a^*(\theta_{-1})) < 1$ implies $b(\theta_{-1}, \theta) = \frac{\delta}{1-\delta}\Pi(\theta)$ for both $\theta = H, L$.

This lemma states that if there exists inefficiency due to a strict under-provision of effort, the principal must, in each state, be pledging all of the future surplus from the relationship as the bonus payments. To see why this must be the case suppose that $b(\theta_{-1}, \theta) < \frac{\delta}{1-\delta}\Pi(\theta)$ for some θ. The principal could then increase $b(\theta_{-1}, \theta)$ and simultaneously request that the agent exert more effort. Since there is under-provision of effort that would improve efficiency.

Now, denote by $\Pi(\theta)^{FB}$ the maximum payoffs that can be obtained without the dynamic enforceability for the state θ. According to Lemma 1 the dynamic enforceability constraint is the only friction preventing from achieving the first-best outcome, and the following monotonicity result obtains:

Proposition 2 (Monotonicity)**.** In any optimal Markovian relational contract the constraint optimal effort and the principal's profits satisfy

(M-1) $a^*(L) \leq a^*(H) \leq (c')^{-1}(1) = a^{FB}$; and

(M-2) $\Pi^*(L) < \Pi^*(H)$, $\Pi^*(L) \leq \Pi(L)^{FB}$ and $\Pi^*(H) \leq \Pi(H)^{FB}$.

Remark 2. There is an intuitive explanation for this result contrasting with the Lemma 1. The latter states that if there exists inefficiency due to a strict under-provision of effort, future surplus from the relationship is bounded away from that in the efficient allocation, which in turn constraints the bonus payments in each state relative to the inefficiency. In contrast, by the part (M-1) of Proposition 2, there is no over-provision of effort. Hence, for each θ_{-1}, we effectively need to analyze just two possibilities: either (1) two constraints $\delta\Pi(H) \geq (1-\delta)b(\theta_{-1}, H)$ and $\delta\Pi(L) \geq (1-\delta)b(\theta_{-1}, L)$ bind at the same time or (2) they both become slack at the same time. This provides a very convenient and powerful tool; the arguments for the proof of the part (M-2) of monotonicity heavily relies on this feature. Moreover, this does not depend on the additive specification assumed for the technology.

4. Main Results

We are now ready to focus on the main contributions of the paper which is to show that this simple yet natural environment has numerous implications that match well several empirical findings.

4.1. **Morale Effect.** Our first main result shows that the dynamic enforcement constraint in the low state becomes more binding. Hence, it depresses the expected bonus payment that can be promised, which in turn limits the amount of effort that can be demanded from the agent and thus reduces the surplus from the relationship.

Proposition 3 (Morale Effect)**.** There exists a $\tilde{\delta} \in (0,1)$ such that for all $0 < \delta < \tilde{\delta}$,

$$\text{(i)}\quad \mathbb{E}(b^*(L,\theta)\,|\theta_{-1} = L) < \mathbb{E}(b^*(H,\theta)\,|\theta_{-1} = H),$$
$$\text{(ii)}\quad a^*(L) < a^*(H).$$

There is an intuitive explanation for this result that highlights the effect of persistent productivity during bad times. When the current state is bad, the future value of the relationship is low and the principal can no longer credibly promise to make large payments. Thus, it can no longer motivate the agent to exert very high effort.

Our model thus rationalizes the "low morale effect" during firm distress, once the profitability is interpreted as a driver of financial distress. In our model, firms reduce the expected bonus payments during difficult times, to which workers respond by exerting less effort. Thus, firms' productivity is further depressed due to de-moralization of the workforce. This mechanism resonates well with multiple labor concession episodes in the domestic steel industry during 1980s (DeAngelo and DeAngelo, 1991).

The paper by Barron et al. (2018) has a similar mechanism at play. Although the shocks in their model are transient, their firms have different levels of leverage which generates persistence. An increase in leverage in the previous period in their model is similar to having experienced a negative persistent shock in our model and leads to a lower labor productivity. They then use data on a large sample of European firms to show that indeed, that in increase in leverage in the previous year leads to a decrease in year t productivity. In terms of magnitude, they report that a one standard deviation increase in contemporary leverage is correlated with a decrease in TFP-R equal to 9-15% of the median within firm standard deviation.

DeAngelo and DeAngelo (1991) illustrates this effect with the labor problems experienced by the US steel plants in the 1980s. This was a particularly challenging time for US steel producers as they faced stiff competition from more productive Japanese counterparts coupled with a decline in global demand. Mas (2008) uses re-sale data for industrial machinery to show how Caterpillar's problems with its workers had a negative and economically significant impact on the quality of the machinery it produced.

Furthermore, we can recast our model as in Fuchs (2015) to rationalize why distressed firms are more likely to lose, or less likely to attract workers. This is consistent with the empirical findings in Baghai et al. (2016), and Brown and Matsa (2016). When a negative productivity shock hits the firm, workers with high outside values would leave the company, because firms cannot promise sufficiently high bonuses to retain them.

4.2. **Rewarding Luck.** The next Proposition shows that when determining the optimal structure of bonus payments *an observable, contemporaneous* shock would not be filtered out, even when the realization of the shock contains no information about possible deviations by the agent in his choice of effort. Hence, it would appear as if employees are partially rewarded for luck.

Proposition 4 (Rewarding Luck)**.** There exists a $\bar{\delta} \in (0,1)$ such that for all $\delta < \bar{\delta}$ *and* $\theta_{-1} \in \Theta$, $b(\theta_{-1}, L) < b(\theta_{-1}, H)$.

Importantly, even though the size of the bonus varies with the persistent state θ, an element outside the agent's control or influence and with no information content, incentives are only really being provided by the expected bonus. Additionally, the bonus is still only paid if the agent delivers on his part of the output. Thus, although luck determines the size of the reward these rewards are still fully driven by incentive motives.

Our model thus provides a transparent intuition why the discretionary bonuses could appear to exhibit "reward for luck": the principal is less tempted to break the implicit promise to pay out bonuses when the ongoing relationship is more valuable ($\theta_t = H$). Since the lack of commitment limits the size of bonuses the firm can pay resulting in underprovision of effort, the principal wishes to promise the largest possible bonuses whenever credible. Given that, the principal can credibly promise to pay out larger amounts when the current shock is revealed to be favorable. This is the case because shocks are persistent. After a good realization the future prospects of the firm are better, hence the enforceability constraint of the firm is relaxed and thus the firm can credibly promise to pay more in such a state.

4.3. **Shock Amplification and Implications for TFP.** We have focused our analysis and discussion on a single firm but our analysis at the micro level has implications from an industry wide and macroeconomic perspective. If we assume that the theta shocks are correlated across firms our model generates an amplification effect of productivity shocks when there is poor contractual enforceability. This is consistent with the empirical findings that show that emerging economies have much higher output volatility than developed economies.[8] Naturally there could be many explanations for this difference and indeed there is a rich literature providing a number of different rationales for these facts.

Our paper contributes to this literature by arguing that the larger reliance on self-enforcing contracts by firms in emerging economies might provide an amplifier effect on exogenous productivity shocks leading to larger business cycle volatility. It is well documented that emerging economies tend to have poorer legal enforcement and a larger underground economy (Porta et al., 1998; Allen et al., 2005). As a result, a larger fraction of firms in emerging markets need to rely on relational contracts relative to fully enforceable ones. Therefore, the amplification effect generated by the enforceability constraints would lead to larger volatility of labor productivity and output in emerging economies vis a vis developed ones.

Proposition 5 (Amplification)**.** For given shocks $\{L, H\}$ firms that rely on relational contracts have a higher dispersion of expected output $Y(\theta)$ as measured by $\frac{Y(L)}{Y(H)}$ relative to firms that can write enforceable contracts and there exists a $\bar{\delta} \in (0,1)$ such that for all $\delta \in \left(0, \bar{\delta}\right)$ the relationship is strict.

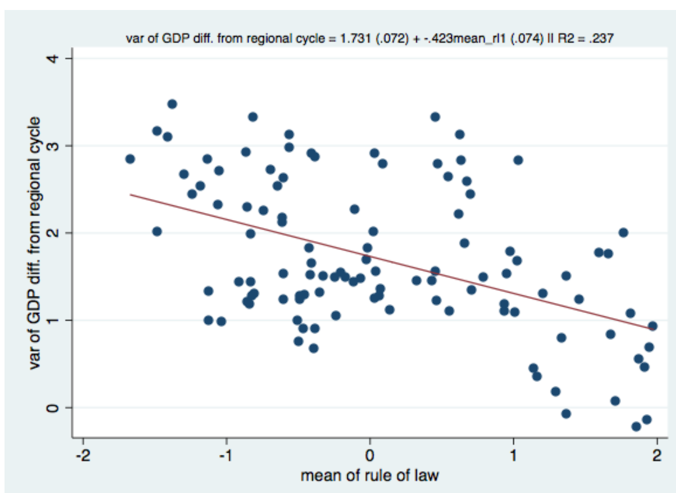
Unfortunately, there are no good direct measures of what fraction of firms rely on formal vs informal contracts. Thus, to look at how this prediction holds in the data we must first find a proxy for what fraction of firms in the economy are likely to be relying on relational contracts. Fortunately, there are a number of well established indicators for how strong the rule of law is in a given country and how well the court system works. Since formal contracts require a functional court system we believe these measures are likely to correlate well with the degree of reliance on

[8]See for example Aguiar and Gopinath (2007)

self-enforcing contracts. We use the world wide governance indicators from the World Bank in the results presented below.

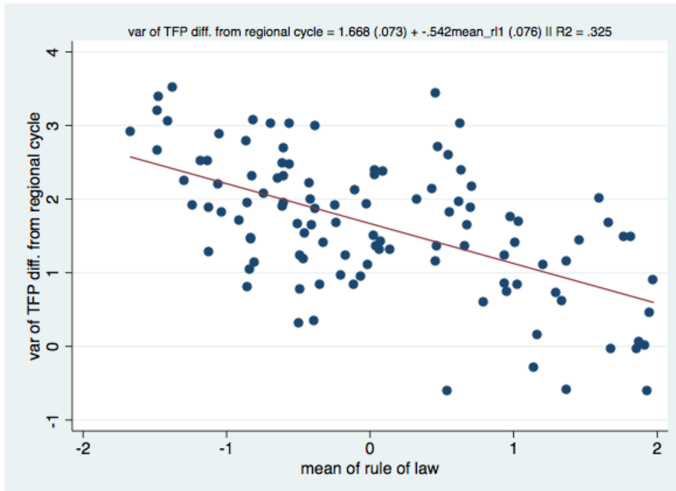
In Figures 2 and 3 below we plot this measure of rule of law against the GDP and TFP volatility over the last 20 years for a large sample of countries. [9] As predicted by theory, the more a country relies on formal contracts (i.e. the better the legal environment) the lower the TFP and GDP volatility it has experienced over the last 20 years. This, of course, is just a correlation and we are conscious that there are many other determinants of the legal environment and TFP and GPD volatility of a country. We partly address this by using regional dummies but we are conscious this might not capture all other sources of variation. We are hopeful that, with better data, future research might be able to establish a clear causal link

Figure 2. Rule of Law and LN GDP Volatility



[9]See the Data Appendix for a detailed description of the data used and its sources.

FIGURE 3. Rule of Law and LN TFP Volatility



Figures 2 3 plot a measure of GDP (TFP) country volatility and the average rule of law from 1996 to 2016. Volatility is measured as the variance (in logs) of the difference between each country's GDP (TFP) and the regional yearly average, using data from Total Economy Database (conference-board.org). Rule of law measures agent's confidence in contract enforceability, property rights, police, courts and the likelihood of crime and violence (using data from the World Bank)

In addition, the same mechanism has implications for the spread of the cross section distribution of TFP. Namely, the amplification effect, implies that firms in countries with poor contract enforcement would have a larger dispersion of TFP at any given point in time as well. The comparison between the findings by Syverson (2004) for the US and Hsieh and Klenow (2009) for India and China are clearly in line with our predictions. While for the US there is a twofold difference in productivity between the 10th and 90th percentile firm, in China and India there is a fivefold difference.

Figures 4 5 plot the productivity spread between 10th and 90th percentile firm within a country and rule of law measure using firm-level TFP data from 65 countries from the Enterprise Survey and 13 countries in the OECD. Unfortunately, there is no systematic study following the same methodology for a large set of firms in a large set of countries. We are currently trying to expand on this comparison by looking at a larger set of countries.

Figure 4. Rule of Law and TFP Dispersion (Enterprise Surveys & OECD)

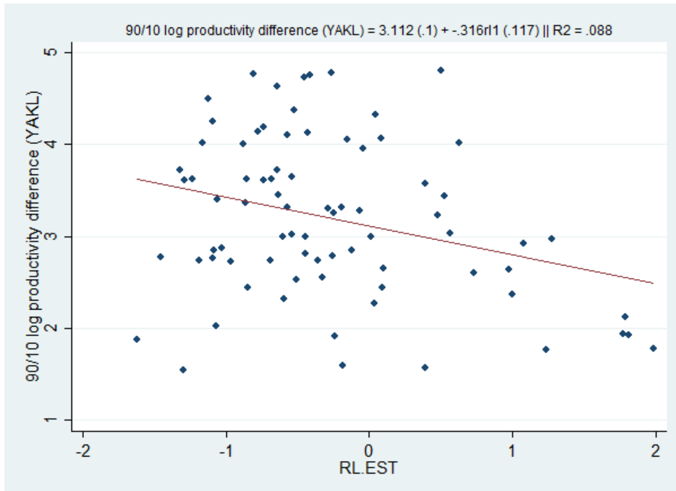
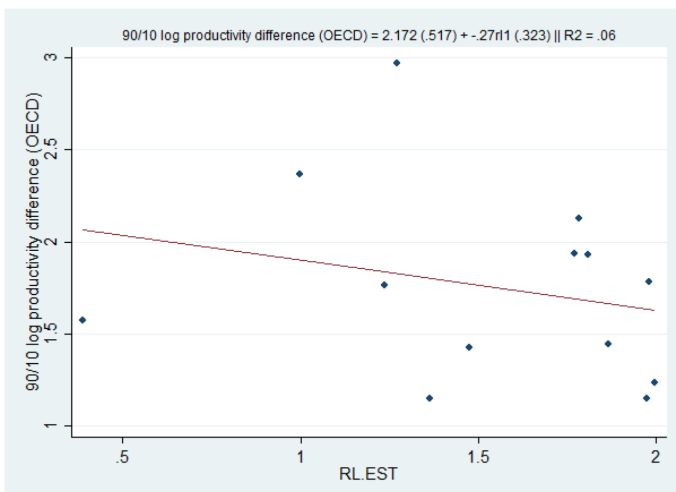


Figure 5. Rule of Law and TFP Dispersion (OECD only)



5. Conclusion and Suggestions for Future Research

[To be included]

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Data Appendix

A. Rule of Law Measure

This measure directly comes from the Worldwide Governance Indicators (WGI) project website. The sample period spans from 1996-2016, and covers around 216 countries.

B. Country-Level GDP / TFP Growth Rate Volatility

We use the adjusted version of the database Growth Accounting and Total Factor Productivity, 1950-2016 from the Conference Board (TCB) Total Economy Database website. The number of sample countries is 123. The original data calculates growth rates as log changes, after deflating levels using price deflators. We obtain deviations from regional yearly average using the following procedure. There are eight different regions according to TCB: Africa; Asia; Central and Eastern Europe, and Central Asia; Latin America; Middle East; North America; Oceania; Western Europe. We then calculate the regional average by taking simple average of each rate every year. Finally, we obtain deviations by subtracting the regional averages from the original variable.

We run the following regressions to obtain figures in the main text:

$$LNVOLDEV_i = \alpha + \beta AVGRL_i + \epsilon_i$$

where $LNVOLDEV_i$ denotes the logarithm of variance of deviations from the regional average for a country i, and $AVGRL$ denote the average rule of law of the country i. The result is qualitatively similar without logging the variance, and using the raw data instead of the deviation from the regional average.

C. Within Country-Level TFP Dispersion (In Progress)

We estimate the country's productivity dispersion as the difference between the firm-level log-TFP at the top 10 percentile and log-TFP at the top 90 percentile within a country. Here, the Enterprise Surveys (ES) estimate the TFP using the VAKL methodology for 65 countries (World Bank Group, 2017) while the OECD measures "multifactor productivity" for 13 countries (Berlingieri et al., 2017). Unfortunately, the sample countries in ES consist mostly of developing countries, some of whose firm-level data are not always reliable. Moreover, ES's and OECD's TFP estimation methodologies are slightly different. To address such concerns, we plan to use firm-level TFP estimates for OECD countries in the future.

Appendix for Proofs

Proof of Proposition 1. Suppose that the optimal dynamic contract generates surplus $s^*(\theta)$ starting from the initial state θ and that it specifies after the public history h^{t-1} and state θ_{t-1} the fixed wage payments $w_t(h^{t-1}, \theta_{t-1})$ and contingent payments $b_t(h^{t-1}, \theta_{t-1}, y_t, \theta_t)$ which induce from Agent effort $a_t(h^{t-1}, \theta_{t-1})$ and yields him continuation expected value $v_t(h^{t-1}, \theta_{t-1}, y_t, \theta_t)$.

Fix θ and let $v^*(\theta) \in [\overline{u}, s^*(\theta) - \overline{\pi}]$ be Agent's value from the optimal contract. We now construct from the optimal contract a Markovian relational contract that transfers any variation in the continuation value into the bonus payments that implements $e_t(h^{t-1}, \theta_{t-1})$ and yields Agent value $v^*(\theta_{t-1})$.

Define the history-dependent bonus payments:

$$\widehat{b}_t(h^{t-1}, \theta_{t-1}, y_t, \theta_t) = b_t(h^{t-1}, \theta_{t-1}, y_t, \theta_t) + \frac{\delta}{1-\delta}\Big(v_t(h^{t-1}, \theta_{t-1}, y_t, \theta_t) - v^*(\theta_{t-1})\Big)$$

and the fixed payments:

$$\widehat{w}_t(h^{t-1}, \theta_t) = v^*(\theta) + c\Big(a_t(h^{t-1}, \theta_{t-1})\Big) - \mathbb{E}_{y_t, \theta_t}\Big[\widehat{b}_t(h^{t-1}, \theta_{t-1}, y_t, \theta_t) | a_t(h^{t-1}, \theta_{t-1}), \theta_{t-1}\Big]$$

Notice that the contract $\Big(\widehat{w}_t(h^{t-1}, \theta_t), \widehat{b}_t(h^{t-1}, \theta_t, y_t, \theta_{t+1})\Big)$ induces the same effort level $a_t(h^{t-1}, \theta_{t-1})$ as in the optimal contract and yields Agent his outside value $v^*(\theta_{t-1})$.

Now define the Markovian bonus payments $b^{*\prime}(\theta_{t-1}, y_t, \theta_t)$ as the bonus payments $\widehat{b}$ that maximize the expected profit, which is equal to the expected per-period joint surplus, for each state:

$$b^{*\prime}(\theta_{-1}, y, \theta) \quad = \quad \underset{\widehat{b}_t(h^{t-1}, \theta_{-1}, \cdot;\cdot), h^{t-1}, t}{\text{argmax}} \quad \mathbb{E}_\theta[\theta | \theta_{-1}] + \underbrace{a_t(h^{t-1}, \theta_{-1})}_{= \mathbb{E}_y\left[y \middle| a_t(h^{t-1}, \theta_{-1})\right]} - c\Big(a_t(h^{t-1}, \theta_{-1})\Big)$$

$$s.t. \quad a_t(h^{t-1}, \theta_{-1}) = \underset{a, \theta'}{\text{argmax}}\, \mathbb{E}_{y, \theta'}\Big[\widehat{b}_t(h^{t-1}, \theta, y, \theta') | a, \theta_{-1}\Big] - c(a)$$

Let $a'(\theta)$ be the solution

$$a'(\theta_{-1}) = \underset{a}{\text{argmax}}\, \mathbb{E}_{y, \theta}\left[b^{*\prime}(\theta_{-1}, y, \theta) | a, \theta_{-1}\right] - c(a)$$

and define the state-dependent wages:

$$w^{*\prime}(\theta_{-1}) = v^*(\theta_{-1}) + c(a'(\theta_{-1})) - \mathbb{E}_{y, \theta'}\left[b^{*\prime}(\theta_{-1}, y, \theta) | a'(\theta_{-1}), \theta_{-1}\right] \qquad (2)$$

By construction the Markovian contract $(w^{*\prime}, b^{*\prime})$ is self-enforcing, yields to Agent value $v^*(\theta_{t-1})$ and to Principal $s^*(\theta_{t-1}) - v^*(\theta_{t-1})$. Finally, because this Markovian contract repeats in each following period, the continuation contract is self-enforcing. Thus the constructed contract is self-enforcing and gives a per-period expected surplus $s^*(\theta_{t-1})$.

Notice that taking $v^* \in [\overline{u}, \min\{s^*(H) - \overline{\pi}, s^*(L) - \overline{\pi}\}]$, a state-independent value for Agent, and setting ${w^*}'(\theta_{-1}) = v^* - \mathbb{E}_{y,\theta}\left[{b^*}'(\theta_{-1}, y, \theta)|a'(\theta_{-1}), \theta_{-1}\right] + c(\theta_{-1})$ for all θ_{-1} redistributes the surplus from the optimal Markovian relational contract and establishes that the property (R-1) holds in any optimal Markovian contract.

Now, to establish the property (R-2) we start by noticing that by (R-1) the optimal Markovian relational contract reduces to solving the following equations recursively (we illustrate for the lagged state is H and the arguments follow analogously for the state L):

$$\Pi(H) = \max_{\{b_H(1,\theta), b_H(0,\theta)\}, a} (1-\delta)\Bigg(\mathbb{E}_\theta[\theta|\theta_{-1} = H] + a(1 - \lambda b_H(1,H) - (1-\lambda) b_H(1,L)) \\ - (1-a)(\lambda b_H(0,H) + (1-\lambda) b_H(0,L)) - w(H)\Bigg) \\ + \delta[\lambda \Pi(\theta_H) + (1-\lambda)\Pi(\theta_L)]$$

$$\text{[IC]} \quad c'(a) = \lambda(b_H(1,H) - b_H(0,H)) + (1-\lambda)(b_H(1,L) - b_H(0,L));$$

$$\text{[PC's]} \quad \Pi(H) \geq \overline{\pi}, \quad v^* \geq \overline{u};$$

$$\text{[DEPH]} \quad b_H(1,H) \leq \frac{\delta}{1-\delta}\Pi(H);$$

$$\text{[DEPL]} \quad b_H(1,L) \leq \frac{\delta}{1-\delta}\Pi(L);$$

$$\text{[DEAH]} \quad -b_H(0,H) \leq \frac{\delta}{1-\delta}v^* - \overline{u}.$$

$$\text{[DEAL]} \quad -b_H(0,L) \leq \frac{\delta}{1-\delta}v^* - \overline{u}.$$

Notice that by (R-1) without loss we set $u^*(\theta) = \overline{u}$ and hence ensuring the participation constraints can safely be dropped and that since $\overline{u} = 0$ the dynamic enforcement constraints [DEPH] and [DEPL] reduce to respectively, $b_H(0,H) \geq 0$ and $b_H(0,L) \geq 0$. Furthermore, using 2 we have $w_H = \overline{u} + c(a) - a(\lambda b_H(1,H) + (1-\lambda) b_H(1,L)) - (1-a)(\lambda b_H(0,H) + (1-\lambda) b_H(0,L))$ and the principal's contracting problem further simplifies to solving:

$$\Pi(H) = \max_{\{b_H(1,\theta'), b_H(0,\theta')\}_{\theta'}, a} (1-\delta)\left[\mathbb{E}_{\theta'}[\theta'|\theta_H] + a - c(a) - \overline{u}\right] + \delta\left[\lambda\Pi(\theta_H) + (1-\lambda)\Pi(\theta_L)\right]$$

s.t.

$$\text{[IC]} \quad c'(a) = \lambda(b_H(1,H) - b_H(0,H)) + (1-\lambda)(b_H(1,L) - b_H(0,L));$$

[DEPH] $b_H(1,H) \leq \dfrac{\delta}{1-\delta}\Pi(H);$

[DEPL] $b_H(1,L) \leq \dfrac{\delta}{1-\delta}\Pi(L);$

[DEA] $b_H(0,H) \geq 0$ and $b_H(0,L) \geq 0$

Notice that only the spread in bonus payments $b_H(1,\theta) - b_H(0,\theta)$ matter for incentives. The constraint set in contracting problem therefore further simplifies to:

[IC] $c'(a) = \lambda(b_H(1,H) - b_H(0,H)) + (1-\lambda)(b_H(1,L) - b_H(0,L));$

[DEH] $b_H(1,H) - b_H(0,H) \leq \dfrac{\delta}{1-\delta}\Pi(H).$

[DEL] $b_H(1,L) - b_H(0,L) \leq \dfrac{\delta}{1-\delta}\Pi(L).$

Inspecting [IC] and [DE]'s we see that without loss Principal uses contracts that do not involve paying a bonus following low output: $b_H(0,\theta) = 0$, and $b_L(0,\theta) = 0$ follows from analogous arguments. This establishes (R-2). □

Proof of Lemma 1. Notice from the principal's reduced form of relational contracting problem (1) that the incentive compatibility condition is equivalent to:

[IC'] $a(\theta_{-1}) = g[\mathbb{E}_\theta[b(\theta_{-1},\theta)|\theta_{-1}]]$

where $g := (c')^{-1}$ is the inverse of the marginal effort cost function. Now, using [IC'] we form a Lagrangian based on the dynamic enforceability constraint:

$$\Pi(\theta_{-1}) = \max_{b(\theta_{-1},\theta)} (1-\delta)(g[\mathbb{E}(b(\theta_{-1},\theta)|\theta_{-1})] + \mathbb{E}(\theta|\theta_{-1}) - c(g(\mathbb{E}_\theta[b(\theta,\theta_{-1})|\theta_{-1}])))$$
$$+\delta\mathbb{E}_\theta[\Pi(\theta)|\theta_{-1}] + \mathbb{E}_\theta[L(\theta_{-1},\theta)\big(\delta\Pi(\theta) - (1-\delta)b(\theta_{-1},\theta)\big)|\theta_{-1}]$$

Notice that $\frac{\partial \mathbb{E}(b(\theta_{-1},\theta)|\theta_{-1})}{\partial b(\theta_{-1},\theta)} = \mathsf{Prob}\,(\theta|\theta_{-1})$ and $g'(x) = \frac{1}{c''(g(x))}$.[10] This implies that the first-order condition with respect to bonus payment $b(\theta_{-1},\theta)$ from the objective function yields:

$$0 = \frac{(1-\delta)\mathsf{Prob}\,(\theta|\theta_{-1})}{c''(g(\mathbb{E}(b(\theta_{-1},\theta)|\theta_{-1})))} - \frac{(1-\delta)\mathsf{Prob}\,(\theta|\theta_{-1})c'(g(\mathbb{E}(b(\theta_{-1},\theta)|\theta_{-1})))}{c''(g(\mathbb{E}(b(\theta_{-1},\theta)|\theta_{-1})))}$$
$$-L(\theta_{-1},\theta)(1-\delta)\mathsf{Prob}\,(\theta|\theta_{-1})$$

where $L(\theta_{-1},\theta) \geq 0$ is the Lagrange multiplier. Because the common factor $(1-\delta)\mathsf{Prob}\,(\theta|\theta_{-1})$ is strictly positive, and $g(\cdot)$ is the inverse function of $c'(\cdot)$ the first-order condition is equivalent to

$$0 = \frac{1}{c''(g(\mathbb{E}(b(\theta_{-1},\theta)|\theta_{-1})))} - \frac{\mathbb{E}(b(\theta_{-1},\theta)|\theta_{-1})}{c''(g(\mathbb{E}(b(\theta_{-1},\theta)|\theta_{-1})))} - L(\theta_{-1},\theta)$$

[10] The fact that $x = c'(g(x))$ follows from the definition of $g(x)$. Taking derivative with respect to x and applying the chain rule we have $1 = c''(g(x))g'(x)$ and hence $g'(x) = \frac{1}{c''(g(x))}$.

(FOC)

Solving for the Lagrange multiplier from the first-order condition (FOC) we have:

$$L(\theta_{-1},\theta) = \frac{(1 - [\mathbb{E}(b(\theta_{-1},\theta)|\theta_{-1}])}{c''(g[\mathbb{E}(b(\theta_{-1},\theta)|\theta_{-1}])} \tag{FOC$'$}$$

This together with the enforceability constraint and complementary slackness conditions form necessary conditions for optimality:

$$L(\theta_{-1},\theta) \geq 0;\ \delta\Pi(\theta) \geq (1-\delta)b(\theta_{-1},\theta) \text{ and}$$

$$L(\theta_{-1},\theta)(\delta\Pi(\theta) - (1-\delta)b(\theta_{-1},\theta)) = 0$$

If the condition $c'(a(\theta_{-1})) < 1$ in the statement of the current lemma holds, then it implies from strict monotonicity of $c'(x)$ that $\mathbb{E}_\theta[b(\theta_{-1},\theta)|\theta_{-1}] < 1$. Thus, since the cost function is strictly convex $c'' > 0$ everywhere, when $c'(a(\theta_{-1})) < 1$, (FOC$'$) implies that the Lagrange multiplier is strictly positive: $L(\theta_{-1},\theta) > 0\ \forall\theta \in \Theta$. Now a strictly positive Lagrange multiplier in the complementary slackness conditions implies that dynamic enforcement constraints in both states bind: $\delta\Pi(H) = (1-\delta)b(\theta_{-1},H)$ and $\delta\Pi(L) = (1-\delta)b(\theta_{-1},L)$. Rearranging the latter we complete the proof. □

Proof of Proposition 2. Define $\gamma := \frac{1-\lambda\delta}{1-\lambda\delta+(1-\lambda)\delta} > \frac{1}{2}$. Observe the following relationship holds under Markovian contracts:

$$\begin{bmatrix}\Pi(H)\\ \Pi(L)\end{bmatrix} = \begin{bmatrix}\gamma & 1-\gamma\\ 1-\gamma & \gamma\end{bmatrix}\begin{bmatrix}a^*(H) - c(a^*(H)) + \mathbb{E}(\theta|\theta_{-1} = H)\\ a^*(L) - c(a^*(L)) + \mathbb{E}(\theta|\theta_{-1} = L)\end{bmatrix}$$

subject to :

$$\text{IC}\quad a^*(\theta_{-1}) = c'[\mathbb{E}(b^*(\theta,\theta_{-1})|\theta_{-1})]$$

$$\text{DE}\quad \delta\Pi^*(\theta) \geq (1-\delta)b^*(\theta_{-1},\theta) \qquad \forall\theta_{-1},\theta \in \Theta$$

To establish (M-1), we start with showing that $\max_{\theta\in\Theta}\{a^*(\theta)\} \leq g(1)$ under the optimal contract. Suppose to the contrary that there exists a state with $a^*(\theta_0) > g(1)$. Now, from the incentive compatibility condition $a(\theta_{-1})^* = g[\mathbb{E}(b^*(\theta,\theta_{-1})|\theta_{-1})]$, strict monotonicity and continuity of c and c', decreasing either both $b^*(\theta_0,\theta)$ by arbitrarily small $\epsilon > 0$ would result in a new effort level $\tilde{a}^*(\theta_0)$, which is still higher than $g(1)$ but strictly less than $a^*(\theta_0)$. Since the function $x - c(x)$ is concave and attains its maximum at $g(1)$, the expected current surplus at state θ_0 $a^*(\theta_0) - c(a^*(\theta_0)) + \mathbb{E}(\theta|\theta_{-1} = \theta_0)$ strictly increases. From the system of equations given above, we see that a strict increase in the expected current surplus at state θ_0 strictly increases both levels of principal's surpluses would $\Pi^*(H), \Pi^*(L)$ as well.

Now we construct a new contract using the bonus payment $\tilde{b}^*(\theta_{-1},\theta) = b^*(\theta_{-1},\theta) + \epsilon \times 1(\theta_{-1} = \theta_0)$. Notice that these bonus payments are higher compared to the original contract, while surpluses have strictly increased in all states. This implies that the dynamic enforceability is also satisfied under the new contract if the original optimal contract does. Yet, this new contract obtains strictly higher joint payoffs in both states compared to initial optimal contract, which contradicts the optimality of the latter. It therefore follows that $\max_{\theta\in\Theta}\{a^*(\theta)\} \leq g(1)$.

Next, we start with showing that $\max_{\theta\in\Theta}\{a^*(\theta)\} \leq g(1)$ under the optimal contract. Suppose to the contrary that there exists a state θ_0 with $a^*(\theta_0) > g(1)$. Now, decrease $b^*(\theta_0,\theta)$ by arbitrarily small $\epsilon > 0$. A new effort level $\tilde{a}^*(\theta_0)$ under $b^*(\theta_0,\theta)-\epsilon$ would result in an effort level higher than $g(1)$ but strictly less than $a^*(\theta_0)$, because the incentive compatibility condition $a^*(\theta_{-1}) = g[\mathbb{E}(b^*(\theta,\theta_{-1})|\theta_{-1})]$ determines the new effort level. Since the function $x - c(x)$ is concave and attains its maximum at $g(1)$, decrease in effort would increase $a^*(\theta_0) - c(a^*(\theta_0))$. From the system of equations given above, a strict increase in $a(\theta_0) - c(a^*(\theta_0))$ would strictly increase both levels of principal's surpluses would $\Pi^*(H), \Pi^*(L)$ as well.

Now we construct a new contract using the bonus payment $\tilde{b}^*(\theta_{-1},\theta) = b^*(\theta_{-1},\theta) - \epsilon \times 1_{\theta_{-1}=\theta_0}$. Notice that these bonus payments never exceed bonus pays under the original contract, while principal's surpluses have strictly increased in all states. Hence, the left hand side of the dynamic enforceability condition is higher under the new contract, while the right hand side is lower. Hence the dynamic enforceability would be also satisfied under the new contract if the original optimal contract does. Yet, this new contract obtains strictly higher joint payoffs in both states compared to initial optimal contract, which contradicts the optimality of the latter. It therefore follows that $\max_{\theta\in\Theta}\{a^*(\theta)\} \leq g(1)$.

Next, we show that $a^*(H) \geq a^*(L)$ under the optimal contract. Suppose to the contrary and $a^*(H) < a^*(L)$ under the optimal contract. Now, as we have observed in the previous part of the current proof that $\max_{\theta\in\Theta}\{a^*(\theta)\} \leq g(1)$ must hold, it must be that $a^*(H) < a^*(L) \leq g(1)$. Denote optimal principal's surpluses by $(\Pi^*(H), \Pi^*(L))$.

From the agent's incentive compatibility condition, expected bonus payments should be equal to the agent's cost of effort. Thus,

$$\mathbb{E}(b(L,\theta)|\theta_{-1} = L) = c'(a^*(L)) > c'(a^*(H)) = \mathbb{E}(b(H,\theta)|\theta_{-1} = H)$$

where the inequality follows from the assumption that $a^*(L) > a^*(H)$ and $c'' > 0$.

Now, construct a new payment scheme $\hat{b}$ as follows:

$$
\begin{aligned}
\hat{b}(H,H) &= b(L,L) - \frac{1-\lambda}{\lambda}\left(\frac{\delta}{1-\delta}\Pi^*(L) - b(L,H)\right) \\
\hat{b}(H,L) &= \frac{\delta}{1-\delta}\Pi^*(L) \\
\hat{b}(L,\theta) &= b(L,\theta) \quad \text{ for all } \theta \in \Theta
\end{aligned}
$$

It is routine to verify that $\mathbb{E}(\hat{b}(\theta,\theta_{-1})|\theta_{-1}) = \mathbb{E}(b(L,\theta)|\theta_{-1} = L)$ for both $\theta_{-1} \in \Theta$. Denote the principal's surplus from this new contract by $(\hat{\Pi}^*(H), \hat{\Pi}^*(L))$. Recall that $x - c(x)$ is concave and attains maximum at $g(1)$. Since the recommended effort in the high state $a(H)$ is strictly less than $g(1)$, the new contract increases the effort level $a(H)$ compared to the initial contract, due to the incentive compatibility condition. Hence, arguing similarly as in the first part of the proof, both surpluses would strictly increase under new payment $\hat{b}$.

Summarizing the observations thus far, we have the following relationships hold:

$$
\hat{\Pi}^*(H) > \hat{\Pi}^*(L) > \Pi^*(L), \hat{\Pi}^*(H) > \Pi^*(H)
$$

where the first inequality follows from $\mathbb{E}(\theta|\theta_{-1} = H) > \mathbb{E}(\theta|\theta_{-1} = L)$ and that principal's surpluses in all states would strictly increase under the new payment. The other inequalities merely state that the principal's surpluses increased. Now, it remains to verify that dynamic enforceability constraints are satisfied under the new payment system. It is routine to check:

$$
\begin{aligned}
\delta\hat{\Pi}^*(H) &> \delta\Pi^*(H) \geq (1-\delta)b(L,H) = (1-\delta)\hat{b}(L,H) \\
\delta\hat{\Pi}^*(L) &> \delta\Pi^*(L) \geq (1-\delta)b(L,L) = (1-\delta)\hat{b}(L,L) \\
\delta\hat{\Pi}^*(L) &> \delta\Pi^*(L) = (1-\delta)\hat{b}(H,L)
\end{aligned}
$$

which follow from the definition of $\hat{b}$, the system of equations at the beginning of the proof, or dynamic enforceability constraints of the initial optimal contract. Now,

$$
\begin{aligned}
\delta\hat{\Pi}^*(H) &> (2 - \frac{1}{\lambda})\delta\Pi^*(L) + (\frac{1}{\lambda} - 1)\delta\Pi^*(H) \\
&\geq (2 - \frac{1}{\lambda})\delta\Pi^*(L) + (\frac{1}{\lambda} - 1)(1-\delta)b(L,H) \\
&= \delta\Pi^*(L) + (1 - \frac{1}{\lambda})\delta\Pi^*(L) + (\frac{1}{\lambda} - 1)(1-\delta)b(L,H) \\
&\geq (1-\delta)b(L,L) + (1 - \frac{1}{\lambda})\delta\Pi^*(L) + (\frac{1}{\lambda} - 1)(1-\delta)b(L,H) \\
&= (1-\delta)\hat{b}(H,H)
\end{aligned}
$$

where the first inequality follows from the observation $\hat{\Pi}^*(H) > \max\{\Pi^*(L), \Pi^*(H)\}$ and $2 > \frac{1}{\lambda}$ (which is equivalent to $\lambda > \frac{1}{2}$), the second and the third inequality from dynamic enforceability constraints of the original contract. Finally, the last equality follows from the definition of $\hat{b}(\cdot)$. As in the first part of the proof to establish $\max_{\theta \in \Theta}\{a^*(\theta)\} \leq g(1)$, the existence of this new payment scheme defies the optimality of the originally proposed contract. These two contradictions establish the part (M-1).

Applying the result (M-1), $a^*(H) - c(a^*(H)) + \mathbb{E}(\theta|\theta_{-1} = H) > a^*(L) - c(a^*(L)) + \mathbb{E}(\theta|\theta_{-1} = L)$. From the system of equations, this implies that the joint payoff is strictly higher under the favorable state and hence it establishes (M-2). □

Proof of Proposition 3. Let $\bar{\delta} = \frac{1}{1+\Pi(H)^{FB}}$ so that the following relationship holds:

$$\frac{\bar{\delta}}{1-\bar{\delta}}\Pi(H)^{FB} = 1$$

Choose any $\delta < \tilde{\delta}$. Using incentive compatibility condition,

$$c'(a^*(H)) = \mathbb{E}(b^*(H, \theta)|\theta_{-1} = H) \leq \frac{\delta}{1-\delta}\Pi(H)^{FB} < 1$$

where the second inequality holds from dynamic enforceability condition, and the last strict inequality from $\delta < \tilde{\delta}$.

Now, due to weak monotonicity in the effort level and the observation above, it must be that both states' effort levels fall strictly below the first best level $(c')^{-1}(1)$. This implies that $b^*(\theta_{-1}, \theta) = \frac{\delta}{1-\delta}\Pi(\theta)\ \forall\ \theta_{-1}, \theta \in \Theta$ due to Lemma 1. Using the incentive compatibility once more,

$$\begin{aligned} c'(a^*(H)) = \mathbb{E}(b^*(H, \theta)|H) &= \frac{\delta}{1-\delta}\mathbb{E}(\Pi(\theta)|H) \\ &> \frac{\delta}{1-\delta}\mathbb{E}(\Pi(\theta)|L) = \mathbb{E}(b^*(L, \theta)|L) = c'(a^*(L)) \end{aligned}$$

where the strict inequality in the middle follows from the strict monotonicity in payoffs, establishing the part (i). Finally, the strict monotonicity in c' would imply that $a^*(H) > a^*(L)$, establishing the part (ii). □

Proof of Proposition 4. The existence of such $\bar{\delta} = \tilde{\delta}$ follows from analogous arguments as Proposition 3. Now applying the Lemma 1 yields:

$$b(\theta_{-1}, L) = \frac{\delta}{1-\delta}\Pi(L) < \frac{\delta}{1-\delta}\Pi(H) = b(\theta_{-1}, H)$$

□

Proof of Proposition 5. Let $\bar{L} = E[\theta | \theta_{-1} = L]$ and $\bar{H} = E[\theta | \theta_{-1} = H]$, then the expected output with limited enforcement can be written as: $Y^*(L) = \bar{L} + a^*(L)$ and $Y^*(H) = \bar{H} + a^*(H)$ where $a^*()$ denotes the equilibrium effort level in each state. Instead, the first best expected outputs are given by $Y^{fb}(L) = \bar{L} + a^{fb}(L)$ and $Y^{fb}(H) = \bar{H} + a^{fb}(H)$.

$$\frac{Y^*(L)}{Y^*(H)} = \frac{\bar{L} + a^*(L)}{\bar{H} + a^*(H)} \leq \frac{\bar{L} + a^*(H)}{\bar{H} + a^*(H)} \leq \frac{\bar{L} + a^{fb}(L)}{\bar{H} + a^{fb}(H)} = \frac{Y^{fb}(L)}{Y^{fb}(H)}$$

where the inequalities follow from Proposition 2 and as shown there are strict for low δ.

□